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Dynamic Analysis of Cable Roofs Under Transient Wind: A Comparison Between Time Domain and Frequency Domain Approaches

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Abstract: At present, high-speed computing capabilities and advanced nonlinear dynamic finite element procedures enable detailed dynamic analysis of cable structures. Although deterministic approaches require considerable analysis time and effort in relation to modeling, running, and data processing, they seem to be the only alternative to obtain high accuracy. Detailed dynamic analysis of cable roof networks is sophisticated and requires advanced modeling expertise. This paper presents a comparison between detailed nonlinear dynamic analysis and a simplified frequency domain approach to estimate the maximum probable response of weakly nonlinear cable roofs. The approach can be considered as alternative to detailed time-domain analysis in the preliminary design phase, or can be used to validate results obtained from more elaborated numerical models. The proposed method is illustrated with two examples of cable net roofs that were also analysed in the time domain.

Key words: weakly nonlinear structures; frequency domain analysis; wind spectrum

Introduction

Large cable roof structures are frequently associated with memorable events. They take advantage of high-strength steel with members in tension in order to provide an elegant structural system with large column-free areas. These days, cable roofs have a wide field of application and have been used to cover such diverse buildings as stadia and sports halls, swimming pools and water reservoirs, concert halls and theatres, cooling towers, hangars, warehouses and factories. Figure 1 depicts the North Carolina State Fair Arena in Raleigh, USA, the first large-span cable roof structure, which was built in 1953.

Cable roof structures are, in general, lighter and more flexible than most other forms of roof constructions. As a result, they are inherently more resistant to earthquake excitations but more sensitive to turbulent winds. Moreover, cable roofs with spans exceeding 25 m typically have their dominant lower frequencies within the high-energy range of the wind spectral density function. Hence the need for dynamic analysis under wind loads should not be readily dismissed for these roof constructions. Also, the nonlinearity of the structure will add to the complexity of the analysis[1].

For highly nonlinear cable networks, such as cable roofs with cable edge elements or without a stiffening cladding, dynamic analysis should be performed with a time-marching incremental nonlinear scheme[2]. However, such deterministic approaches for wind analysis of cable networks are complicated due to modeling considerations. Simplified frequency domain analytical approaches provide an acceptable practical alternative for weakly nonlinear cable networks, especially for preliminary design.

This paper presents a simplified procedure which

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combines a linear frequency-domain analysis of the fluctuating component of wind effects and a nonlinear static analysis under mean wind. The proposed method is applied to two case studies, and the results are compared with those obtained by detailed time-domain nonlinear dynamic analysis[3].

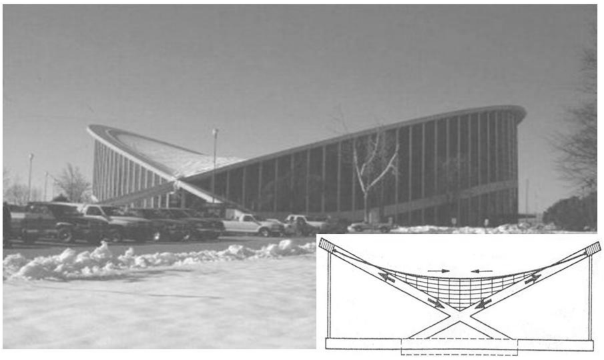

Fig. 1 North Carolina State Fair Arena in Raleigh, USA (http://www.wikipedia.org/)

1 Case Studies

Two saddle-shaped cable roof geometries are studied here, one is rectangular (see Fig. 2), and the other has circular boundaries.

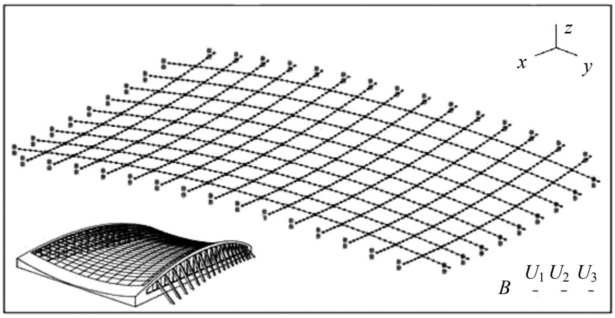

Fig. 2 Rectangular saddle-shaped cable network-cable links and restraints

In order to define the geometry of the networks and calculate the joint coordinates in the undeformed roof configuration, the general equation of saddle-shaped surfaces is employed:

$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2} \tag{1}$$

where the origin of the coordinate system is placed at the center of the network, and a and b are the corresponding sags in x and y directions, respectively.

The circular roof has a diameter of 120 m and the sag of the pretensioning and the suspension cables is 3% of the roof projected diameter. The net was modeled with 127 mm diameter stranded bridge cable links at 10 m center to center, the material properties of which are listed in Table 1. The self weight of the roof, including net and cladding, is taken as 0.6 kN/m^2.

