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Information Sharing in a Multi-Echelon Inventory System*

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Abstract: The influence of different information sharing scenarios in a single supplier-single retailer supply chain is analyzed. The five information sharing scenarios are centralized information sharing, full information sharing, supplier-dominated information sharing, retailer-dominated information sharing, and non-information sharing. Iterative procedures are developed to obtain the inventory policies and the system costs at equilibrium points. Numerical examples show that the cost of a centralized inventory system is about 20% - 40% lower than that of a decentralized system with non-information sharing. Furthermore, a higher information sharing level does not always lead to a lower system cost in a decentralized supply chain due to inventory competition.

Key words: information sharing; inventory ordering policy; Markov decision process

Introduction

In the past decade, a number of papers have discussed optimal policies for two-stage serial inventory systems. Most works have focused on supply chains with centralized control. A central decision maker determines inventory policies for every stage so as to obtain optimal system performance in the entire supply chain. Clark and Scarf[1] first introduced the echelon inventory concept. Echelon inventory policies are optimal for serial systems with stochastic customer demands under periodic review with set-up costs only at the highest echelon. Federgruen and Zipkin[2] extended the work to the infinite-horizon case for both discounted and average cost criteria. Chen[3] introduced a simple heuristic policy which is 94% effective if set-up costs are charged in both stages.

However in real supply chain systems, the members in the supply chain may be independent and even competing. Hence, centralized control of the supply chain may be impossible and the supply chain has to be decentralized. Therefore, the objective of each stage is to optimize its own performance instead of the total supply chain performance. However, to achieve the best performance, the members of the supply chain might also be willing to cooperate. Many studies have focused on the design of the coordination mechanisms (e.g., price discounting, additional cost, contracts, etc.) to achieve the system equilibrium[4,5]. Cachon and Zipkin[6] analyzed the non-cooperative game behavior and the joint optimum of a two-echelon serial supply chain given that all supply chain members have full information. Wang et al.[7] extended their model to a one supplier-multiple retailers configuration. Chen et al.[8] showed that with proper coordination the same optimal profits can be achieved in a decentralized system as in a centralized system.

Studies of both centralized and decentralized serial supply chains normally assume complete information sharing in the system. At each decision epoch, the supply chain system states are fully known to each stage in the supply chain. However, in a real decentralized system, each stage plays the role of a player in a game and each player may not know the states of the other

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players due to technical reasons or business confidentiality. They then make decisions based only on their local states and historical data. In addition, some supply chain systems seek to partially share information between different stages. Therefore, four typical information sharing scenarios can be identified in a two-stage decentralized supply chain.

(1) Full information sharing (FIS). The supplier has information about the retailer's state and customer demands at each decision point and the retailer also knows the supplier's state.

(2) Supplier-dominated information sharing (SDIS). The supplier has information concerning the retailer, but the retailer does not know the state of the supplier. Therefore, the retailer makes decisions based only on its local states.

(3) Retailer-dominated information sharing (RDIS). The retailer has information about the state of the supplier, but the supplier does not know about the retailer.

(4) Non-information sharing (NIS). The retailer and the supplier do not share information with each other.

This paper considers these different information sharing scenarios in a two-stage supply chain composed of a supplier and a retailer. Unlike many prior studies that assume each stage has a structural inventory policy like (s, S) or (r, Q)[9,10], this study investigates the equilibrium policies of the supply chain for general inventory conditions. The analysis considers decentralized supply chains with these information sharing scenarios as well as a centralized supply chain. In a decentralized supply chain, each stage independently chooses its ordering policy to minimize the expected long-run average operating cost. Each stage has inventory holding costs, lost-sale penalty costs, and fixed and variable ordering costs. Iterative algorithms are developed to calculate the equilibrium policies and the total system costs for each information sharing scenario. The system costs for the various information sharing scenarios are analyzed to show their impact on the supply chain performance. The results provide insights into planning information sharing scenarios.

1 Model Formulation

The supply chain consists of one supplier and one retailer. Inventory is reviewed periodically and the customers' demands only occur at the retailer. The inventory is in units. Customers' demand D at the retailer in one period is a stochastic variable, with a probability mass function $f(x)=P\{D=x\}$. An outside supplier with infinite capacity replenishes goods to the supplier. The sequence of events is as follows: (1) At the end of each period, the supplier reviews its inventory quantity and places an order to the outside supplier. (2) The retailer reviews its inventory and places an order to the supplier. (3) At the beginning of the next period, a shipment arrives at the supplier. (4) If the on-hand inventory of the supplier is larger than the ordering quantity from the retailer, the supplier completely satisfies the retailer; otherwise, the supplier delivers the entire on-hand inventory to the retailer. (5) The retailer receives the shipment and the consumers express demands. There is no backlogging at either the supplier or the retailer, so unsatisfied consumer demands and retailer orders are lost in each period.

