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Numerical Analysis of Moisture Influential Depth in Concrete and Its Application in Durability Design*

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Abstract: Determining the moisture influential depth in concrete under drying-wetting cycles is of great interest for investigation of chloride ion transport and thus the initiation of reinforcement corrosion. In this paper, the moisture transport processes during drying and wetting are modeled by diffusion and absorption. A predictor-corrector implicit scheme of finite difference method is used to solve the partial differential equations. The stability of moisture influential depth is then analyzed with the available numerical tool for both initially saturate and unsaturated concretes. The concept of equilibrium time ratio is proposed for drying-wetting cycles by the balance between water loss and intake during drying and wetting. According to this time ratio, drying-wetting cycles are classified into drying-dominated, wetting-dominated, and equilibrium ones. For drying-dominated cycles, the drying front will penetrate gradually into material while the influential depth is determined by wetting; for wetting-dominated cycles, the wetting front will progress into material while the influential depth is determined by drying. This classification has strong engineering implication and can give a more rational division of convection and diffusion zones of chloride ion transport. The case of concrete in marine splash zone is investigated to illustrate the application of influential depth in durability design of concrete structures.

Key words: concrete; chloride; dying-wetting cycles; influential depth; finite-difference method; service life

Introduction

The drying-wetting cycles are identified as the most unfavorable environmental condition for concrete deterioration processes. Concrete is a porous material of very small permeability with 10^{-14}-10^{-12} m/s for saturate permeability coefficient[1]. Therefore, for typical natural drying and wetting duration, the moisture loss and intake affect usually a rather limited depth into concrete surface[2,3]. This depth is commonly noted as influential depth of moisture transport, which has been noticed and investigated by several authors[4-6].

However, the available results are far from enough to predict the moisture transport under natural drying-wetting cycles because some determinant factors are not integrated: (1) different moisture transport mechanisms are mobilized during drying and wetting, so the concrete diffusivity cannot be the same for drying and wetting; (2) under natural climate, wetting is dominated by liquid water accumulation on concrete surface by atmospheric precipitation or sea tide splashing[7], and natural cycles can have very different drying and wetting durations. This paper, on the basis of different moisture transport mechanisms during drying and wetting as well as their durations, attempts to give a comprehensive modeling of moisture transport. With the built numerical tool, the influential depth is analyzed

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numerically and its important application is illustrated through a case study in marine environment.

1 Mechanism and Modeling of Moisture Transport

1.1 Main mechanisms of moisture transport in drying and wetting process

Moisture transfers through the pore network in concrete. Multiple moisture transport models have been proposed on the basis of three main mechanisms: absorption, diffusion, and permeation[1]. When concrete is exposed to atmosphere, the humidity difference between the surface and its micro-climate will cause moisture exchange at their interface. If the atmospheric humidity is lower than the equilibrium humidity of surface concrete, the liquid water in the surface pores will evaporate to the atmosphere, thus causing a capillary suction increase and a Darcian flow towards the surface[8]. If the atmospheric humility is higher, the opposite process may occur. When the unsaturated concrete is exposed to liquid water, the capillary pressure in the surface pores vanishes, thus causing a considerable capillary pressure gradient in surface concrete and consequently an intensive Darcian flow into the concrete. In this paper, wetting only means capillary absorption for the liquid-saturated-surface condition since it is the main moisture supply mechanism in natural environment and contributes substantially to the transport of corrosive ions into concrete.

1.2 Global nonlinear transport model

A complete transport model should include the transport of vapor, liquid water, and air in concrete[1,8]. However, to simplify the analysis process and to utilize the existing experimental data from the literatures, a unified nonlinear diffusion-form equation is adopted to describe the global moisture transport in concrete[9-12],

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(D(\theta)\frac{\partial \theta}{\partial x} \right) \tag{1}$$

where θ is water saturation degree in concrete pores, t the time (s), and $D(\theta)$ the saturation-dependent moisture diffusivity ($m^2 \cdot s^{-1}$). In a one-dimensional case, the initial moisture content is described by an initial moisture profile $\theta_{ini}(x)$,

$$\theta(x, t=0) = \theta_{ini}(x) \tag{2}$$

and the Dirichlet boundary condition can be expressed, for a changing moisture condition, as

$$\theta(x=0, t>0) = \theta_s(t) \tag{3}$$

with $\theta_s(t)$ as the prescribed changing water saturation degree at the boundary x=0. To simulate a semi-infinite case, an additional boundary condition is needed,

$$\theta(x=\infty, t>0) = \theta_{ini}(\infty) \tag{4}$$

In this paper, we assume a uniform initial condition, which means $\theta_{ini}(x) = \theta_{ini}$.