Table 1 Spiral-strand Bridge cables

Diameter (mm)	Minimum breaking load (kN)	Weight (kN/100m)	Steel area (mm^2)	Elastic Modulus (GPa)
116.0	10487	66.01	7862	147.2
127.0	13371	78.33	9450	147.2

The rectangular roof is 100 m wide and 160 m long with 116 mm diameter stranded bridge cables in a grid of 10 m center to center in the two principal directions.

Both networks are prestressed to 30% of the ultimate strain of the cables. Moreover, for both case studies, the mean horizontal wind speed at the average height of the roof is assumed as $U(z)=37$ m/s. All the loading assumptions regarding the wind effects on the network and the self weight of the roof are also considered to be the same.

2 Modeling Considerations

A nonlinear elastic material model has been used for the cables: a tension-only behaviour is introduced through a bilinear diagram with a tensile elastic modulus $E=1.47\times10^{11}$ N/m^2, $\nu=0.3$, and $\rho=7850$ kg/m^3. The stress-strain diagram is shown in Fig. 3. The pretension force has been modeled as its corresponding initial strain in each cable element.

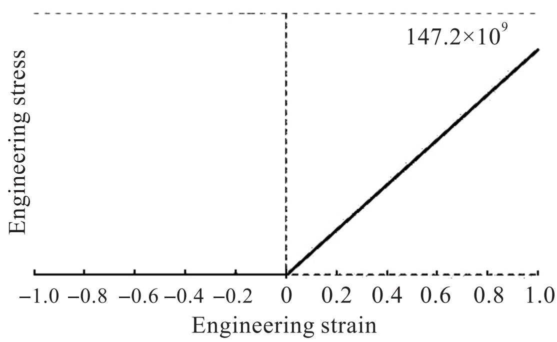

Fig. 3 Tension-only stress-strain diagram for the cables

Among the truss elements in ADINA[4], the 3-node parabolic element has been employed for it provides a good compromise in terms of accuracy and numerical effort. Approximately five cable elements are required to model the first five transverse modes of a cable segment so a mesh based on 2 m elements is selected[5].

A nonlinear static analysis under roof self-weight and cable initial strain was performed to obtain the deformed shape of the structures. These deformed configurations were considered as the initial shapes of the structures when applying the external wind forces.

3 Dynamic Analysis of Nonlinear Cable Networks: Time Domain Approach

The common practice in nonlinear dynamic analysis of structures is to calculate the response using incremental matrix updates and direct integration in time. In this approach, iterations are employed to establish the equilibrium of the forces at the end of each time increment, and algebraic extrapolation is used to evaluate kinematic parameters.

3.1 Nonlinear analysis

As an alternative to forms of Newton-Raphson iteration, which are based on the assumption that the solution for the discrete time $t+\Delta t$ is established based on the equilibrium configuration at time t, a class of methods known as matrix update methods or quasi-Newton methods have been developed for iteration on nonlinear systems of equations. These methods involve updating the coefficient matrix to provide a secant approximation to the matrix from iteration $(i-1)$ to (i)[6]. That is defining a displacement increment

$$\boldsymbol{\delta}^{(i)} = {}^{t+\Delta t}\boldsymbol{U}^{(i)} - {}^{t+\Delta t}\boldsymbol{U}^{(i)} \tag{2}$$

and an increment in the out-of-balance loads

$$\boldsymbol{\gamma}^{(i)} = \Delta\boldsymbol{R}^{(i-1)} - \Delta\boldsymbol{R}^{(i)} \tag{3}$$

the updated matrix ${}^{t+\Delta t}\boldsymbol{K}^{(i)}$ should satisfy the quasi-Newton equation

$${}^{t+\Delta t}\boldsymbol{K}^{(i)}\boldsymbol{\delta}^{(i)} = \boldsymbol{\gamma}^{(i)} \tag{4}$$

These schemes provide a compromise between the full reformation of the stiffness matrix and the use of a stiffness matrix from a previous configuration. Among the quasi-Newton methods available, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method was found effective. Use of line searches, with the default settings of ADINA, is also useful to accelerate the rate of convergence. An energy-based convergence criterion was used to avoid difficulties associated with displacement or force convergence criteria. The cable displacements induced by wind effects are quite large; but since their extensions are small, large displacements but small strains were assumed, which is justified in cable roof applications[6].

In this study, the cable pretension has been modeled as the corresponding initial strain in each cable element. As such, the stiffness matrix of the structure remains always non-singular.

