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Models and Algorithm for Stochastic Network Designs*

Anthony Chen[1,**], Juyoung Kim[2], Seungjae Lee[3], Jaisung Choi[3]

1. Department of Civil and Environmental Engineering, Utah State University, Logan, Utah 84322-4110, USA;
2. Center for National Transport Database, The Korea Transport Institute, 2311 DaehwaDong, Ilsan-Gu, Goyang City, Korea;
3. Department of Transportation Engineering, University of Seoul, Dongdaemoon-Ku, Seoul, Korea

Abstract: The network design problem (NDP) is one of the most difficult and challenging problems in transportation. Traditional NDP models are often posed as a deterministic bilevel program assuming that all relevant inputs are known with certainty. This paper presents three stochastic models for designing transportation networks with demand uncertainty. These three stochastic NDP models were formulated as the expected value model, chance-constrained model, and dependent-chance model in a bilevel programming framework using different criteria to hedge against demand uncertainty. Solution procedures based on the traffic assignment algorithm, genetic algorithm, and Monte-Carlo simulations were developed to solve these stochastic NDP models. The nonlinear and nonconvex nature of the bilevel program was handled by the genetic algorithm and traffic assignment algorithm, whereas the stochastic nature was addressed through simulations. Numerical experiments were conducted to evaluate the applicability of the stochastic NDP models and the solution procedure. Results from the three experiments show that the solution procedures are quite robust to different parameter settings.

Key words: user equilibrium; traffic assignment; network design; bilevel program; stochastic program

Introduction

The network design problem (NDP) is one of optimizing the improvement of a transportation network with respect to a system-wide objective while considering the route choice behavior of network users[1]. It has been extensively studied by engineers, mathematicians, operations research analysts, and planners. It is considered as one of the most difficult and challenging problems in the transportation field (see Boyce[2], Magnanti and Wong[3], Friesz[4], and Yang and Bell[5] for a review of the modeling, algorithm development, and applications on this topic). The NDPs can be categorized into three types based on the design variables.

(1) Discrete NDPs: the addition of new links[6,7], selection of one-way and two-way streets[8], and design of links and facility locations[9];

(2) Continuous NDPs: capacity enhancement[10-14], road pricing[15-17], signal timing[18-23], and ramp metering[24];

(3) Mixed NDPs: combined land-use-network design problems[25], simultaneous location-network design problems[26], and cordon-based network congestion pricing in discrete and continuum networks[27,28].

Most of the NDP models in the literature have been posed as deterministic problems where all the relevant inputs are assumed to be known with certainty. For example, travel demands are often assumed to be known exactly in the future, but there is no guarantee

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** To whom correspondence should be addressed.
E-mail: achen@engineering.usu.edu; Tel: 1-435-797-7109

that the travel demand forecast would precisely materialize due to uncertainties. Because travel demand forecasts are affected by many factors such as economic growth, land-use patterns, and socioeconomic characteristics, all these factors cannot be measured accurately, but can only be roughly estimated. Evaluation of network performance without accounting demand uncertainty can potentially lead to biased investment decisions[29]. To account for demand uncertainty, a few recent studies have extended the NDP to consider the uncertainty regarding future travel demands by defining a number of possible future scenarios. These include optimizing the expected performance of the system[30,31], optimizing the mean-variance performance of the system[32], maximizing the probability of achieving a predefined threshold of the system performance[33-35], and optimizing the α-quantile of the system performance[36]. This paper gives three stochastic models for the network design problem with demand uncertainty. These three stochastic NDP models are an expected value model, chance-constrained model, and dependent-chance model in a bilevel programming framework using different criteria to address the demand uncertainty. These stochastic models can be considered as a subset of uncertain programming[37] which has been developed for a variety of applications, including topological optimization[38], capacitated location-allocation[39], redundancy optimization[40], project scheduling[41], and path finding[42,43]. This paper focuses on capacity enhancement by adopting different criteria to develop three stochastic models and the development of a simulation-based genetic algorithm for solving the stochastic NDP models.

1 Notations and Model Formulations

This section describes the stochastic network design problem for optimal capacity enhancement with demand uncertainty.