1.3 Diffusivities for drying and wetting

It is more reasonable to attribute different diffusivities for concrete during drying and wetting, i.e., different D values in Eq. (1) for drying and wetting,

$$D(\theta) = \begin{cases} D_d(\theta), & \text{drying;} \\ D_w(\theta), & \text{behind wetting front, wetting} \end{cases} \tag{5}$$

For the diffusivity during drying, a well established relation between the diffusivity and the water saturation degree can be written as[9,10]

$$D_d(\theta) = D_d^s \left(\alpha_0 + \frac{1-\alpha_0}{1+\left(\dfrac{1-\theta}{1-\theta_c}\right)^N} \right) \tag{6}$$

where D_d^s is the concrete diffusivity at total saturation ($m^2 \cdot s^{-1}$), α_0, θ_c, and N are experimental parameters.

The diffusivity during wetting can only be deduced indirectly since it is intended to represent an absorption process by the diffusion equation. Hall deduced the wetting diffusivity from capillary absorption data into the following form[11],

$$D_w(\theta) = D_w^0 \exp(n\theta) \tag{7}$$

with D_w^0 denoting the wetting diffusivity at a totally dry state ($m^2 \cdot s^{-1}$), and n a regression coefficient. Hall[11] attributed n=6-8 for building materials while Leech et al.[12] suggested n=6 for concrete on the basis of NMR imaging analysis of wetting front. Moreover, an explicit relation between concrete sorptivity and diffusivity is given by Lockington et al.[13],

$$\left(\frac{S}{\varphi}\right)^2 = D_w^0 \left[\exp(n)\left(\frac{2}{n} - \frac{1}{n^2}\right) - \left(\frac{1}{n} - \frac{1}{n^2}\right) \right] \tag{8}$$

where S is the concrete sorptivity at a totally dry state ($m \cdot s^{1/2}$) and φ the capillary porosity. The material data used in this paper are from Wongs (Table 1)[9].

Table 1 Concrete characteristics and properties retained for numerical analysis

Material	W/C	D_d^s / (10^{-10} m^2·s^{-1})	α_0	θ_c	N	D_w^0 / (10^{-10} m^2·s^{-1})	n
Concrete I	0.40	2.00	0.025	0.792	6	3.22	6
Concrete II	0.50	3.15	0.025	0.792	6	4.24	6
Concrete III	0.60	4.23	0.025	0.792	6	9.45	6

2 Numerical Solution of Moisture Transport

Equations (1)-(7) give a complete modeling for a concrete subject to drying-wetting cycles. In this modeling the following important assumptions are made: (1) the dying-wetting processes are isothermal; (2) for the moisture intake during wetting both the external hydraulic pressure and gravity are neglected compared to the capillary suction; and (3) hysteresis phenomena are not taken into account.

With the drying-wetting cycles expressed into a changing moisture boundary condition in Eq. (3), the solution of the time-dependent moisture content profile is performed by the finite difference method. A predictor-corrector implicit scheme[14] is used to solve numerically Eq. (1) with Eq. (2) as the initial condition and Eqs. (3) and (4) as boundary conditions. This scheme can assure 2nd-order accuracy with no need to solve a system of nonlinear equations. The central idea of this scheme is, for a given time interval, to discretize temporally Eq. (1) into two substeps and solve the water saturation degree consecutively. With the discretized initial condition of Eq. (2) and boundary conditions in Eqs. (3) and (4), a generalized minimal residual method (GMRES) with a precondition technique is employed to solve the systems of linear equations[15], and the water saturation degree θ is calculated for all the discretized nodes in the space domain and all instants in the temporal domain. The numerical accuracy is achieved by decreasing progressively the element size and time interval until converging to an expected tolerance. Detailed algorithms can be referred to Ref. [14].

