TSINGHUA SCIENCE AND TECHNOLOGY
ISSN 1007-0214 26/67 pp158-164
Volume 13, Number S1, October 2008

# Fuzzy Set-Based Risk Evaluation Model for Real Estate Projects*

SUN Yijian (孙艺键)**, HUANG Rufu (黄如福)†,
CHEN Dailin (陈岱林)†, LI Hongnan (李宏男)

School of Civil and Hydraulic Engineering, Dalian University of Technology, Dalian 116024, China;
† Institute of Building Engineering Software, China Academy of Building Research, Beijing 100013, China

**Abstract:** With the rapid development of residential real estate market, risk evaluation has been an important task in the process of project. This paper describes a risk evaluation method for residential real estate projects based on fuzzy set theory which uses linguistic variables and respective fuzzy numbers to evaluate the factors. The primary weights of factors and evaluation of alternatives are determined by applying linguistic variables and fuzzy numbers. The notion of Shapley value is used to determine the global value of each factor in accomplishing the overall objective of the risk evaluation process, so the primary weights are revised, thus the importance of factors can be reflected more precisely. A major advantage of the method is that it allows experts and engineers to express their opinions on project risk evaluation in linguistic variables rather than crisp values. An illustration is presented to demonstrate the application of the method in risk evaluation. The results are consistent with the results calculated by conventional risk evaluation method. The research demonstrates that the method is objective and accurate, and is of an application value in the risk evaluation for residential real estate project.

**Key words:** real estate; project risk; fuzzy evaluation; Shapley value

## Introduction

The real estate industry has been a support industry in the national economy with the rapid development of housing market since the welfare housing allocation system has been canceled in China. The real estate is an industry with high cost, high profit and high risk, with more and more enterprises paying attention to the risk evaluation.

Risk evaluation is concerned with evaluating the probability and impact of individual risk; the risk evaluation methods which are widely used include expert grade method, Monte Carlo method and analytic hierarchy process (AHP)[1,2]. Despite the successful application of above risk evaluation methods, many problems remain: expert grade method is mainly based on the subjective judgments by the experts and the conclusions are approximate; Monte Carlo method is difficult to identify the correlations among risk factors, and is based on the model selection, thus the model selection has a deep influence on the precision of calculation; meanwhile there is a great calculation amounts and usually the computer is needed to finish the calculation. The AHP has some issues in the application: the imposed inconsistency due to the restriction of pairwise comparisons to a 1-to-9 scale and to the problematic correspondence between the verbal and the numeric scales[3]; the variation in the verbal expressions from one person to another, as well as their dependence on the type of elements involved in the comparison[4]. The application of the methods is limited.

Received: 2008-05-30
∗ Supported by the Ministry of Construction of China (No. 06-K9-22)
∗∗ To whom correspondence should be addressed.
E-mail: sunyijian0508@163.com; Tel: 86-13717771830

# 1 Fuzzy Risk Evaluation

The risk evaluation for real estate projects is to evaluate the risk scale and the degree of their influence. It takes the entire project risk, inter-infection and interaction among risk, and their infection on the project into consideration, as well as the risk capacity of the project. Due to the properties of real estate, the risk must be evaluated to win a better profit of the projects.

The fuzzy risk evaluation is a process based on evaluation model, and combines the fuzzy set with real estate risk evaluation to evaluate the risk comprehensively. Figure 1 shows a process model of fuzzy risk evaluation.

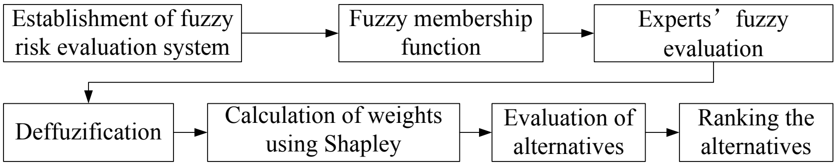


**Fig. 1 Process of fussy risk evaluation**

## 1.1 Establishment of a fuzzy risk evaluation system

A theoretical model for risk evaluation is presented to evaluate the risk of a project, as shown in Fig. 2.