1.1 Notations

A: set of links in the network

$\overline{A}$: set of capacity enhancement links in the network

W: set of origin-destination (O-D) pairs

R_w: set of paths between O-D pair, $w \in W$

f_r^w: flow on path $r \in R_w$ between O-D pair, $w \in W$

$\boldsymbol{f}$: vector of path flows $\boldsymbol{f} = (\dots, f_r^w, \dots)^{\mathrm{T}}$ in the lower-level subproblem

v_a: link flow on link, $a \in A$

$\boldsymbol{v}$: vector of link flows $\boldsymbol{v} = (\dots, v_a, \dots)^{\mathrm{T}}$ in the lower-level subproblem

u_a: capacity enhancement of link, $a \in \overline{A}$

$u_a^{\max}$: upper bound of capacity enhancement on link, $a \in \overline{A}$

$\boldsymbol{u}$: vector of link capacity enhancements $\boldsymbol{u} = (\dots, u_a, \dots)^{\mathrm{T}}$ in the upper-level subproblem

$t_a(v_a, u_a)$: travel time on link $a \in A$, which is a function of link flow v_a and link capacity enhancement u_a

c_r^w: travel time on path $r \in R_w$ between O-D pair, $w \in W$

π_w: minimum travel time between O-D pair, $w \in W$

Q_w: random demand between O-D pair, $w \in W$

q_w: realization of Q_w

$\boldsymbol{Q}$: vector of random variables Q_w

$\boldsymbol{q}$: vector of realization q_w

$g_a(u_a)$: capacity expanding cost for link, $a \in \overline{A}$

δ_{ar}^w: 1 if path r of O-D pair w uses link a, and 0 otherwise

B: a fixed improvement budget

α: confidence level in the chance-constrained model

TTTB: total travel time budget in the chance-constrained model

TTTR: total travel time requirement in the dependent chance model

1.2 Stochastic bilevel mathematical program

The NDP is generally formulated as a bi-level optimization problem to reflect the different aims of the two decision makers who are the network users and the planner. The network users are free to choose their routes such that their individual travel costs are minimized, whereas the planner aims to make the best use of limited resources to optimize network performance (e.g., reducing congestion, minimizing environmental impact, and maximizing throughputs), taking into account users' route choice behavior. The general stochastic bilevel mathematical program can be formulated as follows:

$$\text{(UP)}\quad \underset{u}{\text{Minimize}}\ F(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{\varepsilon})) \tag{1}$$

$$\text{subject to}\quad G(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{\varepsilon}))\leqslant 0 \tag{2}$$

where $\boldsymbol{v}(\boldsymbol{u}, \boldsymbol{\varepsilon})$ is implicitly defined by

$$\text{(LP)}\quad \underset{v}{\text{Minimize}}\ f(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{\varepsilon})) \tag{3}$$

$$\text{subject to}\quad g(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{\varepsilon}))\leqslant 0 \tag{4}$$

where F is the objective function and $\boldsymbol{u}$ is the design vector of the upper-level subprogram (UP), G is the constraint set of UP, f is the objective function and $\boldsymbol{v}(\boldsymbol{u}, \boldsymbol{\varepsilon})$ is the decision variable vector of the lower-level subprogram (LP) as a function of the design vector $\boldsymbol{u}$ as well as a random vector $\boldsymbol{\varepsilon}$, and g is the LP constraint set. The upper-level subprogram describes the leader or planner problem, and the lower-level subprogram represents the follower or user's behavioral problem.

This paper considers the continuous NDP, where link capacity enhancements are treated as continuous design variables $\boldsymbol{u}$. In the capacity enhancement NDP, the upper-level subprogram determines the optimal capacity enhancements $\boldsymbol{u}$ in a transportation network by optimizing a system-wide objective with demand uncertainty $\boldsymbol{Q}$, while the lower-level subprogram determines the route choice behavior of network users for a given capacity enhancement with demand uncertainty $\boldsymbol{v}(\boldsymbol{u}, \boldsymbol{Q})$. The system-wide objective function is to minimize the total travel time (i.e., reducing congestion) defined as

$$F(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{Q}))=\sum_{a\in A}t_a(u_a,v_a(\boldsymbol{u},\boldsymbol{Q}))\ v_a(\boldsymbol{u},\boldsymbol{Q}) \tag{5}$$

where $t_a(u_a,v_a(\boldsymbol{u},\boldsymbol{Q}))$ and $v_a(\boldsymbol{u},\boldsymbol{Q})$ are the travel time and the flow on link a for the design vector $\boldsymbol{u}$ and the random demand vector $\boldsymbol{Q}$. Hence, the performance measure is a random variable.

Let $\boldsymbol{Q}=(\ldots,Q_w,\ldots)^{\mathrm{T}}$ be the random demand vector defined on the probability space $(\Omega,\Theta,\mathrm{Pr})$ where Ω is a set of all outcomes of a random experiment (a non-empty set), Θ is called a σ-algebra, and Pr is referred to as a probability measure. For each $\boldsymbol{\omega}\in\Omega$, $\boldsymbol{q}=\boldsymbol{Q}(\boldsymbol{\omega})$ is a realization of the random demand vector $\boldsymbol{Q}$. In the three stochastic models to be described later, the lower-level subprogram is modeled as a standard user equilibrium traffic assignment problem[44]. For a given design vector determined by the upper-level subprogram $\boldsymbol{u}$ and for each realization of the random demand vector $\boldsymbol{q}$, the lower-level subprogram solves the following traffic assignment problem:

$$\min_{v}\sum_{a\in A}\int_0^{v_a}t_a(x,u_a)\mathrm{d}x \tag{6}$$

$$\text{subject to}\quad \sum_{r\in R_w}f_r^w=q_w,\quad \forall w\in W \tag{7}$$

$$v_a=\sum_{w\in W}\sum_{r\in R_w}f_r^w\delta_{ar}^w,\quad \forall a\in A \tag{8}$$

$$f_r^w\geqslant 0,\ \forall r\in R_w,\quad w\in W \tag{9}$$

where Eq. (6) is the objective function for the user-equilibrium (UE) traffic assignment problem (i.e., the sum of the integrals of the link cost function), Eq. (7) is the flow conservation constraint, Eq. (8) represents the link-path flow relationship, and Eq. (9) ensures the non-negativity of the path flows. The optimal solution $\boldsymbol{f}^*=(\ldots,f_r^w,\ldots)^{\mathrm{T}}$ to the problem satisfies the following user equilibrium conditions:

$$c_r^w(\boldsymbol{f}^*)-\pi_w(\boldsymbol{f}^*)\begin{cases}=0, & \text{if } f_r^w>0,\\ \geqslant 0, & \text{if } f_r^w=0,\end{cases}\quad \forall r\in R_w, w\in W \tag{10}$$

where $c_r^w(\boldsymbol{f}^*)=\sum_a t_a(v_a)\delta_{ar}^w$ is the travel time on path $r\in R_w$ between the O-D pair, $w\in W$, and $\pi_w(\boldsymbol{f}^*)=\min\{c_r^w(\boldsymbol{f}^*),\forall r\in R_w\}$ is the minimum travel time between the O-D pair, $w\in W$. When the travel time on path r is larger than or equal to the minimum travel time, the flow on that path is zero or the path is not used. When the travel time on path r is equal to the minimum, its flow is greater than zero or the path is used. For simplicity, this widely used UE model is used as the lower-level subprogram to model users' route choice behavior in this paper. The stochastic bi-level mathematical program framework can also accommodate other route choice models (e.g., stochastic user equilibrium (SUE), extended logit-based SUE model, generalized user equilibrium models, reliability-based user equilibrium models, etc.).

1.3 Expected value model

The expected value model (EVM) is perhaps the most commonly used method for handling demand uncertainty in the network design problem[30,31]. The main idea is to optimize the expected value of a linear (or additive) system-wide objective function subject to the budget constraint and the limit constraints on the decision variables.

$$\min_{u}\quad \mathrm{E}[F(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{Q}))] \tag{11}$$

$$\text{subject to} \quad \sum_{a \in \bar{A}} g_a(u_a) \leqslant B \tag{12}$$

$$0 \leqslant u_a \leqslant u_a^{\max}, \quad \forall a \in \bar{A} \tag{13}$$

In the expected value network design model, the objective function in Eq. (11) is to minimize the expected total travel time in the networks, Eq. (12) ensures the links selected for capacity enhancements do not exceed the available budget, and Eq. (13) sets the lower and upper bounds of the possible link capacity enhancements.

The expected value model only considers the average total travel time while its variability is totally ignored. Under this model, the planner (network designer) would consider two capacity enhancement plans that have equal expected total travel time but different total travel time variabilities to be equal. The capacity enhancement plan identified in this model can be risky since it may select a plan with higher total travel time variability. Such a plan is suboptimal for the planner who is concerned with the total travel time reliability.

1.4 Chance constrained model

The chance constrained model, originally developed by Charnes and Cooper[45], models stochastic decision systems with the assumption that the constraints will hold at least α times, where α is referred to as the confidence level provided as an appropriate safety margin by the decision-maker. Its focus is on the system's ability to meet the chance constraints (risk measures) with a certain reliability under uncertainty. Charnes and Cooper[45] suggested three different types of objective functions: (1) a function that optimizes the expected value of the objective function (the E model), (2) a function that minimizes the generalized mean square of the objective function (the V model), and (3) a function that maximizes the probability of satisfying an aspiration level of the objective function (the P model). The original chance constrained model requires the users to specify both the threshold and the confidence level. However, it is sometimes difficult to determine the appropriate threshold in advance. Hence, a variant of the chance constrained model proposed by Liu[37] was used to determine the minimum threshold required to satisfy the chance constraint at a confidence level α. The modified chance-constrained model is formulated as

$$\min_{u} \ \text{TTTB} \tag{14}$$

$$\text{subject to} \quad \Pr(F(\boldsymbol{u}, \boldsymbol{v}(\boldsymbol{u}, \boldsymbol{Q})) \leqslant \text{TTTB}) \geqslant \alpha \tag{15}$$

Eqs. (12) and (13),

where TTTB is the total travel time budget required to satisfy the chance constraint at least α times. The objective function in Eq. (14) minimizes TTTB subject to the chance constraint in Eq. (15) that guarantees the probability that the total travel time less than TTTB is greater than or equal to the predefined confidence level α, subject to the budgetary constraint in Eq. (12) and the limit constraints in Eq. (13) on the set of link capacity enhancements. TTTB is a variable in the modified chance constrained model. The planner only needs to specify the confidence level α. A more risk averse planner can specify a higher α to control risk.