3 Equilibrium Time Ratio and Influential Depth

3.1 Equilibrium time ratio

The natural drying-wetting cycles are idealized as periodical drying-wetting alternative cycles with a drying period t_d and a wetting period t_w. The moisture boundary condition in Eq. (3) can be specified as

$$\theta(x=0,t>0)=\theta_s(t)=\begin{cases}\theta_d, & \text{drying;}\\ 1.0, & \text{wetting}\end{cases} \tag{9}$$

The drying is idealized by imposing a constant water saturation degree θ_d at the concrete surface and the wetting by a constant saturation equal to 100%. With these specified boundary conditions, we investigate the influential depth of moisture transport.

Firstly, an equilibrium state is discussed for the moisture loss during drying and the moisture intake during the subsequent wetting. In such an equilibrium state, the moisture loss is equal to the moisture intake; accordingly one can expect that the drying front created at the end of t_d is just totally restored during wetting period t_w and the moisture content beyond this drying front will keep undisturbed. The term "undisturbed" is numerically defined as the moisture change is less than 1% compared to the initial water content for initially saturate concrete or to the stable value in initially unsaturated cases. For a specific drying-wetting cyclic scheme, the drying-wetting time ratio at the equilibrium state is defined as

$$\tau_{eq}=t_d/t_w \tag{10}$$

From a factor analysis of the constitutive equations Eqs. (1)-(7) and (9) this time ratio τ_{eq}, if existed, will be governed by several factors: concrete diffusivity of drying, diffusivity of wetting, initial water saturation profile, and external drying water saturation degree,

$$\tau_{eq}=\tau_{eq}(D_d,D_w;\theta_{ini},\theta_d) \tag{11}$$

where the diffusivities are intrinsic properties of concrete and the water saturation degrees are external factors.

This "equilibrium time ratio" is identified numerically and confirmed unambiguously for both initially saturate and unsaturated cases. Moreover, this ratio is independent on the period $t_{d\text{-}w}$ ($t_{d\text{-}w}=t_d+t_w$, Fig. 1). The equilibrium time ratios of initially saturate concretes with different W/C under different humidity gradients $\Delta\theta = \theta_{\text{ini}} - \theta_{\text{d}}$ are illustrated in Fig. 2.

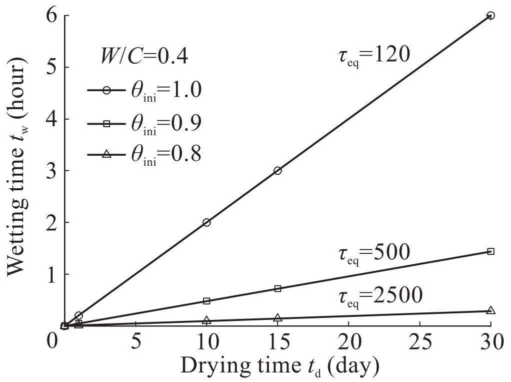

Fig. 1 Equilibrium wetting periods in terms of drying periods

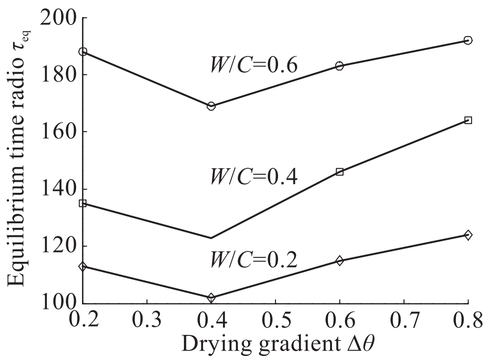

Fig. 2 Equilibrium time ratio in terms of drying gradient and *W/C* in initially saturate cases

3.2 Influential depth of drying-wetting cycles and its stability

The equilibrium time ratio can help to evaluate the influential depth of moisture transport as well as its evolution. This depth measures the extent of concrete surface subjected alternatively to drying and wetting. The influential depths under the equilibrium schemes are illustrated in Fig. 3. In fact, the equilibrium time ratio gives a first judgment on the evolution of influential depth.

