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Attitude Determination for MAVs Using a Kalman Filter

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Abstract: This paper presents a Kalman filter to effectively and economically determine the Euler angles for micro aerial vehicles (MAVs), whose size and payload are severely limited. The filter uses data from a series of micro-electro mechanical system sensors to determine the selected 3 variables of the direction cosine matrix and the bias of the rate gyro sensors as state elements in a dynamic model, with the gravitational acceleration to build a measurement model. For high speed maneuvers, rigid motion equations are used to correct the measurements of the gravitational acceleration. The filter is designed to automatically tune its gain based on the dynamic system state. Simulations indicate that the Euler angles can be determined with standard deviations less than 3°. The algorithm was successfully implemented in a miniature attitude measurement system suitable for MAVs. Aerobatic flights show that the attitude determination algorithm works effectively. The attitude determination algorithm is effective and economical, and can also be applied to bionic robofishs and land vehicles, whose size and payload are also greatly limited.

Key words: attitude determination; Kalman filter; micro-electro mechanical system (MEMS); micro aerial vehicle (MAV)

Introduction

Micro aerial vehicles (MAVs) are being developed worldwide. To realize miniature attitude determination systems for MAVs, micro-electro mechanical system (MEMS)-based sensors and global positioning systems (GPS) have been introduced into inertial navigation systems (INSs) and attitude and heading reference systems (AHRSs)[1-3]. However, these algorithms are not effective on MAVs as the performance of the very small micro-controllers used on MAVs is limited. Generally, the computations for the Kalman filter and the quaternion integration[4,5] in the algorithms must be reduced to get sufficient output frequency. Another problem is that the measurement update frequency is reduced in the microsystems. Because of the agility of MAVs and the limited size and payload, gravitational acceleration measurements are strongly affected by the kinetic accelerations. In addition, the geomagnetic field is disturbed by the ferrous metals and electric motors in the very small MAVs[6].

This paper presents a Kalman filter with a series of MEMS sensors to accurately estimate the attitude angles for MAVs. The filter uses the 3 variables of the direction cosine matrix as state elements. The rigid body motion equations are introduced to correct the gravitational acceleration measurements during high speed maneuvers. The pitch and roll angle can then be calculated with the Kalman filter. The geomagnetic field is used only to determine the yaw angle. Therefore, magnetic disturbances can not affect the output of the pitch and roll. Simulation results and light test results are presented to verify validity of the attitude algorithm.

1 Euler Angles and Direction Cosine Matrix

In the attitude determination system, two orthogonal

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coordinate frames are often defined as the body frame, which is fixed to the vehicle's body and denoted by superscript "b", and the inertial frame, which is denoted by superscript "n". The attitude can be described as a set of Euler angles, which are the pitch angle (θ), roll angle (ϕ), and yaw angle (ψ). The transformation matrix from the inertial frame to the body frame is expressed as the direction cosine matrix:

$$\boldsymbol{R}^{\mathrm{n2b}} = \begin{bmatrix} \cos\psi\cos\theta & \sin\psi\cos\theta & -\sin\theta \\ \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \cos\theta\sin\phi \\ \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi & \cos\theta\cos\phi \end{bmatrix} \tag{1}$$

Given $\boldsymbol{R}^{\mathrm{n2b}}$, the Euler angles can be calculated as

$$\theta = -\arcsin r_{1,3}^{\mathrm{n2b}} \tag{2}$$

$$\phi = \begin{cases} \arctan\left(\dfrac{r_{2,3}^{\mathrm{n2b}}}{r_{3,3}^{\mathrm{n2b}}}\right), & r_{3,3}^{\mathrm{n2b}} \geqslant 0 \\ \pi + \arctan\left(\dfrac{r_{2,3}^{\mathrm{n2b}}}{r_{3,3}^{\mathrm{n2b}}}\right), & r_{3,3}^{\mathrm{n2b}} < 0,\ r_{2,3}^{\mathrm{n2b}} \geqslant 0 \\ -\pi + \arctan\left(\dfrac{r_{2,3}^{\mathrm{n2b}}}{r_{3,3}^{\mathrm{n2b}}}\right), & r_{3,3}^{\mathrm{n2b}} < 0,\ r_{2,3}^{\mathrm{n2b}} < 0 \end{cases} \tag{3}$$

$$\psi = \arccos\left(\frac{r_{1,1}^{\mathrm{n2b}}}{\sqrt{1-(r_{1,3}^{\mathrm{n2b}})^2}}\right) \tag{4}$$

Therefore, the attitude determination problem is equivalent to determining the four elements of $\boldsymbol{R}^{\mathrm{n2b}}$: $r_{1,1}^{\mathrm{n2b}}, r_{1,3}^{\mathrm{n2b}}, r_{2,3}^{\mathrm{n2b}}$, and $r_{3,3}^{\mathrm{n2b}}$.