1.5 Dependent chance model

The dependent chance model, first introduced by Liu[46], maximizes the chance function of some events in an uncertain environment. In the network design problem, the planner specifies a goal to be attained (i.e., the congestion level or level-of-service) and the underlying philosophy of the dependent chance model is to select the optimal design with maximum chance to meet the specified objective.

$$\max_{u} \ \Pr(F(\boldsymbol{u}, \boldsymbol{v}(\boldsymbol{u}, \boldsymbol{Q})) \leqslant \text{TTTR}) \tag{16}$$

subject to Eqs. (12) and (13),

where TTTR is the total travel time requirement given by the planner. The objective function in Eq. (16) determines a vector of capacity enhancement links (i.e., design variables) that will maximize the total travel time reliability which is defined as the probability that the total travel time is less than TTTR (a predetermined threshold). The solution to the dependent chance model can be regarded as the most reliable NDP for a given total travel time requirement. The constraints are the same as the expected value model (i.e., budgetary constraint and limit constraints on the capacity enhancement links).

2 Simulation-Based Genetic Algorithm

Stochastic bi-level programs are generally difficult to solve by traditional calculus-based optimization methods. These network design models with demand uncertainty were solved using a solution procedure consisting of a traffic assignment algorithm, genetic algorithm, and Monte-Carlo simulation to handle the different complexities involved in solving the stochastic network design models in this paper. The demand uncertainty is addressed by the stochastic simulation. The nonlinear and nonconvex nature of the bilevel program is handled by the genetic algorithm. Bilevel mathematical programs are generally difficult to solve because evaluation of the upper-level objective function requires solving the lower-level subprogram. Here a standard traffic assignment algorithm (known as the Frank-Wolfe algorithm) is used to solve the lower-level subprogram[44].

2.1 Computing uncertain functions

Stochastic (or Monte Carlo) simulations are an important tool for performing sampling experiments on stochastic system models[37]. The simulations are based on sampling random variables from probability distributions to compute the uncertain functions. The uncertain functions used in the stochastic NDP models assume that a set of designs (i.e., capacity enhancements) have been determined by the genetic algorithm procedure. The three uncertain functions to be computed are the expected value function, chance constrained function, and probability function.

2.1.1 Expected value function

The objective function for the expected value model minimizes:

$$U_1:\ (\boldsymbol{u},\boldsymbol{v}) \to E[F(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{Q}))] \qquad (17)$$

This is computed using the following stochastic simulation procedure:

Step 1 Set $U_1(\boldsymbol{u},\boldsymbol{v})=0$.

Step 2 Generate $\boldsymbol{\omega}$ from Ω according to the probability measure Pr.

Step 3 For each $\boldsymbol{Q}(\boldsymbol{\omega})$, solve the lower-level subproblem in Eqs. (6) to (9) and calculate $F(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{Q}(\boldsymbol{\omega})))$.

Step 4 $U_1(\boldsymbol{u},\boldsymbol{v}) \leftarrow U_1(\boldsymbol{u},\boldsymbol{v}) + F(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{Q}(\boldsymbol{\omega})))$.

Step 5 Repeat the second to fourth steps for N times, where N is sufficiently large.

Step 6 Return $U_1(\boldsymbol{u},\boldsymbol{v})/N$.

2.1.2 Chance constrained function

The objective function for the chance constrained model minimizes

$$U_2:(\boldsymbol{u},\boldsymbol{v}) \to \{\text{TTTB} \mid \Pr(F(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{Q})) \leqslant \text{TTTB}) \geqslant \alpha\} \qquad (18)$$

The steps in the stochastic simulation procedure are as follows:

Step 1 Generate $\boldsymbol{\omega}_1,\boldsymbol{\omega}_2,\ldots,\boldsymbol{\omega}_N$ from Ω according to the probability measure Pr, where N is sufficiently large.

Step 2 For each $\boldsymbol{Q}(\boldsymbol{\omega}_k)$, solve the lower-level subproblem in Eqs. (6) to (9) and denote the total travel time by $F_k(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{Q}(\boldsymbol{\omega})))$ for $k=1,2,\cdots,N$.

Step 3 Set N' as the integer part of αN.

