Input-Output Feedback Stability and Robustness, 1959-85

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The literature on input-output feedback is too large to do justice to in a single article. Here we will concentrate on the formative years ending in 1985 and leave the subsequent story to be told elsewhere.

History is not objective. To a cinema buff, the parable of the Japanese movie Rashomon comes to mind. Four people observe the same event, but their later accounts of it differ radically from each other. For observers living in various countries and at different times, the disparity in perspective can be even greater. The account that follows is written from the perspective of a viewer who was based in Cambridge, MA, in the 1970s and Montreal afterwards. It is not a comprehensive survey of the literature or even of the most important papers. Rather, it is an attempt to describe events that marked the turning points.

The period under scrutiny can be divided into roughly two parts; interest in nonlinear stability dominated the first part and robustness the second.

Stability, 1959-75

The Wiener Representation School

The input-output system theory, like so many innovations, goes back to Norbert Wiener. In the late 1950s, Wiener was working on a very general representation theory for nonlinear systems. Black boxes were to be represented by Volterra integral series, which were then decomposed into Hermite-Laguerre functionals. These functionals had a magical property of "statistical orthogonality," which enabled the decomposition coefficients to be found explicitly by an averaging process involving Gaussian noise. The method promised to solve all the problems of filtering. Wiener's longtime collaborator, Y.W. Lee, proclaimed this to be the future and assembled a group of graduate students at MIT to work out the details and find applications.

Lee had already graduated two students, Henry Singleton [1] and Amar Bose [2], whose Sc.D. theses on nonlinear filtering were widely praised. By 1957, when the author joined it, the group had greatly expanded, eventually including M. Brilliant [3], Don George [4], Irwin Jacobs, Bob Wernikoff, Harry Van Trees, and M. Schetzen [5], among others, most of whom were asked to devote at least some of their effort to the Wiener Theory.

(A systems background is evidently good preparation for business. Singleton, Bose, Jacobs, Van Trees, and Wernikoff eventually founded or became chief executives of companies—including Teledyne and its many subsidiaries, Bose Corp., Linkabit, Qualcom, and Comsat—that between them control quite a few hundreds of enterprises.)

The Shift to Analysis

The Wiener representation looked great on paper. But enthusiasm for it was tempered by the gradual realization that it required astronomical numbers of additions and multiplications. By 1960 no one had succeeded in computing even a single nontrivial example. In Wiener's time the distinction between a feasible and a computationally feasible solution had not yet gained much currency!

Anyway, it became apparent to the author that great generality and accuracy were conflicting requirements for a representation scheme, at least if complexity were to remain tractable. Schemes that sought to achieve both would remain impractical for the foreseeable future. Rather than work with representations, it seemed fruitful to concentrate on analysis of the more qualitative aspects of system behavior. For the purposes of control design, gross qualitative properties such as robustness can be analyzed and predicted without depending on accurate models or syntheses. Mathematical analysis provides topological tools that are very well suited for this purpose, such as compactness, contraction, and fixed-point methods. Furthermore, in control design, where there is lots of model uncertainty, it is often more important to be able to gauge qualitative behavior (robustness, stability, existence of oscillations) than to compute exactly.

Whether for this or other reasons, the Wiener-Lee group's research on representations and accurate optimization of nonlinear systems petered out around 1960, and analysis of feedback started taking off.

From Companding to the Small Gain Theorem

The input-output theory of nonlinear feedback began, oddly enough, with a problem in communications theory known as the companding problem. When a bandlimited signal is companded, i.e., filtered by a memoryless nonlinearity, its bandwidth usually increases. On the other hand, if the nonlinearity is invertible, the number of degrees of freedom of the signal does not increase, as the original signal can be recovered from a sampling of the companded one at the Nyquist rate. It is natural, therefore, to wonder if the extra bandwidth is redundant for the purposes of recovering the original signal. It turns out to be redundant. This was first shown in 1959 in [6]. The recovery scheme is a feedback system whose convergence and robustness properties were established using the Contraction Mapping Principle.