3.2 Dynamic considerations

The constant-average acceleration Newmark-Beta method was selected (with parameters $\gamma=0.5$ and $\beta=0.25$) for direct integration of the incremental equilibrium equations. For accuracy considerations, it is recommended to consider $\omega_{\text{co}}\Delta t \leqslant 0.20$, in which ω_{co} is the highest frequency of interest in the dynamic response. The analyses performed in the current work are based on the assumption that the highest frequency of interest is 30 Hz. Therefore, $\Delta t = 0.001\text{sec}$ was selected[7].

3.3 Modeling of damping

In order to investigate the sensitivity of the structural response to different representations of internal damping, the following three damping models were considered: (1) numerical damping introduced by the direct integration scheme, (2) structural viscous damping modelled by individual linear dashpots in parallel with cable elements, and (3) no damping at all.

When no damping was introduced, it was found that the response increased abnormally, presumably due to spurious high frequencies of the discrete model. Algorithmic damping was introduced to filter this high frequency noise by setting the parameters of the Newmark method to $\gamma=0.55$ and $\beta=0.30$.

A similar representation of numerical damping could be modeled with the Wilson method. Surprisingly, the response of structure was further increased, and it was concluded that the higher mode vibrations were not spurious. A Fast Fourier Transform (FFT) analysis of the input wind load functions - one example is presented in Fig. 4, indicated that except for the narrow lowest frequency range, which contains considerable energy, the other parts of the frequency content of the loading had fairly uniform energy. As a result, structural resonance under the wind spectrum was inevitable, and it was deemed necessary to introduce a proper structural damping mechanism in the form of viscous dashpot elements in parallel with cable elements. This is illustrated schematically in Fig. 5.

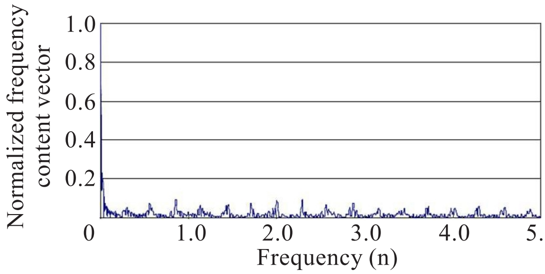

Fig. 4 Amplitude of Fast Fourier Transform of loading related to wind history 3

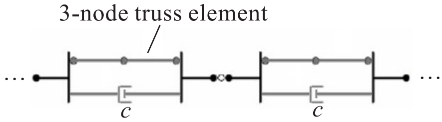

Fig. 5 Cable modelling — Each cable truss element is coupled in parallel with a linear viscous dashpot

A parametric study on the sensitivity of structural response to the amount of equivalent viscous damping was carried out, and it was concluded that varying the damping ratio between 0.5 to 5% critical does not have a significant effect on the total response. Finally, an equivalent translational viscous damper with 2% of the critical viscous damping was selected.

3.4 Loading

In this study, five random time histories of horizontal wind pressure are generated through Fourier series. The duration of the generated time histories is taken as 60 s[8]. In addition, the delay in the arrival times of the loading in different portions of the roof, following the propagation path of the horizontal wind across the structure, was used to model the correlation of wind histories[9].

The generated horizontal wind pressure histories were projected to the structure according to different angles of attack, and it was found that the worst direction was the longitudinal direction for the rectangular roof (and corresponding direction for circular network) with the suspension cables experiencing synchronised wind gusts.

3.5 Results and discussion

The results obtained from time history analysis of the roofs for the maximum vertical displacement of the central point are listed in the second column of Tables 2 and 3. It should be mentioned that for calculation of the mean value of the response to different wind histories, which is considered for the design, RMS is applied as in Eq. 5.

$$\mathrm{RMS}=\sqrt{(x_1^2+x_2^2+\cdots+x_n^2)/n} \tag{5}$$

Table 2 Maximum vertical displacements of middle point of circular cable network

Wind time history No.	X_{max} (Middle point)	RMS	Difference with FD (%)
1	0.47		
2	0.47		
3	0.46	0.46	24.4
4	0.45		
5	0.46		

Table 3 Maximum vertical displacements of middle point of rectangular cable network

Wind time history No.	X_{max} (Middle point)	RMS	Difference with FD
1	0.46		
2	0.47		
3	0.47	0.46	5.6%
4	0.44		
5	0.47		

The response time history of the circular cable network due to wind history 3 is presented in Fig. 6.