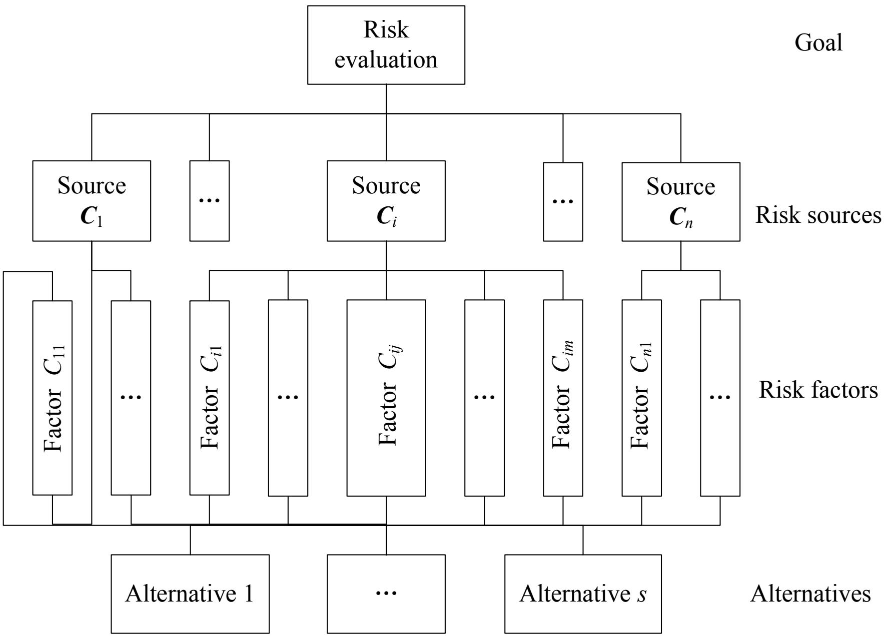


**Fig. 2 Theoretical model of risk evaluation**

The model consists of levels of goal, risk sources, risk factors and alternatives. The level of goal is to explain the goal of risk evaluation, the risk factors are divided into several groups according to their sources, and every risk source contains some factors. Combined with the alternatives which will be evaluated, an evaluation model is established.

In the model, there are $n$ kinds of risk sources which can be represented as $\boldsymbol{C}=\{\boldsymbol{C}_1, \boldsymbol{C}_2, \ldots, \boldsymbol{C}_i, \ldots, \boldsymbol{C}_n\}$, and there are $m$ factors in the risk source $c_i$, which can be represented as $\boldsymbol{C}_i=\{C_{i1}, C_{i2}, \ldots, C_{ij}, \ldots, C_{im}\}$.

When a model is established, the principles below should be followed.

(1) Systemic principle. The system is established according to the reality of the project, so the reliability of the evaluation is guaranteed.

(2) Both the quantitative and qualitative factors are taken into consideration, so the objectivity and maneuverability can be guaranteed in the course of evaluation.

(3) Independence principle. The establishment of factors should avoid containing each other among the factors, so the essential factors must be grasped.

(4) Facility principle. The comprehensive factors should be selected, for it is convenient to experts.

The model of risk evaluation aims at the residential real estate development in large cities in China, such as Beijing, Shanghai, etc. After investigating the markets of residential real estate in these cities, the experts of real estate select some risk factors and divide them into four risk sources which are political risk, social risk, economical risk and technical risk by their own experiences. Thus, a hierarchy structure of risk evaluation for residential projects is established, as shown in Fig. 3.

