Modeling Air Traffic Throughput and Delay with Network Cell Transmission Model

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Outline

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Background

- Evaluate tradeoff of throughput and delay in ATM
- In US, flights pushed into system & congestion is managed accordingly
  - TMIs (e.g. MIT) implemented in tactical manner
  - Potentially more throughput at cost of increased delay
- On the other hand, if flights depart only when open path is available between origin & destination
  - Potentially less delay, but at cost of lower throughput
- Desire way to model delay & throughput and tradeoff
- Inspired by the Cell Transmission Model from highway transportation

Cell Transmission Model

- Originates from highway transportation
  - Determine propagation of delays
  - Optimize metering entry of onramp traffic
- Simulate highway traffic movement

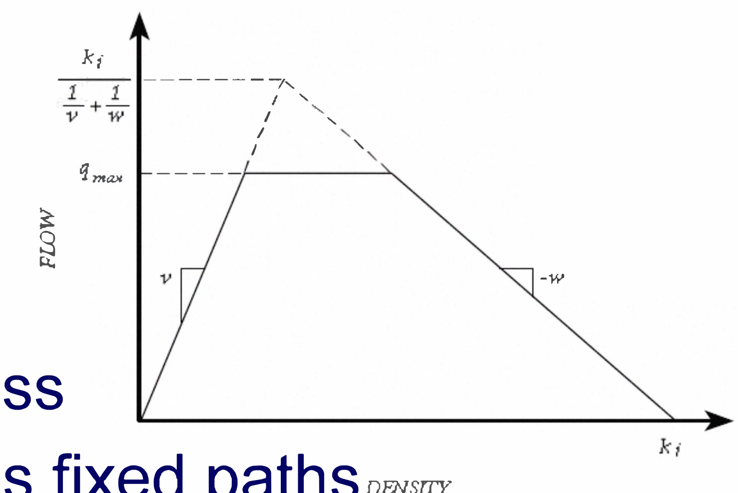
- Vehicles move from cell to cell in time increments

$$n_i(t+1) = n_i(t) + y_i(t) - y_{i+1}(t)$$

- Flow is determined by discredited version of PDEs from LWR model

$$y_i(t) = \min\left\{n_{i-1}(t), Q_i(t), N_i(t) - n_i(t)\right\}$$

- CTML model, with capacities across multiple cells, min delay only & has fixed paths



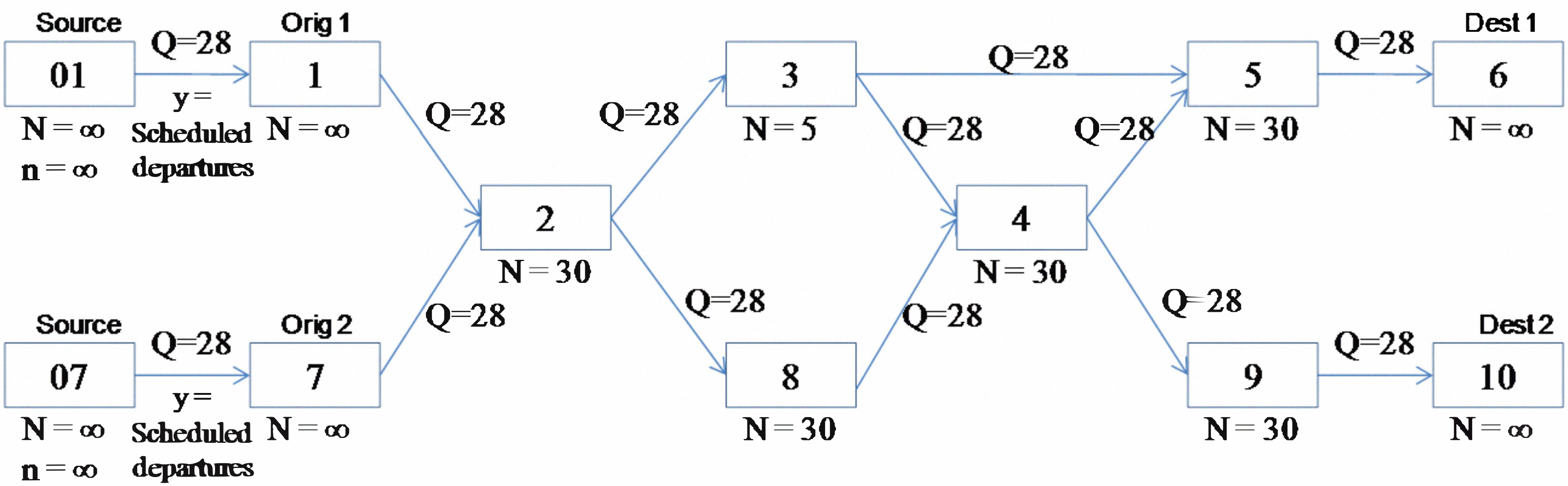
Motivation

- Model that dynamically routes traffic through congested areas and optimizes delay and throughput.
- Leverages the framework of the Cell Transmission Model, but with routing capabilities and two objectives.
- Air traffic flow models typically assume flow is steady between nodes.
  - In reality, a bottleneck can cause backups and increased miles in trail restrictions on aircraft heading to the area.
- Provides intuition into the dynamics of the congestion along each segment.
  - Where, when, how many aircraft impacted by congestion downstream, and their speed reductions or vectors are important in the management of the traffic.