2 Reference Vector Measurements

The gravitational acceleration and geomagnetic vector are used as reference vectors to calculate the elements. The gravitational acceleration in the body frame can be written as

$$\boldsymbol{G}^{\mathrm{b}} = \begin{bmatrix} g_x^{\mathrm{b}} \\ g_y^{\mathrm{b}} \\ g_z^{\mathrm{b}} \end{bmatrix} = \boldsymbol{R}^{\mathrm{n2b}} \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} = g \begin{bmatrix} r_{1,3}^{\mathrm{n2b}} \\ r_{2,3}^{\mathrm{n2b}} \\ r_{3,3}^{\mathrm{n2b}} \end{bmatrix} \tag{5}$$

where g is the magnitude of the gravity acceleration. The magnetic declination is zero and the geomagnetic vector in the body frame can be written as

$$\boldsymbol{M}^{\mathrm{b}} = \begin{bmatrix} m_x^{\mathrm{b}} \\ m_y^{\mathrm{b}} \\ m_z^{\mathrm{b}} \end{bmatrix} = \boldsymbol{R}^{\mathrm{n2b}} \boldsymbol{M}^{\mathrm{n}} = m\boldsymbol{R}^{\mathrm{n2b}} \begin{bmatrix} \cos\beta_{\mathrm{mag}} \\ 0 \\ \sin\beta_{\mathrm{mag}} \end{bmatrix} \tag{6}$$

where m is the magnitude of the geomagnetic field and β_{mag} is the magnetic inclination which can be measured. Then, m_x^{b} can be expressed as

$$m_x^{\mathrm{b}} = m(r_{1,1}^{\mathrm{n2b}} \cos\beta_{\mathrm{mag}} + r_{1,3}^{\mathrm{n2b}} \sin\beta_{\mathrm{mag}}) \tag{7}$$

In the static mode, the gravitational acceleration can be measured by accelerometers on the three axes of the body frame. If there are no magnetic disturbances, the magnitude of the geomagnetic field can be measured by magnetometers. The pitch and roll can then be calculated using Eqs. (2), (3), and (5). Then, the yaw can be obtained using Eqs. (4) and (7).

However, in the dynamic mode, the accelerometer measurements are strongly affected by kinetic accelerations. The relationship can be expressed using the rigid motion equation as

$$\begin{bmatrix} a_x^{\mathrm{b}} \\ a_y^{\mathrm{b}} \\ a_z^{\mathrm{b}} \end{bmatrix} = \begin{bmatrix} g_x^{\mathrm{b}} \\ g_y^{\mathrm{b}} \\ g_z^{\mathrm{b}} \end{bmatrix} - \begin{bmatrix} \dot{v}_x + \omega_y v_z - \omega_z v_y \\ \dot{v}_y + \omega_z v_x - \omega_x v_z \\ \dot{v}_z + \omega_x v_y - \omega_y v_x \end{bmatrix} \tag{8}$$

where ω_x, ω_y, and ω_z are the angular rotation rates measured by rate sensors on the x, y, and z axes in the body frame and v_x, v_y, and v_z are the projections of the velocity vector. For most conventional fixed-wing aircrafts, v_x can be easily measured by the airspeed meter, while v_y and v_z are much less than v_x and difficult to measure. If these are neglected, the gravitational acceleration can be calculated as

$$\begin{bmatrix} g_x^{\mathrm{b}} \\ g_y^{\mathrm{b}} \\ g_z^{\mathrm{b}} \end{bmatrix} \cong \begin{bmatrix} a_x^{\mathrm{b}} + \dot{v}_x \\ a_y^{\mathrm{b}} + \omega_z v_x \\ a_z^{\mathrm{b}} - \omega_y v_x \end{bmatrix} \tag{9}$$

In addition, noisy sensor measurements should be pre-filtered before introducing them into the system, especially if the noise is electronic or high-frequency vibrational disturbances above the bandwidth of the vehicle performance. Furthermore, sensor misalignment, temperature drift, and center of gravity (CG) offsets should be included based on calibration data[7]. Furthermore, with high speed maneuvers, the Kalman filter is usually utilized to fuse the multiple sensor data.