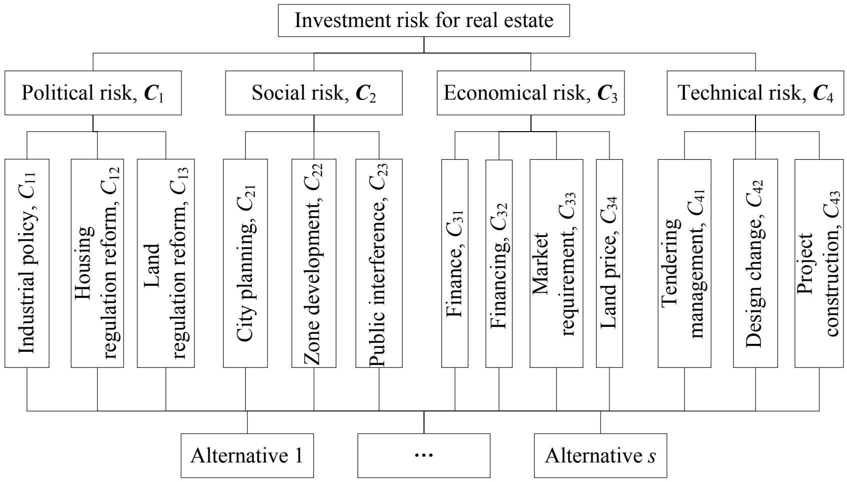


**Fig. 3 Hierarchy structure of risk evaluation for residential projects**

## 1.2 Linguistic variables and membership function

The fuzzy set is a kind of mathematical expression which deals with some phenomenon with vagueness[5].

A mapping on the Universe $X$ is given below:

$$u_A : X \to [0,1]$$
$$x \mapsto u_A(x)$$

So $u_A$ confirms a fuzzy subset $\underset{\sim}{A}$ on the Universe $X$, $u_A$ is the membership of $\underset{\sim}{A}$, and $u_A(x)$ is the grade of membership.

That is to say, this grade represents the degree to which that individual is similar or compatible with the concept represented by the fuzzy set. Thus, an individual may belong in the fuzzy set to a greater or lesser degree as indicated by a larger or smaller membership grade. These membership grades are very often represented by real-number values ranging in the closed interval between 0 and 1. The grade of membership is expressed by membership function. Trapezoidal and triangular functions are usually used.

The fuzzy set supplies the concepts of membership function, linguistic variables, and so on. The linguistic variables describe a vague concept. In the model, it is presented that the linguistic variables are used by experts of real estate to evaluate the fuzzy weights of the risk factors, which can be written as a comment set, {very important, important, above average, average, below average, below important, very below important}. So the experts can express their opinions more clearly and eliminate the effects on the weights evaluation caused by the vagueness in thinking and uncertainty.

Linguistic variables are also supplied for engineers to evaluate the alternatives on factors which can be expressed as a comment set, {very good, good, above average, average, below average, poor, very poor}. The better the comment is, the lower the risk is.

For each linguistic variable, there is a fuzzy number correspondingly, for instance, the linguistic variables, such as “important” and “very important”, have fuzzy numbers $(0.6, 0.7, 0.8, 0.9)$, $(0.8, 0.9, 1.0, 1.0)$, respectively. The linguistic variables and trapezoidal fuzzy number are listed in Table 1.

**Table 1 Fuzzy number for linguistic variables**

| No. | Linguistic variables | Fuzzy number |
|---|---|---|
| 1 | very important/very good | (0.8, 0.9, 1.0, 1.0) |
| 2 | important/good | (0.6, 0.7, 0.8, 0.9) |
| 3 | above average | (0.5, 0.6, 0.7, 0.8) |
| 4 | average | (0.4, 0.5, 0.5, 0.6) |
| 5 | below average | (0.2, 0.3, 0.4, 0.5) |
| 6 | below important/poor | (0.1, 0.2, 0.3, 0.4) |
| 7 | very below important/very poor | (0.0, 0.0, 0.1, 0.2) |

### 1.3 Experts' evaluation on factors and fuzzy weights calculation

Based on the hierarchy structure of risk factors, experts evaluate the fuzzy weights of factors with the linguistic variables, and the average fuzzy weights can be calculated according to the fuzzy number of each linguistic variable.

Let $\underset{\sim}{\boldsymbol{A}}=\{a_1,a_2,a_3,a_4\}$, $\underset{\sim}{\boldsymbol{B}}=\{b_1,b_2,b_3,b_4\}$ be any two positive trapezoidal fuzzy numbers, and $\oplus$ is the symbol for fuzzy plus operation, then the fuzzy plus operation is expressed as

$$\underset{\sim}{\boldsymbol{A}}\oplus\underset{\sim}{\boldsymbol{B}}=\{a_1,a_2,a_3,a_4\}+\{b_1,b_2,b_3,b_4\}=\{a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4\} \tag{1}$$