The costs are charged as follows. At the end of each period, a holding cost is charged proportional to the local on-hand inventory, where h_0 is the holding cost per unit of goods held per period at the supplier and h_1 is the holding cost per unit of goods held per period at the retailer. The penalty cost at the retailer is proportional to the number of lost sales, with p_1 being the cost per customer lost per period. The penalty cost at the supplier is proportional to the unsatisfied order quantity for the retailer, with p_0 being the cost per unsatisfied unit per period. The fixed ordering costs of the supplier and the retailer are K_0 and K_1, respectively. The unit purchasing costs of the supplier and the retailer are q_0 and q_1, respectively.

The on-hand inventory at the supplier at the observation epoch (i.e., at the end of each period) is denoted by x_0. The on-hand inventory at the retailer at the observation epoch is denoted by x_1. Assume that the state space at the supplier is limited to U_0 and that at the retailer is limited to U_1. Therefore, the maximum inventory quantity is U_0 at the supplier and U_1 at the retailer.

2 Centralized Information Sharing (CIS)

A centralized system was discussed by Clark and Scarf[1]. The major difference between the present model and Clark and Scarf's model is that fixed ordering costs are charged at both the supplier and the retailer, whereas fixed ordering costs only occur at the highest echelon in Clark and Scarf's model. The assumption of fixed ordering costs is realistic, but results in analysis difficulties. The assumption causes the model to no longer possess structural properties for optimal inventory policies.

The system was modeled as a discrete time Markov decision process (DTMDP) to obtain the optimal inventory policies. The state space of the centralized system is

$$S=\{(x_0,x_1)\mid 0\leqslant x_i\leqslant U_i, i=0,1\} \tag{1}$$

The action space of the system at state (x_0,x_1) is

$$A(x_0,x_1)=\{(a_0,a_1)\mid 0\leqslant a_i\leqslant U_i-x_i, i=0,1\} \tag{2}$$

The one-step transition probability from state (x_0,x_1) to state (x_0',x_1') with action (a_0,a_1) is

$$\begin{aligned}&P\{(x_0',x_1')\mid(x_0,x_1);(a_0,a_1)\}=\\&P\{(x_0',x_1')\mid(\tilde{x}_0,\tilde{x}_1)\}=\\&P\{x_0'\mid\tilde{x}_0\}\cdot P\{x_1'\mid\tilde{x}_1\}\end{aligned} \tag{3}$$

where $\tilde{x}_i$ $(i=0,1)$ is the state of stage i after the shipment from the supplier to the retailer and before the customer receives his order. Since $x_1\leqslant\tilde{x}_1\leqslant x_1+a_1$, then

$$P\{x_0'\mid\tilde{x}_0\}=\begin{cases}1, & \text{if } x_0'=\tilde{x}_0;\\0, & \text{else}\end{cases} \tag{4}$$

For the retailer,

$$P\{x_1'\mid\tilde{x}_1\}=\begin{cases}P\{D=\tilde{x}_1-x_1'\}, & \text{if } 0<x_1'\leqslant\tilde{x}_1;\\P\{D\geqslant\tilde{x}_1\}, & \text{if } x_1'=0;\\0, & \text{if } x_1'>\tilde{x}_1\end{cases} \tag{5}$$

The transition probability only depends on the current state for a given policy.

Denote function $1\{\cdot\}$ as an indicator function. The one-step cost is

$$\begin{aligned}&c((x_0',x_1')\mid(x_0,x_1);(a_0,a_1))=\\&c_0((x_0',x_1')\mid(x_0,x_1);(a_0,a_1))+\\&c_1((x_0',x_1')\mid(x_0,x_1);(a_0,a_1))\end{aligned} \tag{6}$$

where

$$\begin{aligned}&c_0((x_0',x_1')\mid(x_0,x_1);(a_0,a_1))=\\&h_0x_0'+p_0(a_1-\tilde{x}_1+x_1)+\\&q_0a_0+K_0\cdot 1\{a_0>0\}\end{aligned} \tag{7}$$

$$\begin{aligned}&c_1((x_0',x_1')\mid(x_0,x_1);(a_0,a_1))=\\&h_1x_1'+q_1(\tilde{x}_1-x_1)+K_1\cdot 1\{\tilde{x}_1-x_1>0\}+\\&p_1\max\{0,d-\tilde{x}_1\}\cdot 1\{x_1'=0\}\end{aligned} \tag{8}$$

The long-run average cost of the system, given policy $\varphi=\{a_0(x_0,x_1),a_1(x_0,x_1)\in A(x_0,x_1),(x_0,x_1)\in S\}$, is then given by

$$C(\varphi)=\sum_{(x_0,x_1)\in S}\pi(x_0,x_1)\cdot \mathrm{EC}((x_0,x_1);(a_0,a_1)) \tag{9}$$

where $\pi(x_0,x_1)$ is the stationary distribution of state (x_0,x_1) for policy φ and $\mathrm{EC}((x_0,x_1);(a_0,a_1))$ is the expected one-step cost at state (x_0,x_1).

$$\begin{aligned}&\mathrm{EC}((x_0,x_1);(a_0,a_1))=\\&\sum_{(x_0',x_1')\in S}P\{(x_0',x_1')\mid(x_0,x_1);(a_0,a_1)\}\bullet\\&c((x_0',x_1')\mid(x_0,x_1);(a_0,a_1))\end{aligned} \tag{10}$$

Thus the optimal policy of the centralized system is the policy that minimizes the cost in Eq. (10). According to Putterman[11], the policy can be obtained using a standard value iteration algorithm or a policy iteration algorithm.