Step 4 Return the N'-th least element in $\{F_1(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{Q}(\boldsymbol{\omega}))), F_2(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{Q}(\boldsymbol{\omega}))),\cdots,F_N(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{Q}(\boldsymbol{\omega})))\}$.

2.1.3 Probability function

The objective function for the dependent chance model maximizes

$$U_3:(\boldsymbol{u},\boldsymbol{v}) \to \{\Pr(F(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{Q})) \leqslant \text{TTTR})\} \qquad (19)$$

The steps in the stochastic simulation procedure are as follows:

Step 1 Set $N'=0$.

Step 2 Generate $\boldsymbol{\omega}$ from Ω according to the probability measure Pr.

Step 3 For each $\boldsymbol{Q}(\boldsymbol{\omega})$, solve the lower-level subproblem in Eqs. (6) to (9) and calculate $F(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{Q}(\boldsymbol{\omega})))$.

Step 4 If $F(\boldsymbol{u},\boldsymbol{v}(\boldsymbol{u},\boldsymbol{Q}(\boldsymbol{\omega}))) \leqslant \text{TTTR}$, then $N' \leftarrow N'+1$.

Step 5 Repeat the second to fourth steps for N times, where N is sufficiently large.

Step 6 Return N'/N.

2.2 Computing network equilibrium solutions

For each realization of O-D demand $\boldsymbol{q}=\boldsymbol{Q}(\boldsymbol{\omega})$ generated by the Monte Carlo simulation, the Frank-Wolfe algorithm (also known as the convex combinations method) is used to solve the lower-level subprogram in Eqs. (6) to (9). The algorithmic steps are summarized below.

Step 0 Initialization. Set iteration counter $n=1$. Find an initial feasible flow pattern $\{v_a^{(1)}\}$.

Step 1 Link travel time update. Calculate $t_a^{(n)} = t_a(v_a^{(n)}),\ \forall a \in A$.

Step 2 Direction finding. Perform all-or-nothing (AON) assignment based on $t_a^{(n)}$ to obtain a set of auxiliary flows $y_a^{(n)}$.

Step 3 Move size. Find $\alpha^{(n)}$ which minimizes the objective function along its descent direction.

$$\min_{\alpha^{(n)}} \sum_{a\in A} \int_0^{v_a^{(n)}+\alpha^{(n)}(y_a^{(n)}-v_a^{(n)})} t_a(x,u_a)\mathrm{d}x .$$

Step 4 Flow update. Compute $v_a^{(n+1)}=v_a^{(n)}+\alpha^{(n)}(y_a^{(n)}-v_a^{(n)})$, $\forall a \in A$.

Step 5 Convergence test. If a specified criterion (e.g., changes in link flows or maximum iterations) is met, then terminate; otherwise, set $n = n+1$ and go to Step 1.

2.3 Computing optimal designs

For the network design problem, the lower-level subprogram can be analyzed as a set of nonlinear constraints. This often makes the bi-level mathematical programs non-convex and difficult to solve by standard optimization methods[5]. Many heuristic algorithms have been developed to solve the bi-level network design problem. A summary survey is provided by Yang and Bell[5]. To tackle the non-convexity issue in network design problems, Friesz et al.[47] and Meng and Yang[12] used simulated annealing (SA), while Chen and Yang[30], Yin[48], and Chen et al.[49] employed a genetic algorithm (GA). Both meta-heuristics are stochastic search methods that have the potential of obtaining the global optimal solution by providing a means to escape local optima (i.e., accept moves that worsen the objective value). The main mechanism driving the optimization is a simple search operator inspired by different natural based phenomena (i.e., physical annealing of solids for the SA and natural selection based on the principle of evolution-survival of the fittest for the GA). The GA is used in this study to determine the optimal capacity enhancements because it can work with continuous and discrete parameters, differentiable and non-differentiable functions, and uni-modal and multi-modal functions as well as convex and non-convex feasible regions[50]. In addition, GA has been widely applied in many fields because of its globalization, parallelism, and robustness[51]. Typical GA implementations involve coding the design variables in the upper-level subprogram as chromosomes, evaluating the fitness of the chromosomes, and performing the basic GA operators (i.e., reproduction, crossover, and mutation) to evolve the chromosomes to obtain better solutions. This section provides a brief description of the GA implementation. Readers can refer to Goldberg[50] and Gen and Cheng[51] for more details.

2.3.1 Chromosome representation

In general, the two chromosome representations are binary and real. Since the decision variables in the upper-level subprogram are real, the real representation was used to represent the design variables, u_a, with a length equal to the number of capacity enhancement links in the network, $|\bar{A}|$. The value of each gene represents the link capacity expansion, which is limited by the upper and lower limits of the constraint in Eq. (12).