Application to Aviation

- The intent is to model the aircraft flow rather than optimal ground holding strategies.
  - However, results to improve network flow will inherently lead to certain amount of flights being held on the ground in order to optimize delay and throughput in the air.
- To optimize the delay and throughput, we relax the min { } operator to allow $y_i(t)$ to take on values less than all of the terms.
  - Flights can not advance as quickly or remain on ground if there is bottleneck downstream.
- Multiobjective integer program with similar structure to a multicommodity traffic flow program.

Sample Network



- Each aircraft is destined for a particular destination airport
- May take different paths to reach the destination to avoid congestion and delays.

Sample network. A bottleneck exists in cell 3, and causes the majority of traffic to divert around that cell.

Model

$$\min \sum_{i \neq Gs} c_i \sum_{d} \sum_{t} n_i^d(t) - w_f \sum_{d} \sum_{t} \sum_{j \in R_d} y^d_{l(d),l(d)+1}(t)$$

where:

$n_i^d(t)$	num vehicles in cell *i* at time *t* going to dest *d*
$y_{i,j}^d(t)$	flow from cell *i* to *j* in time (*t*,*t*+1) destined for *d*.
c_i	cost factor for cell *i* (larger for air, less for ground cells)
w_f	weighting factor
$l(d)$	Index of the cell at destination *d*.
G_s	Set of sinks.
R_i	Set of cells from which flights can flow into cell *i*.

Model

Subject to:

$$n_i^d(t+1) = n_i^d(t) + \sum_{k \in R_i} y_{k,i}^d(t) - \sum_{j \in S_i} y_{i,j}^d(t) \qquad \forall d,i,t \qquad (1)$$

$$\sum_{j \in S_i} y_{i,j}^d(t) \le n_i^d(t) \qquad \forall d,i,t \qquad (2)$$

$$\sum_{d} n_i^d(t) \le N_i(t) \qquad \forall i,t \qquad (3)$$

$$\sum_{d} \sum_{j \in R_i} y_{j,i}^d(t) \le Q_i(t) \qquad \forall i,t \qquad (4)$$

$$n_i^d(t), y_{i,j}^d(t) \in \square$$

where:

$N_i(t)$ max num vehicles in cell *i* at time *t*

$Q_i(t)$ max flow into cell *i* from time *t* to *t* +1

R_i Set of cells from which flights can flow into cell *i*.

S_i Set of cells that flights can move to from cell *i*.

Additional Constraints

Ensure flights only go to their intended destination. Could force num of flights arriving to be the desired number of arrivals at each destination:

$$\sum_{t} n^{d}_{l(d)}(t) = A^{d} \qquad \forall d \qquad \text{(5a)}$$

(Requires enough time in model for all flights to reach destination & fixes throughput.)

Alternatively, prevent flights from going to unintended destinations:

$$\sum_{t} \sum_{h \neq d} n^{h}_{l(d)}(t) = 0 \qquad \forall d \qquad \text{(5b)}$$

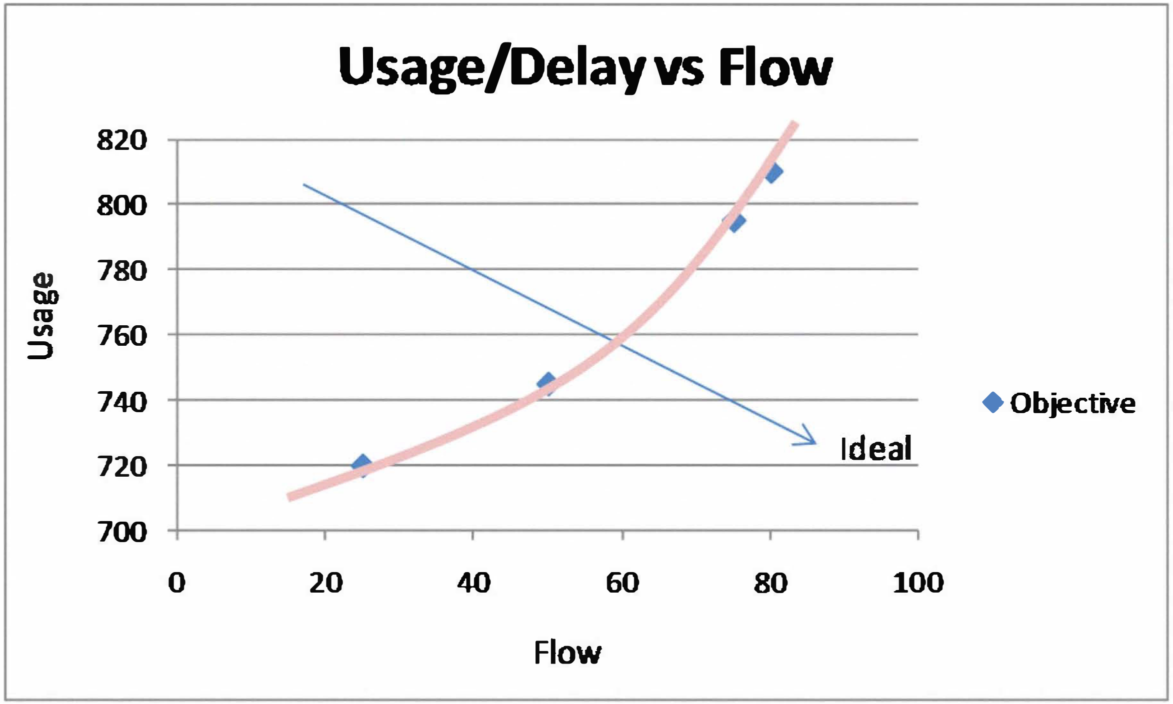
(Does not restrict all flights to actually reach the destination in the time allotted, allowing variations in throughput)