The feedback system used for recovery in [6] was noncausal. Nevertheless, it inspired some important ideas about feedback in general:

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- that contraction mappings were natural for the analysis of feedback robustness;
- that a new type of global linearization was possible that does not require inputs to be small, but instead is valid for nonlinearities in a sufficiently small "sector."

These ideas triggered a concentrated research effort which culminated in the 1960 report Zames [7].

[7] was the first systematic study of feedback by operator-theoretic methods. In them, systems were represented by elements of a normed algebra of input-output mappings. The term *gain*, conventionally used to describe transducer amplification, was introduced to denote the operator norm, which depends on the sector width or Lipschitz constant of the operator. For feedbacks with "loop gain less than one," the effects of open-loop perturbations (distortion, uncertainty, etc.) on closed-loop behavior were bounded. The loop gain is typically large in practice. An effort was made, therefore, to catalog the various loop transformations that could be used to reduce the gain to produce a contraction. These turned out to be combinations of fractional transformations and weightings or "multipliers."

The work in [7] had emphasized robustness. There was no awareness yet of the stability results of Popov or the Lyapunov-based results of Yakubovich and Kalman, which were being obtained around the same time or shortly after that. (At the height of the Cold War, communication across the Iron Curtain was limited.) Then, in 1963, Roger Brockett came to MIT. He had been working on state space stability. He described progress in that field and challenged the author to produce analogous stability results without using the state concept. This started a friendly and eventually very fruitful competition between input-output and state space research that has persisted to this day.

One consequence was that the small gain result was restated as a theorem on stability in 1963 [8] (using finiteness of the operator Lipshitz constant as the indicator of what became known as incremental stability). This "Small Gain Theorem," coupled with loop transformations to achieve small gain, were proposed as a general approach to the problem of nonlinear stability. Beginnings of such a general theory were apparent by the time the paper [9] appeared in 1964. Besides the author's version of the Circle Criterion for incremental stability, [9] introduced the positive-operator-multiplier method, which was used to obtain an operator-theoretic proof of Popov's result.

The great interest in stability that followed the small gain results and Brockett's related state space research had another, possibly less benign, consequence. Feedback robustness, an issue that is perhaps more important for feedback design, was pushed to the back burner, where it remained until its resurgence some 20 years later.

Early Research Groups

During the early '60s, several institutions had become involved in feedback research. Besides MIT, the groups at Bell Telephone Laboratories and University of California at Berkeley were prominent. The Companding Theorem [6] had appeared in 1959. In 1960, Landau at Bell published a similar result [10], evidently unaware of the earlier work. Landau was followed by a succession of researchers at Bell, notably Miranker, Benes, Sandberg, and later Holtzman.

Sandberg produced a succession of papers on input-output feedback beginning in 1963. He, too, started by [11] elaborating on the companding theorem of [6] and what he initially refers to as the "Landau-Miranker-Zames Theory." His versions of the Small Gain and Circle theorems were published in 1964 [12, 13].

Despite the parallel developments at MIT and Bell, there was little contact between them. A few brief interactions in 1963-64 were exceptional. There was even less interaction between researchers in the U.S. and Russia. Some years later it was learned that Tsypkin [14] had obtained another circle theorem (although by different methods and outside the context of any general input-output theory.)

Closer to home, there was substantial contact with and Narendra at Harvard and Desoer at Berkeley. Narendra obtained a state space variant of the Circle Criterion [15]. Desoer had moved earlier to Berkeley from Bell and started a research group. He and his students contributed steadily to feedback analysis [16, 17], later producing the definitive text on that subject.