2.3.2 Reproduction operator

Reproduction is a selection process that selects chromosomes from a population pool based on their fitness for mating. The chromosome fitness implies the number of times each chromosome will be in the mating pool. The most commonly used selection schemes are the roulette wheel and tournament selection. This study uses the roulette wheel selection scheme. After evaluating the fitness of all chromosomes in the population pool, they are ranked in ascending order based on these fitness values. Chromosomes with the highest chance will occupy a larger portion on the roulette wheel. The selection process is based on a random number between 0 and 1, and the solution associated with the intercepted portion of the wheel will enter the mating pool.

2.3.3 Crossover operator

Crossover is a means of exchanging genetic material between two parent chromosomes such that two new offspring chromosomes, containing genetic material from both parent chromosomes, are generated. Crossover occurs with a constant probability, which implicitly indicates the expected number of chromosomes in the mating pool undergoing crossover. There are many crossover schemes in the literature. Since the chromosome in the network design problem is coded using a real-code representation, arithmetic crossover is used. This method is similar to the linear combination of two solution vectors with a random fraction.

2.3.4 Mutation operator

Mutation alters the value of genetic units for the purpose of introducing new genetic structures to the new offspring. All new offspring are subject to the mutation operator with a predefined mutation rate.

Mutation allows the GA to explore new regions of the solution space and helps prevent convergence to a sub-optimal solution.

2.4 Simulation-based genetic algorithm procedure

This section summarizes the major steps of the simulation-based genetic algorithm procedure for solving the stochastic network design models with demand uncertainty.

Step 0 Define input parameters: population size, crossover and mutation rates, maximum number of generations, and maximum number of simulations.

Step 1 Generate an initial population pool and initialize the generation index.

Step 2 Evaluate the fitness of all chromosomes in the population pool using the traffic assignment and stochastic simulation procedures.

Step 3 Check whether the predefined maximum generation number is reached. If yes, go to Step 6; otherwise, go to Step 4.

Step 4 Rank the chromosomes based on their fitness values and use the tournament selection scheme to select parent chromosomes for reproduction.

Step 5 Update the chromosomes using the crossover and mutation operators, increment the generation index, and go to Step 2.

Step 6 Report the best chromosome as the optimal design.

3 Numerical Experiments

Three numerical experiments were used to evaluate the three stochastic NDP models introduced in Section 1. The network used for the three numerical experiments is the Sioux Falls network depicted in Fig. 1. The network consists of 24 nodes, 76 links, and 528 O-D pairs with positive demand. The link characteristics, capacity expansion cost functions, and O-D demands are available in Suwansirikul et al.[13] The random O-D demands were generated according to the triangular distribution (a,b,c), where a and b are the lower and upper limits and c is the mode. The mode c was set equal to the demand specified in Suwansirikul et al.[13],

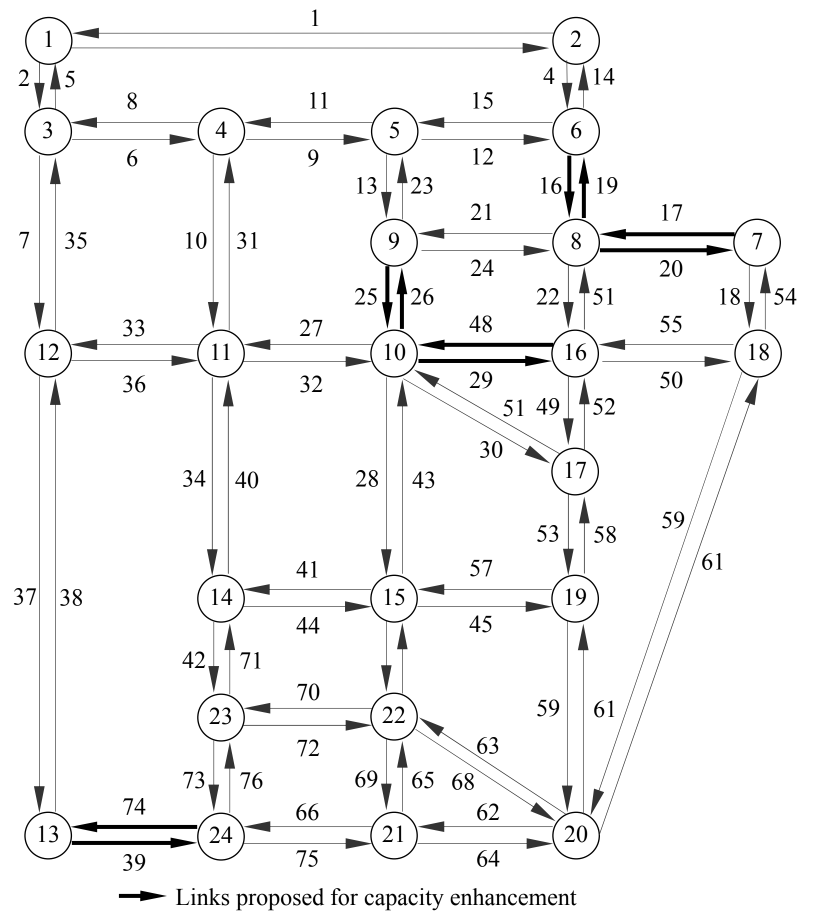

Fig. 1 Sioux Falls network

while a and b are set equal to ±50% of c for all O-D pairs. Ten links were selected for capacity enhancement, i.e., links 16, 17, 19, 20, 25, 26, 29, 39, 48, and 74. The budget was set at 5500 according to Meng and Yang[12].