Force arrivals to move to the adjacent sink at each destination so they don't linger at the destination over time to satisfy arrival constraint.

$$y^{d}_{l(d),l(d)+1}(t) = n^{d}_{l(d)}(t) \qquad \forall d, t \qquad \text{(6)}$$

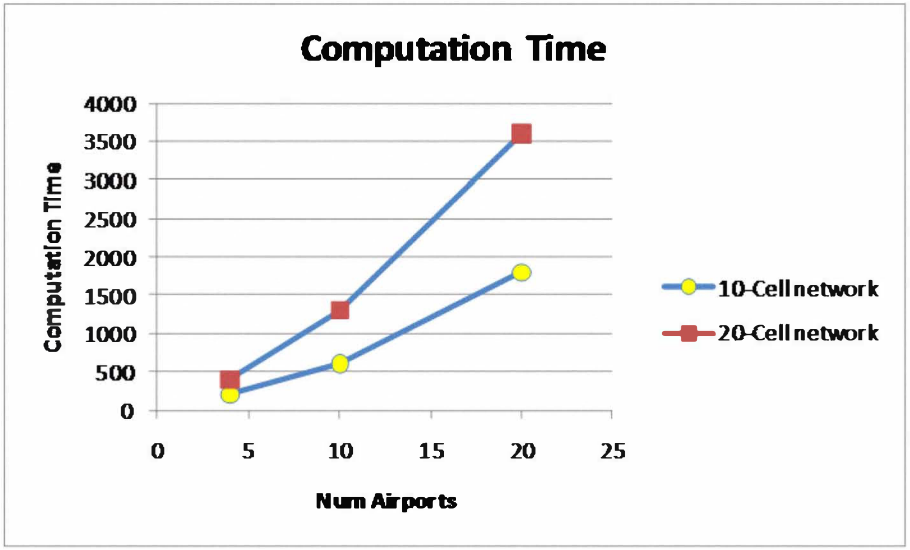
Results

- Tradeoff between delay & throughput by adjusting the weighting factor in the objective



- The “knee” in the curve provides balance between the two objectives.

Results



- Typical multicommodity flow models don't have TU constraint matrices, with the exception of those that have either two or less sources or sinks.

$$\mathbf{A} = \begin{bmatrix} \mathbf{N}_1 & \Upsilon_1 \\ \mathbf{N}_2 & \mathbf{0} \\ \mathbf{0} & \Upsilon_2 \end{bmatrix}$$

- All elements a_{ij} in **A** have $a_{ij} \in \{-1, 0, +1\}$.
- $\mathbf{N}_1$ and Υ_1 columns either have all 0's or have exactly one +1 and one -1.
- $\mathbf{N}_2$ and Υ_2 (capacities across the multiple destinations): $[\mathbf{I} \quad \mathbf{I} \quad \mathbf{I} \quad ...]$ (Does not preserve TU).

- However, this model has more structure that may lead it to yield integer solutions from the relaxation.
- Many cases so far yield integer solutions.

Summary & Future Work

- A model has been developed that models the movement of flights across a network and provides for a tradeoff between minimal delay and maximal throughput.
- Inspired by the Cell Transmission Model, this model captures the movement of aircraft from cell to cell within a network, and determines the optimal routing of aircraft through congested areas.
- Ongoing work to determine efficient integer solutions to the problem & finding cases that yield integer solutions.
- Incorporating stochasticity with inclusion of parameters to model randomness in flight departure times and its effect on delay and throughput.

Thank you

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