The average fuzzy weight is the arithmetical average of all fuzzy weights for factor $C_{ij}$ given by all experts which can be expressed as

$$\underset{\sim}{\boldsymbol{A}}_{ij}=(a_{ij},b_{ij},c_{ij},d_{ij})=\frac{1}{p}\times(\underset{\sim}{A}_1^{ij}\oplus\cdots\oplus\underset{\sim}{A}_k^{ij}\oplus\cdots\oplus\underset{\sim}{A}_p^{ij}) \tag{2}$$

where $a_{ij}=\dfrac{\sum_{k=1}^{p}a_k^{ij}}{p}$, $b_{ij}=\dfrac{\sum_{k=1}^{p}b_k^{ij}}{p}$, $c_{ij}=\dfrac{\sum_{k=1}^{p}c_k^{ij}}{p}$, $d_{ij}=\dfrac{\sum_{k=1}^{p}d_k^{ij}}{p}$, $i=1, 2, \ldots, n$; $j=1, 2, \ldots, m$.

$\underset{\sim}{A}_k^{ij}$ represents the fuzzy weight assigned to factor $C_{ij}$ given by expert $k$; $\underset{\sim}{\boldsymbol{A}}_{ij}=(a_{ij},b_{ij},c_{ij},d_{ij})$ represents the average fuzzy weight assigned to factor $C_{ij}$; $p$ represents the number of experts involved in the process.

### 1.4 Defuzzification

Defuzzification is an operation that produces a crisp value that adequately represents the degree of satisfaction of the aggregated fuzzy number. For a trapezoidal membership function, the defuzzified value $e_{ij}$ for the average fuzzy weight of factor $C_{ij}$ is given by the following equation[6]:

$$e_{ij}=\frac{a_{ij}+b_{ij}+c_{ij}+d_{ij}}{4} \tag{3}$$

where $e_{ij}$ represents the defuzzified value for the average fuzzy weight of factor $C_{ij}$.

The defuzzified values are normalized according to Eq. (4), and the primary weight of factor $C_{ij}$ is reached.

$$\mu(c_{ij})=\frac{e_{ij}}{\sum_{j=1}^{m}e_{ij}} \tag{4}$$

where $\mu(c_{ij})$ represents the primary weight of factor $C_{ij}$.

### 1.5 Calculation of weights

Shapley value represents the expected marginal contribution of a particular player to the overall goal[7]. In the process of risk evaluation, Shapley value is used to revise the primary weights and the weights can be calculated more precisely, so more accurate risk evaluation can be achieved.

For the factors in risk source $i$, different combinations of factors can be made. Experts define the weights of all combinations by their own experience. When the weights are assigned to the combinations, the following regulations should be followed: (1) the weight assigned to the combination of any factor must be at least equal to the sum of the individual weight assigned separately and no more than 1; (2) the weight assigned to the null set is 0. The average can be worked out and then the revised weights can be achieved using Eq. (5).

$$w(C_{ij})=\sum_{q=1}^{v}w(C_{ij})_q=\sum_{q=1}^{v}\left\{\frac{(m-S)!(S-1)!}{m!}[\mu(A_q)-\mu(A_q-C_{ij})]\right\} \tag{5}$$

where $w(C_{ij})$ represents the revised weight for factor $C_{ij}$; $m$ represents the number of factors in risk source $i$; $A_q$ represents any combination of factors containing $C_{ij}$, $q=1, 2, \ldots, v$; $S$ represents the number of factors in combination $A_q$; $v$ represents the number of combination factors; $\mu(A_q)$ represents the average weight of combination $A_q$; $\mu(\Phi)=0$ ($\Phi$ is null set); and

$$\mu(C_{i1},C_{i2},\ldots,C_{ij},\ldots,C_{im})=1.$$

After calculation, the revised weights for all factors in risk source $i$ can be expressed as

$$\boldsymbol{w}_i=[w(C_{i1}), w(C_{i2}), \ldots, w(C_{im})]$$

The weights of all risk sources can also be worked with the method expressed above and expressed as

$$\boldsymbol{w}=[w(c_1), w(c_2), \ldots, w(c_n)]$$

The evaluation model, linguistic variables, the fuzzy numbers and the weights of factors can be stored as a part of the knowledge management of an enterprise for other residential projects' risk evaluation.