This study used the following parameters.

- Population size (Pop-size) is 16-32 chromosomes.
- The maximum number of generations is 400.
- The maximum number of samples is 1000.
- The probability of crossover is 0.3-0.5.
- The probability of mutation is 0.1-0.3.
- The lower and upper bounds for capacity enhancement are [0 veh/h, original link capacity veh/h].

3.1 Expected value model

Experiment 1 uses the expected value model to determine the optimal link capacity enhancements with demand uncertainty. Table 1 presents the optimal capacity enhancements for the ten links in Fig. 1 for the various GA parameters of population size (16 and 32), crossover probability (P_c = 0.3 and 0.5), and mutation probability (P_m = 0.1, 0.2, and 0.3). In addition, the model objective value (Cost) and the percentage error are used to evaluate the GA performance for various parameter settings. The percentage error is computed using the best value among ten runs (i.e., $\text{Error} = \frac{\text{actual value} - \text{best value}}{\text{best value}} \times 100\%$). As can be seen from Table 1, the percentage error among the ten runs with the different parameter settings does not exceed 0.6%. The results suggest that the solution procedure is quite robust to different parameter settings.

Table 1 Comparison of capacity enhancement solutions for the expected value model

No.	Pop_size	P_c	P_m	Optimal capacity enhancement										Cost	Error
				16	17	19	20	25	26	29	39	48	74		(%)
1	16	0.3	0.1	1.279	2.285	2.812	1.966	3.330	13.276	0.972	0.871	1.279	1.279	6930.1	0.6
2	16	0.3	0.3	1.141	1.270	1.369	2.489	3.396	13.629	0.705	1.074	1.075	1.270	6913.2	0.4
3	16	0.5	0.1	1.319	1.663	1.690	1.489	2.072	13.916	0.915	1.289	0.504	1.319	6889.0	0.0
4	16	0.5	0.2	1.342	2.768	2.588	2.122	1.752	13.406	1.342	1.342	0.529	1.346	6927.6	0.6
5	16	0.5	0.3	1.296	2.949	1.837	2.598	13.427	1.296	0.590	1.296	1.120	1.412	6905.5	0.3
6	32	0.3	0.1	1.144	1.260	1.260	1.260	1.670	13.916	0.490	1.260	0.807	1.260	6892.1	0.1
7	32	0.3	0.3	1.164	2.891	3.146	1.164	1.221	13.567	0.696	1.164	1.164	1.164	6921.1	0.5
8	32	0.5	0.1	2.039	1.482	1.484	2.375	2.191	13.817	0.727	1.307	0.541	0.955	6887.3	0.0
9	32	0.5	0.2	1.438	1.438	2.707	1.259	1.438	13.916	0.746	0.921	0.651	1.482	6888.3	0.0
10	32	0.5	0.3	1.442	2.019	1.442	3.931	1.783	13.153	0.903	0.933	0.735	1.813	6901.9	0.2

3.2 Chance constrained model

Experiment 2 uses a variant of the chance constrained model to determine the optimal link capacity enhancements with demand uncertainty. This model can be interpreted as a value-at-risk (VaR) measure to account for the planner risk preferences by using a confidence level of $\alpha = 0.9$ on the total travel time reliability. As with experiment 1, the optimal capacity enhancements for the ten links in Fig. 1 for the different GA parameter settings are listed in Table 2. The percentage error among the ten runs does not exceed 0.9%. The solution procedure appears to be robust and effective in solving the nonlinear and nonconvex probabilistic constraint in the chance constrained model.