3 Decentralized System

In a decentralized system, the supplier and the retailer make their decisions independently. The decision objective of both stages is to minimize their own expected long-run-average costs.

3.1 Full information sharing

If full information sharing is available in the system, the supplier's ordering policies and state are known to the retailer, and vice versa. Therefore, the state spaces of the supplier and the retailer are the same.

$$S_0=S_1=\{(x_0,x_1)\mid 0\leqslant x_i\leqslant U_i, i=0,1\} \tag{11}$$

$$A_0(x_0,x_1)=\{a_0(x_0,x_1)\mid 0\leqslant a_0(x_0,x_1)\leqslant U_0-x_0\} \tag{12}$$

$$A_1(x_0,x_1)=\{a_1(x_0,x_1)\mid 0\leqslant a_1(x_0,x_1)\leqslant U_1-x_1\} \tag{13}$$

Assume that the supplier adopts a stationary policy $\varphi_0=\{a_0(x_0,x_1)\in A_0(x_0,x_1),(x_0,x_1)\in S_0\}$, which is known to the retailer. At state (x_0,x_1), the retailer orders $a_1(x_0,x_1)$ and the retailer knows that the supplier will order $a_0^{\varphi_0}(x_0,x_1)$. Therefore, the one-step transition probability of the retailer from state (x_0,x_1) to state (x_0',x_1') with action $a_1(x_0,x_1)$ is determined by

$$P\left\{(x_0',x_1')\,|\,(x_0,x_1),a_1(x_0,x_1)\right\}=$$
$$\begin{cases}1\left\{x_0'=(x_0+a_0^{\varphi_0}(x_0)-a_1(x_0,x_1))^+\right\}\cdot P\left\{D=x_1+m_1-x_1'\right\}, & \text{if } 0<x_1'\leqslant x_1+m_1;\\ 1\left\{x_0'=(x_0+a_0^{\varphi_0}(x_0)-a_1(x_0,x_1))^+\right\}\cdot P\left\{D\geqslant x_1+m_1\right\}, & \text{if } x_1'=0;\\ 0, & \text{if } x_1'>x_1+m_1\end{cases}\tag{14}$$

where $m_1=\min\left\{a_1(x_0,x_1),x_0+a_0^{\varphi_0}(x_0)\right\}$. The transition probability of the retailer only depends on the current state and action.

From the definition of the DTMDP, with a given policy φ_0 of the supplier, $\left\{(x_0,x_1),a_1(x_0,x_1)\right\}$ is a DTMDP. The one-step transition cost can be calculated as discussed in Section 2. The DTMDP solution gives the retailer's policy $\varphi_1=\{a_1(x_0,x_1)\in A_1(x_0,x_1),(x_0,x_1)\in S_1\}$. Since full information sharing is available, this policy φ_1 is also known to the supplier. The one-step transition probability of the supplier from state (x_0,x_1) to state (x_0',x_1') with action $a_0(x_0,x_1)$ is given by

$$P\left\{(x_0',x_1')\,|\,(x_0,x_1),a_0(x_0,x_1)\right\}=$$
$$\begin{cases}1\left\{x_0'=(x_0+a_0(x_0)-a_1^{\varphi_1}(x_0,x_1))^+\right\}\cdot P\left\{D=x_1+m_0-x_1'\right\}, & \text{if } 0<x_1'\leqslant x_1+m_0;\\ 1\left\{x_0'=(x_0+a_0(x_0)-a_1^{\varphi_1}(x_0,x_1))^+\right\}\cdot P\left\{D\geqslant x_1+m_0\right\}, & \text{if } x_1'=0;\\ 0, & \text{if } x_1'>x_1+m_0\end{cases}\tag{15}$$

With a given policy φ_1 of the retailer, $\left\{(x_0,x_1),a_0(x_0,x_1)\right\}$ is also a DTMDP, with the solution giving a supplier's policy φ'_0. If $\{\varphi_0,\varphi_1\}$ is the equilibrium system policy, then

$$\varphi'_0=\varphi_0\tag{16}$$

An iterative algorithm can be developed to search for the equilibrium policy $\{\varphi_0,\varphi_1\}$. The algorithm is as follows.

Algorithm 1

Step 1: Set an initial supplier's policy φ_0.