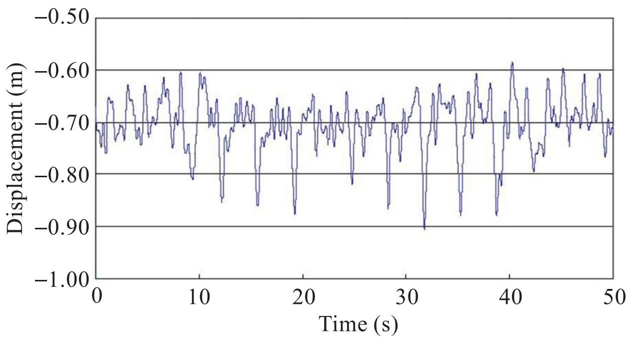

Fig. 6 Time history of vertical displacement of the middle point of the circular cable network under wind history 5

4 Dynamic Analysis of Nonlinear Cable Networks: Frequency Domain Approach

It is proposed to determine the dynamic response of weakly nonlinear cable roof structures to wind loading using spectral analysis. In this method, the response of the structure is divided into two parts: the response under mean wind speed, which is obtained through a nonlinear static analysis, and the response due to the fluctuating component of the wind, which is estimated by spectral analysis. The fluctuating part is obtained by a statistical approach following the determination of the spectral density function of the response of the structure based on the power spectra of the fluctuating component of the wind loading. The fluctuating response is assumed to vary linearly with the fluctuating component of wind pressure. The variance of the fluctuating component of the structural response is calculated using Eq. 6

$$\sigma_x^2 = \int_0^\infty S_x(n)\,\mathrm{d}n = 4\frac{x_s^2}{U^2}\int_0^\infty \mathrm{DLF}^2(n)\,A(n)\,S_u(n)\,\mathrm{d}n \tag{6}$$

The probability density function of the wind speed fluctuations is defined by the Gauss distribution function with zero mean and standard deviation σ_u.

The peak response factor during a wind event of duration T, is calculated as[10]

$$\kappa = [2\ln(f_n T)]^{1/2} + \frac{0.5772}{[2\ln(f_n T)]^{1/2}} \tag{7}$$

For the purpose of estimating the maximum response of the structure, we can use

$$x_{\max} = \kappa\sigma_x \tag{8}$$

Finally, adding the two values will result in the total response. This approach was first proposed by Buchholdt[1], and the present work, which is described in more details in Ref. [11], extends it to involve a larger number of significant degrees of freedom to improve its accuracy. The approach can have direct applications to the dynamic analysis of cable-stayed bridges, cable beams, cable nets and roofs and guyed telecommunication masts. However, its application is limited to weakly nonlinear structures.

Using this approach, we have obtained the maximum vertical displacement of 0.57 m for the circular roof and 0.49 m for the rectangular one when considering the effect of the first mode of excitation only. These values improved while considering more modes in the calculation of the fluctuating portion. For instance, the results show that the accuracy in the estimation of the maximum response has increased significantly when five modes are included – the error is reduced to 15% compared to about 24% for the circular case study when only the first mode is considered. This improved accuracy is deemed reasonable for practical engineering purposes.

5 Conclusions

The results obtained in the two case studies presented show that the proposed simplified frequency domain approach provides a capable tool to estimate the maximum probable wind response of the weakly nonlinear cable roof networks.

Nevertheless, deterministic approaches remain the only reliable analytical method in the case of highly nonlinear cable networks. However, there should be a reasonable balance between the amount of time and effort invested and the level of the reliability of the results.

References

[1] Buchholdt H A. An Introduction to Cable Roof Structures. London, UK: Thomas Telford, 1999.

[2] Stefanou G D. Dynamic response of tension cable structures due to wind loads. *Computers & Structures*, 1992, **43**(2): 365-372.

[3] Ghafari Oskoei S A. Analysis of cable roof structure to dynamic loads due to wind and earthquake in frequency domain [Dissertation]. University of Tehran, 2006.

[4] ADINA R&D Inc. ADINA User Interface Command Reference Manual. Report ARD 04-2 and 04-5. USA: Watertown, 2004.

[5] Guevara E L, McClure G. Nonlinear response of antenna-supporting structures. *Computers & Structures*, 1993, **47**(4/5): 711-724.

[6] Bathe K J. Finite Element Procedure. New Jersey, USA: Prince Hall, 1996.

[7] Wilson E L. Static and dynamic analysis of structures. Computers and Structures, Inc., Berkeley, California, USA, 2006.

[8] Dongyao T, Qingshan Y, Chen Z. Discrete analysis method for random vibration of structures subjected to spatially correlated filtered white noises. *Computers & Structures*, 1992, **43**(6): 1051-1056.

[9] ADINA R&D. ADINA systems 8.2 release notes. USA: Watertown, 2004.

[10] Vickery B J, Majowiecki M. Wind induced response of a cable supported stadium roof. *Journal of Wind Engineering and Industrial Aerodynamics*, 1992, **42**(1-3): 1447-1458.

[11] Ghafari Oskoei S A, McClure G, et al. Dynamic analysis of cable roofs under turbulent wind using a frequency domain approach. *Journal of Wind Engineering and Industrial Aerodynamics*, under review csubmitted 19/08/2008.