For an initially saturate concrete, if $t_{\text{d}}/t_{\text{w}}<\tau_{\text{eq}}$, which means the wetting time is longer than that needed to restore moisture loss during drying, the inner concrete beyond the drying front will always remain saturate, just as $t_{\text{d}}/t_{\text{w}}=\tau_{\text{eq}}$. In this situation, the influential depth is stable and determined by the drying time. If $t_{\text{d}}/t_{\text{w}}>\tau_{\text{eq}}$, the moisture loss in the drying period cannot be restored totally, and the drying front will gradually advance into the inner concrete. Nevertheless, the concept of influential depth also makes sense because it scales the surface depth subjected alternatively to drying and wetting in each cycle.

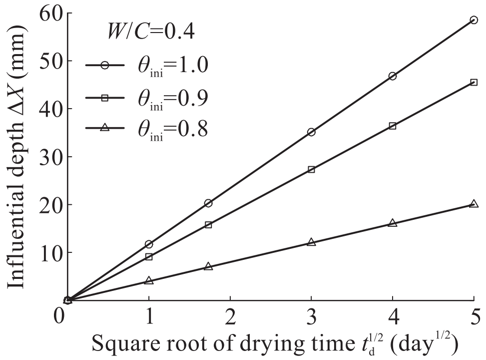

Fig. 3 Influential depth under equilibrium cycles in terms of square root of drying time

For an initially unsaturated concrete, the equilibrium time ratio τ_{eq} can only be identified through trials and errors approach. As the drying and wetting durations satisfy $t_{\text{d}}/t_{\text{w}}=\tau_{\text{eq}}$, the moisture profile can rapidly attain a stable state (Fig. 4). As $t_{\text{d}}/t_{\text{w}}<\tau_{\text{eq}}$, the wetting front will progress after each cycle while if $t_{\text{d}}/t_{\text{w}}>\tau_{\text{eq}}$ the drying front will progress gradually (Figs. 5 and 6).

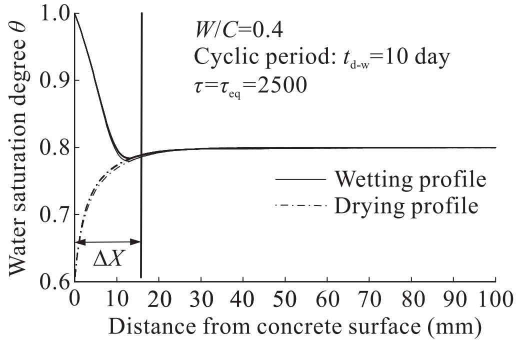

Fig. 4 Moisture profile evolution at equilibrium time ratio for the unsaturated case

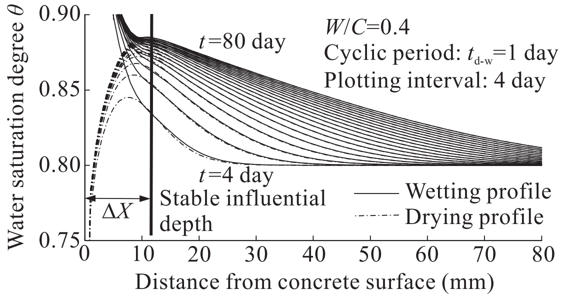

Fig. 5 Moisture profile evolution with time ratio τ = 500 for the unsaturated case

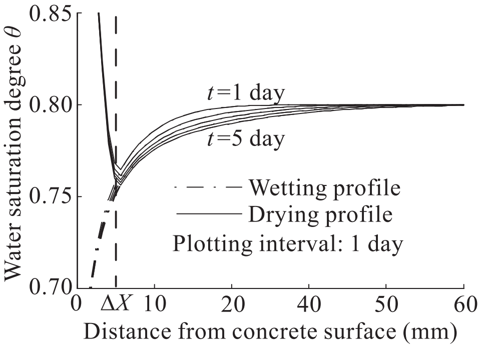

Fig. 6 Moisture profile evolution with time ratio τ = 5000 for the unsaturated case

Still, no matter which phase dominates the drying-wetting cycles, the depth subjected alternatively to drying and wetting in each cycle is rather limited. From the numerical results of Figs. 4-6, the influential depth can be determined by drying time in a wetting-dominated process and can be evaluated by wetting in a drying-dominated one.