Step 2: Solve the DTMDP for the retailer given φ_0 to get the retailer's policy φ_1.

Step 3: Solve the DTMDP for the supplier given φ_1 to get the supplier's policy φ'_0.

Step 4: If $\varphi_0=\varphi'_0$, stop; otherwise let φ'_0 be the new supplier's policy and go to Step 2.

3.2 Non-information sharing

This model has no information sharing in the system due to technical reasons or business confidentiality. The supplier cannot know the states and demands of the retailer, and the retailer cannot know the states and orders of the supplier. Therefore, both the supplier and the retailer can only observe their own inventory states. The state spaces and the action spaces are

$$S_0=\left\{0,1,2,\cdots,U_0\right\}\tag{17}$$

$$A_0(x_0)=\left\{0,1,2,\cdots,U_0-x_0\right\}\tag{18}$$

$$S_1=\left\{0,1,2,\cdots,U_1\right\}\tag{19}$$

$$A_1(x_1)=\left\{0,1,2,\cdots,U_1-x_1\right\}\tag{20}$$

If the retailer orders $a_1(x_1)$ from the supplier, he may receive less than $a_1(x_1)$ from the supplier. An order-delivery rate matrix, $\boldsymbol{M}$, is used to describe the relationship between the order and the delivery to the retailer,

$$\boldsymbol{M}=\begin{bmatrix}m(0,0) & & & \\ m(1,0) & m(1,1) & & \\ \vdots & \vdots & \ddots & \\ m(U_1,0) & m(U_1,1) & \cdots & m(U_1,U_1)\end{bmatrix}\tag{21}$$

where $0\leqslant m(a,b)\leqslant 1$ is the probability that the delivery to the retailer is b if he orders a, and $\sum_{b=0}^{a}m(a,b)=1$, i.e., the summation of any row in the matrix is equal to 1.

With matrix $\boldsymbol{M}$, the one-step transition probability for the retailer from state x_1 to state x_1' with action $a_1(x_1)\in A_1(x_1)$ is given by

$$P\{x_1' \mid x_1, a_1(x_1)\} = \begin{cases} \sum_{b=0}^{a_1} P\{D = x_1 + b - x_1'\} m(a_1, b), & \text{if } x_1' > 0; \\ \sum_{b=0}^{a_1} P\{D \geqslant x_1 + b\} m(a_1, b), & \text{if } x_1' = 0 \end{cases} \quad (22)$$

With a given order-delivery rate matrix $\boldsymbol{M}$, $\{x_1, a_1(x_1)\}$ is a DTMDP.

The retailer's policy $\varphi_1 = \{a_1(0), a_1(1), \cdots, a_1(U_1)\}$ is obtained by solving the DTMDP to minimize the expected long-run average cost. Denote $\Phi_1^{\varphi_1} = \{\pi_1^{\varphi_1}(0), \pi_1^{\varphi_1}(1), \cdots, \pi_1^{\varphi_1}(U_1)\}$ as the stationary distribution for policy φ_1. Furthermore, denote the order distribution of the retailer for policy φ_1 by $O_1 = \{o_1(0), o_1(1), \cdots, o_1(U_1)\}$. The order distribution can be calculated from

$$o_1(j) = \sum_{k=0}^{U_1} \pi_1^{\varphi_1}(k) \cdot 1\{a_1^{\varphi_1}(k) = j\},\ j = 0, 1, \cdots, U_1 \quad (23)$$

In the system, the retailer's order distribution is known to the supplier. With the retailer's order distribution, the one-step transition probability for the supplier from state x_0 to state x_0' with action $a_0(x_0) \in A_0(x_0)$ is given by

$$P\{x_0' \mid x_0, a_0(x_0)\} = \sum_{z=0}^{U_1} 1\{x_0' = (x_0 + a_0(x_0) - z)^+\} \cdot o_1(z) \quad (24)$$

With a given retailer's order distribution O_1, $\{x_0, a_0(x_0)\}$ is a DTMDP. The supplier's ordering policy $\varphi_0 = \{a_0(0), a_0(1), \cdots, a_0(U_0)\}$ and the stationary state distribution $\Phi_0^{\varphi_0} = \{\pi_0^{\varphi_0}(0), \pi_0^{\varphi_0}(1), \cdots, \pi_0^{\varphi_0}(U_0)\}$ for this policy are obtained by solving the DTMDP.

The order-delivery rate matrix is then updated as

$$\boldsymbol{M}' = \begin{bmatrix} m'(0,0) & & & \\ m'(1,0) & m'(1,1) & & \\ \vdots & \vdots & \ddots & \\ m'(U_1,0) & m'(U_1,1) & \cdots & m'(U_1,U_1) \end{bmatrix} \quad (25)$$

where

$$m'(a_1, b) = \sum_{x_0=0}^{U_0} 1\{b = \min[a_1, x_0 + a_0^{\varphi_0}(x_0)]\} \cdot \pi_0^*(x_0) \cdot o_1^{\varphi_1}(a_1) \quad (26)$$

that satisfies $0 \leqslant m'(a_1, b) \leqslant 1$, $\sum_{b=0}^{a_1} m'(a_1, b) = 1$.