The Publication Culture

Publication habits changed greatly around 1960. Until then there had been few engineering journals willing to publish theory. Editors were suspicious of mathematics. Most of the IEEE (then the IRE) society transactions did not yet exist or were new and not taken quite seriously. Institutional publications, such as the MIT Research Laboratory of Electronics "QPRs" or *Bell System Technical Journal* were widely available and played a larger role than they would now. Employees were encouraged to publish in these in-house organs, partly for proprietary reasons, but also to promote their sponsors. To take one example, Y.W. Lee actively discouraged MIT students from submitting their work outside. The best work of the period often appeared in report series or in monographs (cf. Shannon's original information theory report).

All this changed rapidly in the 60s. Journals proliferated. Journal publication became the norm.

(The older culture was not without its advantages. Information was distributed quickly; one did not spend years arguing with pesky referees; library costs were more manageable; and the system had less "noise" in it. There was a downside though. Quality and accuracy were more variable. The system favored the well-connected. In a pinch and with a little help from the boss one could get one's paper out in three months citing company advantage, bypassing referees altogether. Students were less prepared for the real world outside.)

Multipliers

The Popov criterion applies to feedback systems whose open loop consists of the product of a memoryless nonlinearity (MN) and a linear time invariant (LTI) part. The graph of the nonlinear part can lie anywhere in the first and third quadrants. Popov had obtained his criterion in 1960, by an ad hoc and not-very-transparent procedure, which fit none of the established general theories of stability, e.g., Lyapunov's method. It was difficult to see why it worked or how it could be generalized, say to other nonlinearities. For example, if the nonlinearity were monotone, would there be a similar result?

Brockett's challenge had been to find a less opaque proof and a more general theory, but up to 1964 no one, including Popov, knew how to do this. Then the "positive operator" version of stability theory provided a simple solution in the input-output framework. (State space approaches to this problem were also found and are described in the next section.)

The Positive Operator Theorem for stability was derived [9] in 1964 from the Small Gain Theorem by a fractional transformation. It ensures stability whenever the open loop can be factored into two positive operators, one of which is strongly positive. For systems involving LTI-MN pairs, factorization can be achieved if a suitable "multiplier" can be found. The multiplier is an LTI operator that is positive, and remains positive when combined with the nonlinearity. Popov's Criterion was shown [9] to be a special case in which the multiplier is first-order LTI (with Laplace transform $(s+a)^{-1}$). What is more important, the Positive Operator Theorem provided a method of finding stability conditions under a variety of assumptions about the nonlinearity, such as monotonicity, odd symmetry, etc. All one had to do was to find a multiplier that maintained positivity with the given nonlinearity. It was suggested [18] that in this way one could achieve a sequence of sufficient stability conditions, trading off properties of the nonlinearity for properties of the linear part. The search for such multipliers drew considerable attention and effort in the late '60s.

Related State Space Developments

Despite certain popular beliefs, the input-output and state space representations of systems are not interchangeable, either in theory or in practice, for reasons that are quite fundamental. One such reason is that the (approximation) neighborhood of an input-output system may be infinitely difficult to describe in state space terms, and conversely. It is precisely because problems that are difficult or impossible in one framework may be easy in the other that the jousting between them proved to be fruitful. Throughout the 1960s, the results described here were followed, and sometimes preceded, by analogous (if not quite equivalent) state space results.

Yakubovich [19] and Kalman [20] gave the first the first internal or state space description of the external property of positivity. Brockett and (Jacques) Willems [21], who were seeking state space approaches to Popov and multiplier problems, devised a method based on path integrals (along trajectories.)

The approach of factoring the open loop into positive operators originated in the input output theory, where it is a natural extension of the purely external idea of a phase shift of 180°. It was far from clear how to describe this in terms of a state representation. Eventually, Popov [22] abandoned the attempt and adopted the essentially external strategy of factoring into two positive (albeit state-represented) systems, changing only the name of the method to "hyperstability." Yakubovich took a more basic path and went on to develop the theory of linear matrix inequalities, or LMIs. This theory elegantly incorporates many of the multiplier results, although under somewhat more restrictive assumptions. (As we have pointed out, there has been a revival of interest in LMIs recently.) Jan Willems, who had worked extensively with both the state space and input output representations [23], sought to link them. In his work on dissipative systems [24], he related the internal property of energy storage to external energy, dissipation. Apparently inspired by Darlington synthesis, his work foreshadowed some of the current approaches to nonlinear H^∞ synthesis.