### 1.6 Alternative evaluation on factors

The engineers evaluate all the alternatives on every factor with linguistic variables, with the fuzzy evaluation being generated and transferred to fuzzy numbers. Using Eq. (2) for averaging and Eq. (3) for defuzzification, the evaluation for the alternatives on all factors in risk source $\boldsymbol{C}_i$ is achieved which can be expressed as

$$\boldsymbol{R}_i = \begin{bmatrix} x_{i1}^1 & x_{i1}^2 & \cdots & & x_{i1}^S \\ x_{i2}^1 & x_{i2}^2 & \cdots & & x_{i2}^S \\ & & \ddots & & \\ \vdots & \vdots & & x_{ij}^r & \vdots \\ & & & \ddots & \\ x_{im}^1 & x_{im}^2 & \cdots & & x_{im}^S \end{bmatrix}$$

where $\boldsymbol{R}_i$ represents the evaluation matrix for all alternatives on risk source $\boldsymbol{C}_i$; $x_{ij}^r$ represents the evaluation for alternative $r$ on factor $C_{ij}$; and $s$ represents the number of alternatives.

The weights of factors in risk source $\boldsymbol{C}_i$ are multiplied by $\boldsymbol{R}_i$ as follows.

$$\boldsymbol{B}_i = \boldsymbol{w}_i \boldsymbol{R}_i = [w(C_{i1}) \quad w(C_{i2}) \quad \cdots \quad w(C_{im})] \cdot \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1s} \\ x_{21} & x_{22} & \cdots & x_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{ms} \end{bmatrix} = [b_{i1} \quad b_{i2} \quad \cdots \quad b_{is}]$$

This is the evaluation matrix for every alternative on risk source $\boldsymbol{C}_i$.

### 1.7 Ranking the alternatives

Aggregating evaluation matrix for $n$ risk sources, evaluation matrix $\boldsymbol{B}$ for all risk sources can be expressed as

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_1 \\ \boldsymbol{B}_2 \\ \vdots \\ \boldsymbol{B}_n \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1s} \\ b_{21} & b_{22} & \cdots & b_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{ns} \end{bmatrix}$$

The scores of the alternatives are calculated as

$$\mathrm{TS} = \boldsymbol{w}\boldsymbol{B} = \begin{bmatrix} w(\boldsymbol{C}_1) \\ w(\boldsymbol{C}_2) \\ \vdots \\ w(\boldsymbol{C}_n) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1s} \\ b_{21} & b_{22} & \cdots & b_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{ns} \end{bmatrix}$$

The best alternative is the one with the highest score.

## 2 A Case Study

Now there is a real estate project of resident housing. The owner supplies 3 development alternatives, with 5 experts and 5 engineers involved in the risk evaluation. The evaluation of factor weights given by experts is stored as a part of knowledge management in the enterprise, and the engineers evaluate the alternatives on factors, then using the weights to calculate the total scores of the alternatives.

### 2.1 Fuzzy weights evaluation of factors

The experts evaluate the fuzzy weights of factors with the linguistic variables. The results of evaluation are listed in Table 2, with risk source $\boldsymbol{C}_1$ taken as an example.

**Table 2 Evaluation of political risk**

| Expert | Factor | | |
|---|---|---|---|
| | $C_{11}$ | $C_{12}$ | $C_{13}$ |
| Expert 1 | 3 | 2 | 3 |
| Expert 2 | 1 | 1 | 1 |
| Expert 3 | 1 | 2 | 1 |
| Expert 4 | 2 | 2 | 1 |
| Expert 5 | 3 | 2 | 5 |

### 2.2 Calculation of factors' primary weights

First, the results of the evaluation are transferred to fuzzy numbers according to Table 1, and then the average fuzzy weights are calculated based on Eq. (2):

$$\underset{\sim}{A}_{11} = (0.64,\ 0.74,\ 0.84,\ 0.9),$$
$$\underset{\sim}{A}_{12} = (0.64,\ 0.74,\ 0.84,\ 0.9),$$
$$\underset{\sim}{A}_{13} = (0.68,\ 0.78,\ 0.88,\ 0.92).$$

The average fuzzy weights are defuzzified based on Eq. (3) as follows: $e_{11}=0.78$, $e_{12}=0.785$, and $e_{13}=0.815$. The primary weights are obtained based on Eq. (4):

$$\mu(C_{11})=0.33,\quad \mu(C_{12})=0.33,\quad \mu(C_{13})=0.34.$$

### 2.3 Calculation of revised weights

Experts assign the weights to the combinations of factors. The factor combinations consist of ($C_{11}$ $C_{12}$), ($C_{11}$ $C_{13}$), ($C_{12}$ $C_{13}$) and ($C_{11}$ $C_{12}$ $C_{13}$). Table 3 shows the weights assigned by the experts.