If the system reaches equilibrium, then for all $a_1 = 0, 1, \cdots, U_1$, $b = 0, 1, \cdots, a_1$, $m'(a_1, b) = m(a_1, b)$, i.e., $\boldsymbol{M} = \boldsymbol{M}'$. Therefore, an iterative algorithm can be developed to search for the equilibrium order-delivery rate matrix which will then give the equilibrium policy $\{\varphi_0, \varphi_1\}$.

Algorithm 2

Step 1: Set initial values of $\boldsymbol{M}$ and ε.

Step 2: Solve the retailer's DTMDP to obtain the optimal policy φ_1, the stationary distribution Φ^{φ_1} and the order distribution O_1 of the retailer.

Step 3: Solve the supplier's DTMDP to obtain the optimal policy φ_0 and the stationary distribution Φ^{φ_0} of the supplier.

Step 4: Update the order-delivery rate matrix $\boldsymbol{M}'$ according to Eq. (26).

Step 5: If $\Delta(\boldsymbol{M} - \boldsymbol{M}') < \varepsilon$, stop; otherwise let $\boldsymbol{M}'$ be the new order-delivery rate matrix and return to Step 2.

In Step 5, $\Delta(\boldsymbol{M} - \boldsymbol{M}') = \| \boldsymbol{M} - \boldsymbol{M}' \|$, where $\| \boldsymbol{X} \|$ is the norm with $\max_{i,j}\{x_{ij}\} - \min_{i,j}\{x_{ij}\}$.

3.3 Supplier-dominated information sharing

In this case, the supplier knows the retailer's state at each decision epoch, but the retailer only knows its own state. Assume that the supplier also knows the consumer demands to the retailer. The retailer's state space and action space are

$$S_1 = \{0, 1, 2, \cdots, U_1\} \quad (27)$$

$$A_1(x_1) = \{0, 1, 2, \cdots, U_1 - x_1\} \quad (28)$$

As mentioned in Section 3.2, the retailer policy can be obtained from the order-delivery rate matrix if the retailer only knows its own state. Hence, the solution is similar to that in Section 3.2. However, since the supplier knows the retailer's state, the probability that the retailer orders $a_1(x_1)$ at state x_1 and receives b in the next period depends on the retailer's current state x_1. Therefore, the relationship between the order and the delivery to the retailer can be described with a vector of matrices $\vec{\boldsymbol{M}} = [\boldsymbol{M}_{x_1}], x_1 \in S_1$, where

$$\boldsymbol{M}_{x_1}=\begin{bmatrix} m_{x_1}(0,0) & & & \\ m_{x_1}(1,0) & m_{x_1}(1,1) & & \\ \vdots & \vdots & \ddots & \\ m_{x_1}(U_1-x_1,0) & m_{x_1}(U_1-x_1,1) & \cdots & m_{x_1}(U_1-x_1,U_1-x_1) \end{bmatrix} \quad (29)$$

where $m_{x_1}(a,b)$ is the probability that the retailer receives b in the next period if it orders a at state x_1. Therefore, $0 \leqslant m_{x_1}(a,b) \leqslant 1$ and $\sum_{b=0}^{a} m_{x_1}(a,b)=1$ for all $a \in A_1(x_1)$. With a given $\vec{\boldsymbol{M}}$, $\{x_1, a_1(x_1)\}$ is a DTMDP with the same solution procedure as in Section 3.2. The DTMDP solution gives the retailer's ordering policy $\varphi_1=\{a_1(0), a_1(1), \cdots, a_1(U_1)\}$ and stationary state distribution $\Phi_1^{\varphi_1}=\{\pi_1^{\varphi_1}(0), \pi_1^{\varphi_1}(1), \cdots, \pi_1^{\varphi_1}(U_1)\}$.

The state space of the supplier is two-dimensional.

$$S_0=\{(x_0,x_1) \mid 0 \leqslant x_i \leqslant U_i, i=0,1\} \quad (30)$$

$$A_0(x_0,x_1)=\{a_0(x_0,x_1) \mid 0 \leqslant a_0(x_0,x_1) \leqslant U_0-x_0\} \quad (31)$$