3 Kalman Filter Design

3.1 System model

The filter's state elements are defined as

$$\boldsymbol{x}=\begin{bmatrix}\boldsymbol{x}_{\mathrm{r}}\\ \delta\boldsymbol{\omega}\end{bmatrix} \tag{10}$$

where $\boldsymbol{x}_{\mathrm{r}}=[r_{3,1}^{\mathrm{n2b}},r_{3,2}^{\mathrm{n2b}},r_{3,3}^{\mathrm{n2b}}]^{\mathrm{T}}$, which are the third column elements of the cosine matrix[8], and $\delta\boldsymbol{\omega}=[\delta\omega_x,\delta\omega_y,\delta\omega_z]^{\mathrm{T}}$, which are the bias of the rate gyro sensors on each axis in the body frame. The dynamic model of the state vector is

$$\dot{\boldsymbol{x}}=\boldsymbol{A}\boldsymbol{x}+\boldsymbol{w} \tag{11}$$

with

$$\boldsymbol{A}=\begin{bmatrix}\boldsymbol{\Omega} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix},$$

$$\boldsymbol{\Omega}=\begin{bmatrix}0 & (\omega_z+\delta\omega_z) & -(\omega_y+\delta\omega_y)\\ -(\omega_z+\delta\omega_z) & 0 & (\omega_x+\delta\omega_x)\\ (\omega_y+\delta\omega_y) & -(\omega_x+\delta\omega_x) & 0\end{bmatrix} \tag{12}$$

where $\boldsymbol{w}$ presents the process noise. Hence, state elements can be calculated by integrating Eq. (11). In addition, $\boldsymbol{x}_{\mathrm{r}}$ is subject to

$$\boldsymbol{x}_{\mathrm{r}}^{\mathrm{T}}\boldsymbol{x}_{\mathrm{r}}=1 \tag{13}$$

which is used to normalize $\boldsymbol{x}_{\mathrm{r}}$[9]. In view of Eq. (5), the measurement model can be expressed as

$$\boldsymbol{z}=\begin{bmatrix}g_x^{\mathrm{b}}/g\\ g_y^{\mathrm{b}}/g\\ g_z^{\mathrm{b}}/g\end{bmatrix}=\boldsymbol{H}\boldsymbol{x}+\boldsymbol{v} \tag{14}$$

where the gravitational acceleration is used as the observation vector, which is calculated using Eq. (9). Variable $\boldsymbol{v}$ represents the measurement noise. $\boldsymbol{H}=[\boldsymbol{I}_{3\times3}\quad \boldsymbol{0}_{3\times3}]$.

The conventional extended Kalman filter was employed since the system state transition matrix $\boldsymbol{A}$ is time varying. The system states, $\boldsymbol{x}$, are estimated recursively using the measured system output, $\boldsymbol{z}$, and the calculated matrix, $\boldsymbol{A}$, with an assumed process noise covariance, $\boldsymbol{Q}$, and measurement noise covariance, $\boldsymbol{R}$. After the system states, $\boldsymbol{x}$, are calculated, the pitch and roll angles are determined using Eqs. (2), (3), and (4).

Since the measurement matrix is quite simple, the Kalman filter is easy to implement. Some elements of the cosine matrix are used directly as the filter state elements, so the algorithm's computational cost is much less than that for the INS or AHRS algorithms.

3.2 Adaptive tuning design

To get better performance, filter adaptation is used to tune the gain during dynamic maneuvers. According to Eqs. (9) and (14), the observation vector is relative to the measurment of acceleration, velocity, and gyroscope, and then the state measurement noise covariance, $\boldsymbol{R}$, is estimated for simplicity as

$$\boldsymbol{R}=\boldsymbol{R}_{\mathrm{a}}+\boldsymbol{R}_{\mathrm{u}}+\boldsymbol{R}_{\mathrm{o}} \tag{15}$$

where $\boldsymbol{R}_{\mathrm{a}}$ is due to the noise in the accelerometers, $\boldsymbol{R}_{\mathrm{u}}$ is determined from the output noise in the airspeed meter and rate gyroscopes, and $\boldsymbol{R}_{\mathrm{o}}$ is the approximate errors. For optimal performance, the filter adaptation is used to tune the matrix $\boldsymbol{R}_{\mathrm{o}}$ as