Maturation

Toward the end of the '60s, nonlinear input-output stability had become a specialty field actively followed by a small band of aficionados. Two publishing events served to enlarge the audience. One was Willems' monograph outline of the area [25]. The other was Desoer and Vidyasagar's textbook [26] based on courses Desoer had taught at Berkeley. These books made the subject accessible to graduate students and spawned courses on it at many institutions. Both books dealt mainly with feedback stability. Surprisingly, neither devoted any space to nonlinear distortion or robustness, despite the engineering importance of these topics.

During the 1970s, a spate of publications on "large systems" appeared. These dealt with the stability of complex interconnections of devices and were essentially reinterpretations of earlier results. Although some were quite imaginative, on the whole they were more suggestive than substantial. In spite of this burst of activity, research on input-output methods gradually declined. There was only so much interest in analytical methods in what is essentially a synthesis field. Interest did not revive until the end of the '70s decade, when H^∞ provided synthesis tools for robustness optimization.

Robustness

Stabilization is seldom the main objective of using feedback. By the 1970s, it was apparent that if input-output theory was to provide more than entertainment for academics it would have to tackle the harder issues of robustness, feedback performance, and synthesis. Robustness and uncertainty reduction had provided the initial impetus for the Small Gain theory [7, 8], but these issues had lain dormant during the '70s and '80s, with a couple of exceptions that are worth mentioning.

One was Horowitz. He deserves credit for his early emphasis on frequency domain robustness in his 1963 book [27]. However, his work had some serious flaws. His notion of a band of uncertainty was based entirely on frequency response magnitude and completely disregarded phase. Relying on simulation unsupported by analysis, he insisted that phase uncertainty was unimportant in practice and that right half plane zeros imposed no limitations on performance (at that point the small gain robustness results were available, and made it clear that an H^∞ ball of uncertainty is equivalent to a band in the *complex* frequency response). These views he maintained right up to the '80s.

Another exception was Youla et al. [28], following up on the earlier results of Newton, Gould, and Kaiser, who nicely solved the problem of minimizing sensitivity of feedback systems to additive disturbances. Their solutions were based on Wiener quadratic (i.e., L^2) filtering which had been devised for communications rather than control problems. The solutions were well-behaved so long as noises were stationary and there was no plant uncertainty, but could be too sensitive to plant perturbations. Often they were less effective than empirically designed classical compensators, although there were attempts to patch up the L^2 solutions, e.g., by constraints on gain margin (Safonov and Athans [29].)

These exceptions left unanswered the puzzling question: What in mathematical terms is the objective of classical frequency domain design? What criterion is being optimized? In the search for an answer, it was natural to start where robustness analysis had left off a decade earlier, namely with the Small Gain theory. There, systems had been represented as elements of a normed algebra of input-output maps equipped with an operator norm. The interconnection properties of systems could be expressed simply in such an algebra. It was plausible that the shortcomings of L^2 were tied to its not being an algebra and its lacking the multiplicative properties needed to describe cascade products of systems. But normed algebras of operators presented their own difficulties. For LTI systems, only two such algebras were readily available, consisting of systems either with H^∞ frequency responses or L^1 impulse responses. In 1976 not much was known about either. Few mathematicians and even fewer engineers were aware of the existence of an H^∞ optimization theory. (Youla and Saito had used the Pick algorithm to construct a positive real function satisfying a prescribed set of half plane constraints, but these circuit theory results were in a different context, and unknown to most control theorists.) Another apparent stumbling block was that it was known that sensitivity could not be reduced by feedback at some frequencies without being increased at others, as Horowitz had pointed out [27]. This seemed to imply that sensitivity could not be reduced in the $H\infty$ operator norm.