For the SDIS scenario, the supplier knows the retailer's ordering policy and the customer demands to the retailer. The supplier's one-step transition probability from state (x_0,x_1) to state (x_0',x_1') with action $a_0(x_0,x_1)$ can be calculated using Eq. (15). Therefore, with a given ordering policy φ_1, $\{(x_0,x_1), a_0(x_0,x_1)\}$ is a DTMDP. The supplier's ordering policy $\varphi_0=\{a_0(x_0,x_1) \in A_0(x_0,x_1), (x_0,x_1) \in S_0\}$ and stationary state distribution $\Phi_0^{\varphi_0}=\{\pi_0^{\varphi_0}(x_0,x_1), (x_0,x_1) \in S_0\}$ are obtained by solving the DTMDP. With φ_0 and $\Phi_0^{\varphi_0}$, the series of order-delivery rate matrices are updated using

$$m_{x_1}(a,b)=\begin{cases} \dfrac{\sum_{x_0=0}^{U_0} \pi_0^{\varphi_0}(x_0,x_1) \cdot 1\{x_0+a_0^{\varphi_0}(x_0,x_1)=b\}}{\sum_{x_0=0}^{U_0} \pi_0^{\varphi_0}(x_0,x_1)}, & \text{if } b<a; \\[2ex] \dfrac{\sum_{x_0=0}^{U_0} \pi_0^{\varphi_0}(x_0,x_1) \cdot 1\{x_0+a_0^{\varphi_0}(x_0,x_1)>b\}}{\sum_{x_0=0}^{U_0} \pi_0^{\varphi_0}(x_0,x_1)}, & \text{if } b=a \end{cases} \quad (32)$$

The iterative algorithm based on this analysis to search for the order-delivery rate matrices, where $\Delta(\vec{\boldsymbol{M}}-\vec{\boldsymbol{M}}')=\max_{x_1 \in S_1} \| \boldsymbol{M}_{x_1}-\boldsymbol{M}_{x_1}' \|$, is as follows.

Algorithm 3

Step 1: Set initial values of $\vec{\boldsymbol{M}}$ and ε.

Step 2: Solve the retailer's DTMDP to obtain the optimal policy φ_1 and the stationary distribution $\Phi_1^{\varphi_1}$.

Step 3: Solve the supplier's DTMDP to obtain the optimal policy φ_0 and the stationary distribution $\Phi_0^{\varphi_0}$.

Step 4: Update the order-delivery rate matrices, $\vec{\boldsymbol{M}}'$, using Eq. (32).

Step 5: If $\Delta(\vec{\boldsymbol{M}}-\vec{\boldsymbol{M}}')<\varepsilon$, stop; otherwise let $\vec{\boldsymbol{M}}'$ be the new order-delivery rate matrices and return to Step 2.

3.4 Retailer-dominated information sharing

In this case the retailer knows the supplier's state at each decision time, but the supplier only knows its own state. Therefore, the state space and action space of the supplier are the same as in case NIS.

$$S_0=\{0,1,2,\cdots,U_0\} \quad (33)$$

$$A_0(x_0)=\{0,1,2,\cdots,U_0-x_0\}. \quad (34)$$

Since the retailer knows the supplier's state, the order distribution depends on the supplier's state. Therefore, the order distribution observed by the supplier is a two-dimensional matrix

$$\boldsymbol{O}_1=\begin{bmatrix} o_1(0,0) & o_1(0,1) & \cdots & o_1(0,U_1) \\ o_1(1,0) & o_1(1,1) & \cdots & o_1(1,U_1) \\ \vdots & \vdots & \ddots & \vdots \\ o_1(U_0,0) & o_1(U_0,1) & \cdots & o_1(U_0,U_1) \end{bmatrix} \quad (35)$$

where $o_1(x_0,a_1)$ is the probability that the retailer orders a_1 if the inventory state of the supplier is x_0. Hence, for all $x_0 \in S_0$,

$$0 \leqslant o_1(x_0,a_1) \leqslant 1, \qquad \sum_{a_1=0}^{U_1} o_1(x_0,a_1)=1 \quad (36)$$

The supplier's one-step transition probability from state x_0 to state x_0' with action $a_0(x_0)$ can be expressed as

$$P\{x_0' \mid x_0, a_0(x_0)\} = \begin{cases} o_0(x_0, x_0 + a_0(x_0) - x_0'), & \text{if } x_0 + a_0(x_0) \geqslant x_0' > 0; \\ \sum_{b=x_0+a_0(x_0)}^{U_1} o_0(x_0, b), & \text{if } x_0' = 0; \\ 0, & \text{if } x_0' > x_0 + a_0(x_0) \end{cases} \quad (37)$$

Therefore, with a given retailer's order distribution matrix $\boldsymbol{O}_1$, $\{x_0, a_0(x_0)\}$ is a DTMDP. Analogously, the ordering policy $\varphi_0 = \{a_0(0), a_0(1), \cdots, a_0(U_0)\}$ and the state stationary distribution $\Phi_0^{\varphi_0} = \{\pi_0^{\varphi_0}(0), \pi_0^{\varphi_0}(1), \cdots, \pi_0^{\varphi_0}(U_0)\}$ for this policy are obtained by solving the DTMDP.