$$\boldsymbol{R}_{\mathrm{o}}=\Big[k_{\mathrm{g}}\Big|\sqrt{(g_x^{\mathrm{b}})^2+(g_y^{\mathrm{b}})^2+(g_z^{\mathrm{b}})^2}-g\Big|^2+ k_{\omega_x}\omega_x^2+k_{\omega_y}\omega_y^2+k_{\omega_z}\omega_z^2\Big]\bullet\boldsymbol{I}_{3\times3} \tag{16}$$

where k_{g}, k_{ω_x}, k_{ω_y}, and k_{ω_z} are scalar parameters, which could be tuned to achieve optimal performance from flight simulations. Generally, $\boldsymbol{R}_{\mathrm{o}}$ should be much greater than $\boldsymbol{R}_{\mathrm{a}}$ with increasing gravitational acceleration measurement errors and than $\boldsymbol{R}_{\mathrm{u}}$ with increasing angular rotation rates. Therefore, the filter gain can be tuned automatically according to the angular rates and errors of the gravitational acceleration to describe the dynamic state[10,11].

In addition, the state process noise covariance matrix, $\boldsymbol{Q}$, can be determined by the spectral density analysis of the process noise of the gyroscope and its associated drift. However, $\boldsymbol{Q}$ is usually defined with initially chosen values to achieve the best performance.

4 Tests

4.1 Simulations

The attitude determination method was simulated in Matlab. Random initial conditions were given to confirm the convergence. The yaw and roll angles were based on two sine waves with different frequencies and initial phases. Two slope waves were assigned to the pitch angle. The velocity was assumed to be 15 $\mathrm{m}\bullet\mathrm{s}^{-1}$. A uniformly distributed discrete random noise with a 0.3-$m^2 \cdot s^{-4} \cdot Hz^{-1}$ spectral density was added to the accelerations. Discrete random noise with 0.006 $Gauss^2 \cdot Hz^{-1}$ spectral density was added to the magnetometer readings. Discrete random noises with 0.3 $deg^2 \cdot s^{-2} \cdot Hz^{-1}$ spectral density and bias were added to the angular rate signals. The sampling and calculational period was 0.04 s. The results show that the algorithm gives good estimates of the attitude angles, with the standard deviation of yaw being 2.4^o, that of roll 0.8^o, and that of pitch 1.2^o.

To test the method's validity, a popular commercial autopilot model MP2028 with an integrated inertial measurement unit (IMU) was used as a reference. The raw data from a flight was input to the attitude estimation algorithm. The results are compared in Figs. 1 and 2, with error less than 3^o [6,8,10].

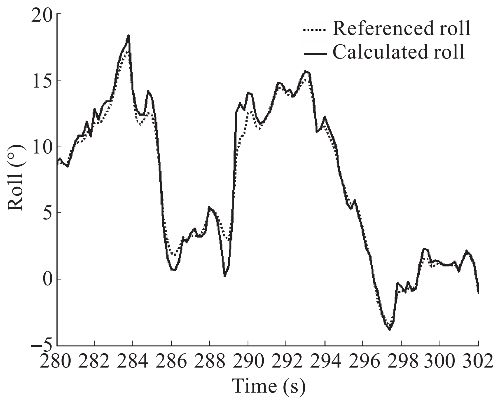

Fig. 1 Roll angle calculated using the filter method and the referenced roll from MP2028

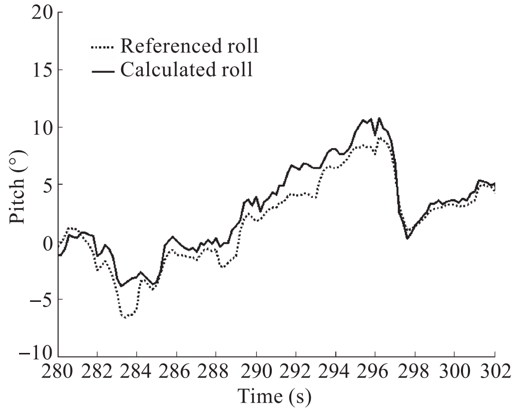

Fig. 2 Pitch angle calculated using the filter method and the referenced pitch from MP2028

4.2 Implementation and flight test

A miniature attitude measurement system was designed using the filter included tri-axial MEMS accelerometers, tri-axial MEMS gyroscopes, tri-axial MEMS magnetometers, an airspeed meter, a 12-bit A/D convertor, and a fixed point digital signal processor. The total mass was less than 20 g, which is suitable for MAVs. The filter algorithm used sampling and calculating frequencies of 200 Hz. The algorithm can be implemented with frequencies of up to 20 Hz with an 8-bit microcontroller.