**Table 3 Weights assigned by experts to different combinations of factors**

| Expert | Factor combinations | | |
|---|---|---|---|
| | $C_{11}$ $C_{12}$ | $C_{11}$ $C_{13}$ | $C_{12}$ $C_{13}$ |
| Expert 1 | 0.75 | 0.80 | 0.85 |
| Expert 2 | 0.80 | 0.85 | 0.85 |
| Expert 3 | 0.70 | 0.70 | 0.70 |
| Expert 4 | 0.75 | 0.80 | 0.85 |
| Expert 5 | 0.80 | 0.70 | 0.90 |

The average weight of each factor combination across all experts can be worked out as follows:

$$\mu(\Phi)=0,\quad \mu(C_{11},C_{12})=0.76,\quad \mu(C_{11},C_{13})=0.77,$$
$$\mu(C_{12},C_{13})=0.83,\quad \mu(C_{11},C_{12},C_{13})=1.$$

There are three factors in the risk source of political risk, so $m=3$. There are four combinations containing factor $C_{11}$, which are ($C_{11}$), ($C_{11}$, $C_{12}$), ($C_{11}$, $C_{13}$) and ($C_{11}$, $C_{12}$, $C_{13}$). The corresponding numbers of the factors contained in the combinations are 1, 2, 2, and 3.

For combination $A_1$=($C_{11}$), the number of factor, $S$=1, and the Shapley value is worked out based on Eq. (5) as

$$w(C_{11})_1=\frac{(m-S)!(S-1)!}{m!}[\mu(A_1)-\mu(A_1-C_{11})]=$$
$$\frac{(3-1)!(1-1)!}{3!}[\mu(C_{11})-\mu(C_{11}-C_{11})]=$$
$$\frac{(3-1)!(1-1)!}{3!}[\mu(C_{11})-\mu(\Phi)]=$$
$$(0.33-0)\times 2/6=0.11.$$

Similarly, Shapley values for combinations $A_2$=($C_{11}$, $C_{12}$), $A_3$=($C_{11}$, $C_{13}$), and $A_4$=($C_{11}$, $C_{12}$, $C_{13}$) are $w(C_{11})_2$ =0.072, $w(C_{11})_3$ =0.072, $w(C_{11})_4$ =0.057, respectively.

So the revised weight for factor $C_{11}$ can be obtained as: $w(C_{11})$＝0.11+0.072+0.072+0.057＝0.311.

Similarly, the revised weights for factors $C_{12}$ and $C_{13}$ can be calculated, and the weights for factors in risk source $\boldsymbol{C}_1$ can be expressed as

$$\boldsymbol{w}_1=[0.311\quad 0.340\quad 0.349].$$

Similarly, the revised weights of other factors and the risk sources can be calculated. The revised weights of each risk source can be worked out as follows.

$$\boldsymbol{w}=[w(\boldsymbol{C}_1)\quad w(\boldsymbol{C}_2)\quad w(\boldsymbol{C}_3)\quad w(\boldsymbol{C}_4)]=$$
$$[0.323\quad 0.224\quad 0.283\quad 0.17].$$

The weights are stored as a part of the knowledge management of an enterprise and can be used when residential projects' risk is to be evaluated by engineers.

## 2.4 Evaluation of alternatives on factors

The satisfactions of alternatives on each of the factors are to be evaluated by the engineers. The linguistic evaluations of the alternatives' satisfactions on the factor $C_{11}$ are listed in Table 4.

**Table 4 Linguistic evaluation of alternatives on factor $C_{11}$**

| Engineer | Alternative | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| Engineer 1 | 2 | 1 | 3 |
| Engineer 2 | 3 | 1 | 2 |
| Engineer 3 | 3 | 2 | 2 |
| Engineer 4 | 2 | 3 | 3 |
| Engineer 5 | 2 | 2 | 3 |

The linguistic evaluations are transferred to fuzzy numbers according to Table 1. Then using Eq. (2) for averaging, the average fuzzy scores for the alternatives 1, 2, and 3 are (0.56, 0.66, 0.76, 0.86), (0.66, 0.76, 0.86, 0.92), and (0.54, 0.64, 0.74, 0.84), respectively. Using Eq. (3) for defuzzification, the scores on factor $C_{11}$, for the alternatives 1, 2, and 3 are 0.71, 0.8, and 0.69, respectively.