4 Case Study: Application in Chloride Ingress Calculation of Splash Zone

The chloride ions transfer into concrete by both diffusion and convection. Naturally, this "convection zone" is determined by the moisture influential depth, beyond which the chloride transport can be modeled by Fick's 2nd law. During drying-wetting cycles, the salty water is sucked into the convection zone when wetting and is concentrated when drying. To take into account this effect as well as to take advantage of the simple form of Fick's law, a formula with a "convection zone" is adopted[16]:

$$C(x,t) = C_0 + (C_{S,\Delta X} - C_0)\operatorname{erfc}\left(\frac{x-\Delta X}{2\sqrt{D_{app,C}t}}\right) \tag{12}$$

where C is chloride content in concrete (% binder mass), C_0 initial chloride content of concrete (% binder mass), $C_{S,\Delta X}$ chloride content at depth ΔX (% binder mass), ΔX depth of convection zone (m), and $D_{app,C}$ apparent chloride diffusivity through concrete ($m^2 \cdot s^{-1}$) which is time-dependent:

$$D_a(t) = D_0\left(\frac{t_0}{t}\right)^m \tag{13}$$

where D_0 ($m^2 \cdot s^{-1}$) is the diffusion coefficient at a reference age t_0 (year), and m an exponent giving the time-dependency of the apparent chloride diffusivity.

It is assumed that $t_d/t_w < \tau_{eq}$ can be satisfied in the splash zone in most cases and $t_{d\text{-}w}$=1 day. From our model and the data in Table 1, the influential depths of the convection zone of initially saturate concretes with W/C of 0.4, 0.5, 0.6 are 11.5 mm, 14.5 mm, 17 mm, respectively. For the chloride ingress calculation, the parameters in Eqs. (12) and (13) are selected considering the recommendations in literatures (Table 2)[16-18].

Table 2 Chloride ingress parameters retained in numerical analysis

W/C	D_0 / (10^{-12} $m^2 \cdot s^{-1}$)	t_0 / year	ΔX / (10^{-3} m)	m	$C_{S,\Delta X}$ / (% binder mass)	C_{cr} / (% binder mass)
0.40	5.6	0.0767	11.5	0.6	6	0.2
0.50	9.0	0.0767	14.5	0.6	6	0.2
0.60	14.9	0.0767	17.0	0.6	6	0.2

The parameters retained in Table 2 are representative for concrete with high volume of fly ash which is recommended for concrete structures in chloride environments. The threshold chloride content C_{cr} is expressed as the proportion between total chloride content and binder mass. The chloride content profile after 100 years' exposition is illustrated in Fig. 7. These results show that to assure a service life of 100 years, the thickness of the concrete cover should not be less than 60, 70, and 95 mm for W/C of 0.4, 0.5, and 0.6, respectively.

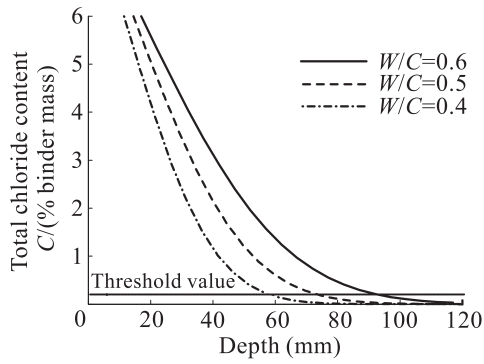

Fig. 7 Chloride content profile after 100 years' exposition

5 Conclusions

The concept of equilibrium drying-wetting time ratio is confirmed to exist in both saturate and unsaturated cases. This concept can help greatly in evaluating the influential depth. It is observed that the influential depth can be determined by drying time in a wetting-dominated process and can be evaluated by wetting time in a drying-dominated one. Our numerical analysis gives a rational way to evaluate influential depth, which, combined with Fick's Law, can give a more consistent approach to predict service life of concrete structures subject to chloride ingress.

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