The attitude determination algorithm was tested in high speed maneuvers in a radio control fixed-wing model airplane. Various aerobatic maneuvers were executed by an experienced operator, such as limbs and dives, flips, and split-S and Immelmann turns. The split-S turn is a maneuver in which an airplane first does half a roll and then executes a half-loop, thus ending flying level in the opposite direction. The Immelmann turn is a maneuver in which an airplane first completes half a loop and then half a roll to simultaneously gain altitude and change the flight direction.

While the angles from the MP2028 are uncertain in these high speed maneuvers, the present system performed quite well, with the results shown in Figs. 3-6. The outputs of roll and pitch are reasonable, even when the pitch is near $\pm 90^o$ as shown in Fig. 4.

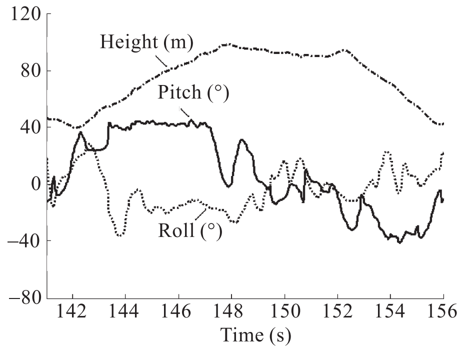

Fig. 3 Attitude estimates during a climb and a dive

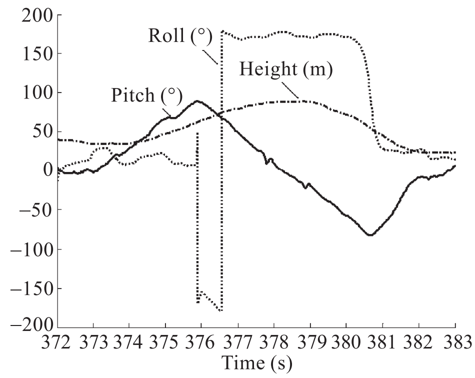

Fig. 4 Attitude estimation during a flip

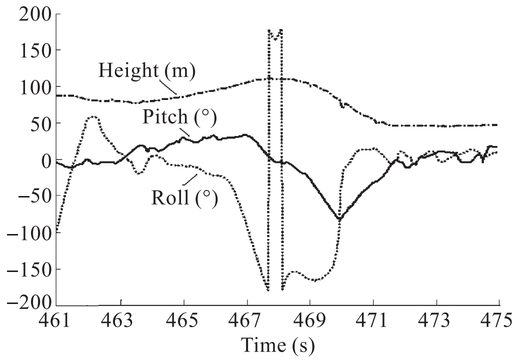

Fig. 5 Attitude estimates during a split-S turn

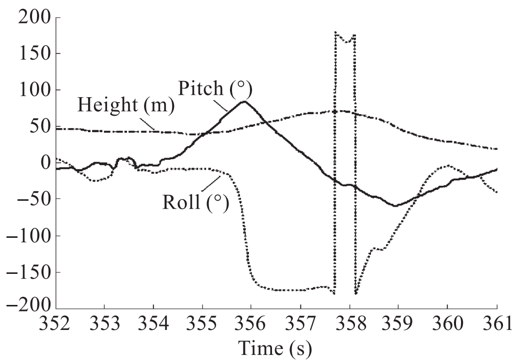

Fig. 6 Attitude estimates during an Immelmann turn

However, the yaw estimates show large variations. The yaw angle, which is estimated mainly from the magnetometer output, is affected greatly by the local magnetic field caused by ferrous metals, batteries, and electric motors. The test results for the yaw angle show accuracies of approximately $\pm 5^{\circ}$, even with some magnetic field shielding. Therefore, for MAVs, a GPS is necessary to get the heading angle.

5 Conclusions

This paper presents an attitude determination algorithm with new state elements and an adaptive tuning method. The model uses the rigid motion equations to determine the kinetic accelerations, which affect the accuracy of attitude determination systems in the classic INS and AHRS algorithm. Simulation and flight test results show that the method is effective for fixed-wing MAVs, even during high speed maneuvers. However, the yaw angle may not be accurate due to magnetic disturbances, so a GPS heading should be used.

This attitude estimation algorithm is easy to implement with low computational cost. The algorithm can be applied in other real-time navigation, guidance, and control applications, such as airships, land vehicles, and missiles, whose sizes and payloads are limited.

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