Similarly, the scores for the alternatives on factors $C_{12}$ and $C_{13}$ can be worked. The evaluation for the three alternatives on risk source $\boldsymbol{C}_1$ can be expressed as

$$\boldsymbol{R}_1=\begin{bmatrix}0.71 & 0.8 & 0.69\\ 0.735 & 0.765 & 0.69\\ 0.56 & 0.62 & 0.56\end{bmatrix}.$$

The first-class evaluation matrix can be obtained as follows.

$$\boldsymbol{B}_1=\boldsymbol{w}_1\boldsymbol{R}_1=[0.311\ \ 0.340\ \ 0.349]\begin{bmatrix}0.71 & 0.8 & 0.69\\ 0.735 & 0.765 & 0.69\\ 0.56 & 0.62 & 0.56\end{bmatrix}=$$
$$[0.667\ \ 0.726\ \ 0.645].$$

These are the scores for each alternative on risk source $\boldsymbol{C}_1$.

Similarly, the scores for each alternative on other risk sources can be worked out, and the score matrix for alternatives on each risk source can be written as

$$\boldsymbol{B}=\begin{bmatrix}0.667 & 0.726 & 0.645\\ 0.667 & 0.721 & 0.667\\ 0.719 & 0.773 & 0.695\\ 0.687 & 0.737 & 0.677\end{bmatrix}.$$

### 2.5 Ranking of alternatives

The aggregated scores for alternatives on risk sources can be represented as

$$\mathrm{TS} = \boldsymbol{wB} = \begin{bmatrix} 0.323 \\ 0.224 \\ 0.283 \\ 0.170 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 0.667 & 0.726 & 0.645 \\ 0.667 & 0.721 & 0.667 \\ 0.719 & 0.773 & 0.695 \\ 0.687 & 0.737 & 0.677 \end{bmatrix} = [0.685 \quad 0.740 \quad 0.670].$$

Hence, the scores for alternatives 1, 2, and 3 are 0.685, 0.740, and 0.670, respectively. The alternative with a higher score has a lower risk, so the best one is alternative 2.

This result is consistent with the result calculated by the analytic hierarchy process which can be read in the project feasibility of the enterprise. Compared with other risk evaluation methods, the fuzzy set-based method is a practical tool for risk evaluation with its simple calculation and reliable results.

## 3 Conclusions

In an actual risk evaluation, it is difficult to judge the weight of each risk factor. So the authors present risk evaluation with linguistic variables. The linguistic variables are transferred to fuzzy numbers, and the primary weights are obtained by defuzzification and then revised by the calculation of Shapley value. The result shows that the model can be used in the risk evaluation of residential project development. The research indicates that the method proposed in this paper has its objectivity and stability, and is of applicable value in the risk evaluation for residential real estate projects.

### References

[1] Liu Xiaojun, Men Fanwen. Application of AHP-fuzzy method in the appraisal of the risk of real estate investment. *Journal of Xi'an University of Architecture & Technology*, 2005, **37**(1): 135-137. (in Chinese)

[2] Yu Jiuru. Risk Evaluation of Investment Project. Beijing: China Machine Press, 1999. (in Chinese)

[3] Belton V, Goodwin P. Remarks on the application of the analytical hierarchy process to judgmental forecasting. *International Journal Forecast*, 1996, **2**(1): 155-161.

[4] Poyhonen M A, Hmalainen R P, Salo A A. An experiment on the numerical modeling of verbal ratio statements. *Journal of Multi Criteria Decision Anal.*, 1995, **6**(1): 1-10.

[5] Xiao Ciyuan. Fuzzy Engineering System. Beijing: Science Press, 2004. (in Chinese)

[6] Kaufmann A, Gupta M M. Introduction to Fuzzy Arithmetic: Theory and Application. New York: Van Nostrand Reinhold, 1988.

[7] Rosenmuller J. Game Theory: Stochastic, Information, Strategies, and Cooperation. Boston: Kluwer Academic Publishers, 2000.