The retailer's state space and action space are

$$S_1 = \{(x_0, x_1) \mid 0 \leqslant x_i \leqslant U_i, i = 0,1\} \quad (38)$$

$$A_1(x_0, x_1) = \{0, 1, \cdots, U_1 - x_1\} \quad (39)$$

For the RDIS scenario, the retailer knows the supplier's ordering policy φ_0 and stationary state distribution $\Phi_0^{\varphi_0}$. Given policy φ_0, the retailer's one-step transition probability from state (x_0, x_1) to (x_0', x_1') with action $a_1(x_0, x_1)$ is

$$P\{(x_0', x_1') \mid (x_0, x_1), a_1(x_0, x_1)\} = \begin{cases} 1\{x_0' = (x_0 + a_0^{\varphi_0}(x_0) - a_1(x_0, x_1))^+\} \cdot P\{D = x_1 + m - x_1'\}, & \text{if } 0 < x_1' \leqslant x_1 + m; \\ 1\{x_0' = (x_0 + a_0^{\varphi_0}(x_0) - a_1(x_0, x_1))^+\} \cdot P\{D \geqslant x_1 + m\}, & \text{if } x_1' = 0; \\ 0, & \text{if } x_1' > x_1 + m \end{cases} \quad (40)$$

where $m = \min\{a_1(x_0, x_1), x_0 + a_0^{\varphi_0}(x_0)\}$.

For a given supplier's ordering policy φ_0, $\{(x_0, x_1), a_1(x_0, x_1)\}$ is a DTMDP. The DTMDP solution gives the ordering policy $\varphi_1 = \{a_1(x_0, x_1) \in A_1(x_0, x_1), (x_0, x_1) \in S_1\}$ and the stationary state distribution $\Phi_1^{\varphi_1} = \{\pi_1^{\varphi_1}(x_0, x_1), (x_0, x_1) \in S_1\}$ of the retailer. If $\{\varphi_0, \varphi_1\}$ is an equilibrium point policy, then the definition of the ordering distribution matrix, $\boldsymbol{O}_1$, leads to

$$o_1(i, j) = \frac{\sum_{b=0}^{U_1} \pi_1^{\varphi_1}(i, b) \cdot 1\{a_1^{\varphi_1}(i, b) = j\}}{\sum_{b=0}^{U_1} \pi_1^{\varphi_1}(i, b)},$$
$$\text{for all } i \in S_0, j \in A_1 \quad (41)$$

Therefore, the iterative algorithm to search for the ordering distribution matrix $\boldsymbol{O}_1$ is as follows.

Algorithm 4

Step 1: Set initial values of $\boldsymbol{O}_1$ and ε.

Step 2: Solve the supplier's DTMDP to obtain the supplier's ordering policy φ_0.

Step 3: Solve the retailer's DTMDP to obtain the retailer's ordering policy φ_1 and the stationary distribution $\Phi_1^{\varphi_1}$.

Step 4: Update the ordering distribution matrix $\boldsymbol{O}_1'$ using Eq. (41).

Step 5: If $\Delta(\boldsymbol{O}_1 - \boldsymbol{O}_1') < \varepsilon$, stop; otherwise let $\boldsymbol{O}_1'$ be the new ordering distribution matrix and return to Step 2, where $\Delta(\boldsymbol{O}_1 - \boldsymbol{O}_1') = \| \boldsymbol{O}_1 - \boldsymbol{O}_1' \|$.

4 Numerical Comparisons and Analyses

The algorithms for the single supplier-single retailer supply chains with various information sharing scenarios were developed to determine their inventory policies. The different information sharing scenarios will result in different total expected long-run-average costs of the supply chains. The algorithms were studied numerically to show the valuation of the total cost with respect to the information sharing scenarios.

The analyses calculated the equilibrium points using randomly generated groups of system parameters. The demand process at the retailer was assumed to follow a Poisson process with parameter λ. The parameters were generated within the ranges listed in Table 1.

Table 1 Parameters' range

h_0	h_1	p_0	p_1	K_0	K_1	λ
[0.1, 3.0]	[h_0, h_0+3.0]	[h_0,15h_0]	[h_1,15h_1]	[0, 60h_0]	[0, 20h_1]	[0.5, 6.5]

The other parameters were fixed for all the numerical tests.

The calculations showed that the iterative algorithms converge for most parameter groups. When the algorithms failed to converge, the results varied cyclically over a fixed set of state-action spaces. The analysis objective is the system long-run-average cost, which may reach its limit for a regular cyclic policy. None of the calculations gave divergent results.

Table 2 presents one group of parameters used in the numerical studies to calculate the total supply chain costs for each scenario. The results are listed in Table 3.