3.3 Dependent chance model

Experiment 3 uses the dependent chance model to determine the optimal link capacity enhancements with demand uncertainty with a total travel time requirement (TTTR = 7150). As with the previous two experiments, the optimal capacity enhancements for the ten links listed in Fig. 1 are presented for the different GA parameter settings in Table 3. The maximum probability (Prob.) is reported instead of the minimum cost. As indicated by the results, the maximum percentage error among the ten runs is less than 3%. Though the percentage error appears to be higher than in the previous two experiments, this is due to the scale (or unit) used to measure the objective function. In term

Table 2 Comparison of capacity enhancement solutions for the chance constrained model

No.	Pop_size	P_c	P_m	Optimal capacity enhancement										Cost	Error
				16	17	19	20	25	26	29	39	48	74		(%)
1	16	0.3	0.1	2.099	1.906	1.147	1.147	1.328	13.916	0.746	0.987	0.761	1.147	7159.4	0.6
2	16	0.3	0.3	1.261	1.349	2.423	1.261	2.102	13.916	1.261	1.261	0.583	1.261	7166.9	0.7
3	16	0.5	0.1	0.749	3.460	1.217	1.678	1.967	13.515	0.960	1.217	0.993	1.217	7179.5	0.9
4	16	0.5	0.2	1.167	2.634	1.167	1.601	1.350	13.916	1.042	1.167	0.605	1.167	7151.3	0.5
5	16	0.5	0.3	1.427	3.270	1.341	1.538	13.585	1.479	0.618	1.173	1.173	1.513	7168.1	0.7
6	32	0.3	0.1	1.333	1.749	1.333	3.394	1.428	13.514	1.026	1.752	0.626	1.006	7155.7	0.6
7	32	0.3	0.3	2.452	3.395	1.485	2.725	2.866	12.905	0.772	1.300	0.579	1.300	7155.2	0.6
8	32	0.5	0.1	1.158	1.373	2.199	1.100	1.739	13.916	0.937	0.783	0.915	1.198	7146.1	0.4
9	32	0.5	0.2	1.104	1.802	3.146	1.104	1.572	13.890	0.428	1.343	0.861	1.104	7134.3	0.3
10	32	0.5	0.3	0.927	0.964	2.084	1.206	13.916	1.206	0.605	1.103	0.516	1.206	7115.5	0.0

Table 3 Comparison of capacity enhancement solutions for the dependent chance model

No.	Pop_size	P_c	P_m	Optimal capacity enhancement										Prob.	Error
				16	17	19	20	25	26	29	39	48	74	(%)	(%)
1	16	0.3	0.1	1.244	4.227	1.423	2.469	1.423	12.962	0.638	1.423	0.978	1.366	89.9	2.4
2	16	0.3	0.3	1.171	2.920	1.413	2.644	2.009	13.467	1.083	1.171	0.618	1.022	90.5	1.7
3	16	0.5	0.1	1.417	1.260	1.833	3.653	1.260	13.511	1.106	0.864	0.672	1.260	90.4	1.8
4	16	0.5	0.2	1.247	1.247	1.607	1.467	2.823	13.916	1.247	1.247	0.408	1.247	89.4	2.9
5	16	0.5	0.3	1.155	3.367	1.196	2.129	1.431	13.601	0.458	1.155	0.589	1.155	89.5	2.8
6	32	0.3	0.1	0.768	1.990	3.462	2.004	3.047	13.369	0.480	1.264	0.843	1.264	91.4	0.8
7	32	0.3	0.3	1.099	3.478	1.187	2.024	1.187	13.567	0.942	1.187	0.426	1.187	91.2	1.0
8	32	0.5	0.1	1.184	1.184	3.276	2.117	1.184	13.836	0.413	1.184	0.611	1.266	91.4	0.8
9	32	0.5	0.2	0.854	1.234	1.569	0.770	1.234	13.916	0.670	1.400	0.484	1.234	92.1	0.0
10	32	0.5	0.3	0.867	2.318	2.129	2.426	3.399	13.266	0.833	0.567	0.683	2.152	91.8	0.3

of the total travel time at the 90 percentile (i.e., the performance measure used to evaluate the objective function), the percentage error does not exceed 0.6%. In engineering applications, this error is considered acceptable.

4 Conclusions and Future Research

This paper discusses three stochastic NDP models to determine optimal link capacity enhancements in road networks with demand uncertainty. These stochastic models used different criteria to account for planner risk preferences on the total travel time reliability. Stochastic bilevel programming formulations were provided in which the upper-level subprogram uses one of the stochastic models to determine the optimal link capacity enhancements and the lower-level subprogram is a user-equilibrium traffic assignment problem subject to demand uncertainty. A simulation-based genetic algorithm procedure was used to solve the stochastic bilevel programming formulations. Numerical experiments were conducted to evaluate the applicability of the stochastic NDP models and the solution procedure. Several directions for future research are possible. On the modeling side, only the demand uncertainty is considered in the current study. Future research should also consider supply uncertainty (i.e., the degradation of network capacity) and route choice uncertainty (i.e., responses of road users to supply and demand uncertainty) in designing reliable roadway networks. In addition, multiple objectives should be considered in the NDP to account for the needs of various stakeholders in making infrastructure investment decisions. On the computational side, the efficiency of the simulation-based genetic algorithm procedure must be enhanced to solve large-scale problems. Finally, the mathematical properties and their relationships of the stochastic NDP models need to be further analyzed.

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