Table 2 Parameters for numerical example I

h_0	h_1	p_0	p_1	K_0	K_1	q_0	Q_1	U_0	U_1	λ	ε
2.5	3.6	8.6	16.5	36.0	2.5	1.0	1.0	19	9	4.5	1×10^{-7}

Table 3 Cost comparison for numerical example I

	Supplier's cost	Retailer's cost	Total cost
CIS	23.43	16.33	39.76
FIS	27.07	15.03	42.10
SDIS	27.55	14.74	42.29
RDIS	27.24	15.83	43.07
NIS	39.38	16.71	56.09

The numerical results show that the total cost in the centralized system is 5.9% lower than the FIS cost, 6.4% lower than the SDIS cost, 8.3% lower than the RDIS cost, and 41.1% lower than the NIS cost. Although the CIS system cost is the lowest, the CIS retailer's cost is not the lowest, but is greater than the retailer's costs with the FIS, SDIS, and RDIS scenarios in this example.

This example shows that the total system cost decreases with increasing information sharing. However, a higher information sharing level does not necessarily result in a lower system cost in a decentralized system. Table 4 shows the parameters for such an example.

Table 4 Parameters for numerical example Ⅱ

h_0	h_1	p_0	p_1	K_0	K_1	q_0	q_1	U_0	U_1	λ	ε
0.6	2.9	5.4	37.7	30.0	23.2	1.0	1.0	19	9	6.3	1×10^{-7}

In example Ⅱ the FIS system cost is higher than the SDIS system cost, as shown in Table 5. This result illustrates that information sharing does not always reduce the system cost in a decentralized system due to competition.

The various information sharing levels may also result in the same equilibrium points and the same costs. Table 6 lists the parameters for such an example.

Table 5 Cost comparison for numerical example Ⅱ

	Supplier's cost	Retailer's cost	Total cost
CIS	20.12	38.46	58.58
FIS	21.01	38.46	59.47
SDIS	21.05	38.16	59.21
RDIS	23.39	37.28	61.67
NIS	35.41	39.70	75.11

Table 6 Parameters for numerical example Ⅲ

h_0	h_1	p_0	p_1	K_0	K_1	q_0	q_1	U_0	U_1	λ	ε
2.6	3.3	31.2	16.5	30.6	25.4	1.0	1.0	19	9	5.5	1×10^{-7}

In example Ⅲ, the SDIS and FIS supply chains have the same equilibrium point, as shown in Table 7, which means that the retailer will follow the same inventory policies with the FIS and SDIS scenarios. Since the retailer's inventory policy with the SDIS scenario is a local policy (the policy only depends on the local inventory state), the retailer also follows a local inventory policy with the FIS scenario.

Table 7 Cost comparison for numerical example Ⅲ

	Supplier's cost	Retailer's cost	Total cost
CIS	22.09	35.67	57.76
FIS	24.54	34.78	59.32
SDIS	24.54	34.78	59.32
RDIS	30.66	35.42	66.08
NIS	35.60	39.45	75.05

5 Conclusions

A two-stage supply chain system with a single supplier and a single retailer was analyzed for various information sharing scenarios: centralized information sharing, full information sharing, suppliers dominated information sharing, retailer-dominated information sharing, and non information sharing. Numerical examples show that the system cost with the NIS scenario is much higher than that with the CIS scenario. Furthermore, higher information sharing levels do not always generate a lower system cost in a decentralized supply chain due to inventory competition. Depending on the system, the total cost of the FIS scenario can be larger than, equal to, or smaller than the cost of the SDIS scenario.

References

[1] Clark A, Scarf H. Optimal policies for a multi-echelon inventory problem. *Management Science*, 1960, **6**(4): 475-490.

[2] Federgruen A, Zipkin P H. Computational issues in an infinite-horizon, multiechelon inventory model. *Operations Research*, 1984, **32**(4): 818-836.

[3] Chen F. 94%-effective policies for a two-stage serial inventory system with stochastic demand. *Management Science*, 1999, **45**(12): 1679-1696.

[4] Axsater S. A framework for decentralized multi-echelon inventory control. *IIE Transactions*, 2001, **33**: 91-97.

[5] Cachon G P. Stock wars: Inventory competition in a two-echelon supply chain with multiple retailers. *Operations Research*, 2001, **49**(5): 658-674.

[6] Cachon G P, Zipkin P H. Competitive and cooperative inventory policies in a two-stage supply chain. *Management Science*, 1999, **45**(7): 936-953.

[7] Wang H W, Guo M, Efstathiou J. A game-theoretical cooperative mechanism design for a two-echelon decentralized supply chain. *European Journal of Operational Research*, 2004, **157**: 372-388.

[8] Chen F, Federgruen A, Zheng Y S. Coordination mechanisms for a distribution system with one supplier and multiple retailers. *Management Science*, 2001, **47**(5): 693-708.

[9] Lau J S K, Huang G Q, Mak K L. Impact of information sharing on inventory replenishment in divergent supply chains. *International Journal of Production Research*, 2004, **42**(5): 919-941.

[10] Feng Y, Xiao B. A new algorithm for computing optimal (*s*, *S*) policies in a stochastic single item/location inventory system. *IIE Transactions*, 2000, **32**(11): 1081-1090.

[11] Putterman M L. Markov Decision Process. New York: John Wiley & Sons, 1984.