Nevertheless, in 1976 Zames [30], relying on abstract reasoning, and noting that this stumbling block could be overcome by introducing a frequency weighting, proposed that the objective of feedback robustness design could be captured in terms of the minimization of sensitivity in a weighted operator norm such as H^∞ or L^1. At the time this was thought to be a speculative undertaking, requiring the creation of new mathematics and with no assurance of eventual success.

The outlook changed shortly afterwards following a chance meeting with Bill Helton at a conference. At that point the sensitivity minimization problem had been reduced to an interpolation problem, namely that of finding the smallest $H\infty$ function satisfying a set of right half plane interpolation constraints. Asked for ideas about how to solve such a problem, Helton pointed to the (then not well known) results of Pick and Nevanlinna, who had solved the scalar interpolation problem around World War I. This meant that the sensitivity minimization problem admitted explicit and even closed form solutions! It opened the path to a practical theory of robustness optimization. The broad outlines of such a theory were presented in a 1979 paper [31] which attracted unprecedented attention and enthusiasm, among others from Bruce Francis.

Francis coauthored the next few articles. The first of these provided the full solution to the scalar sensitivity minimization problem [32] in 1981. It was presented at a NATO Lecture Series that stands out as a memorable event of that period. The lectures were organized by Honeywell, represented by John Doyle and Gunther Stein. They brought together leading researchers in multivariable control, who spent two weeks traveling together through Turkey, Holland, and Norway, presenting their pet theories and arguing passionately in between.

The papers [30, 31, 32] were the first ones to claim that robustness was a quantity that could be optimized. For a plant without sensor noise, the optimal robustness was shown to be equal to the optimal weighted H^∞ sensitivity, which was proposed as a measure of feedback performance and denoted by the letter μ.

μ^Z and μ^D

A multivariable plant lying in an ε-ball or band in H^∞ has an error vector which is arbitrary as to direction. This may be overstating the error, e.g., in problems that are constrained because some directions may correspond to node pairs between which there is no transmission, and therefore no error. Doyle [33] and Safonov [34] singled out such optimization constraints on uncertainty for special attention, though others thought that they were not particularly different from an endless list of possible constraints found in practice. At the 1981 NATO lectures, Doyle argued that block-diagonally constrained plant perturbations were especially important, speculating that instabilities produced by them might act like generalized spectra, and might be intrinsic system characteristics.

At this time he still had no notion of optimality. Later [35], he introduced a measure of optimal robustness for which he also used the symbol μ (thereby creating some confusion with the earlier measure of optimal sensitivity. To distinguish them, let us call his μ^D.) Subsequently he introduced a formulation of the robustness problem, involving block-diagonal perturbations and using the Small Gain idea to estimate μ^D, which, he argued, was general enough to include all others including μ^Z.

The practical value of computing μ^D subject to block diagonal constraints was quickly accepted. However, the theoretical merits of the μ^D formulation became an object of controversy, which has persisted, roughly for the following reasons:

- Numerical methods of computing μ^D were proposed for specific problems, but no general method of proving their convergence was found. Some cases led to nonconvex optimizations whose numerical solution was a chance matter.
- The speculation that μ^D might have intrinsic analytical properties remained unsubstantiated.
- The problem of optimizing robust performance, as opposed to finding a specified level of suboptimality, does not appear to fit nicely into the small gain formulation.

At this stage it is not clear in which way the block diagonal constraint is special as compared to the many other constraints encountered in design.

Multivariable and Multi-Block Optimization

The Pick algorithm for the scalar case was decades old, but the theory of multivariable H^∞ optimization had been completed by Adamjan, Arov, Krein, and Sarason as recently as 1970, and there were no ready-made methods for computation. Helton approached Francis and Zames at the '81 CDC and proposed a collaboration to develop computation tools. A starting point was Ball and Helton's reformulation of interpolation involving the use of the Krein indefinite inner product. A very general computable solution was constructed making use of a basis of "Krein-orthogonal" vectors. The results were published in [36], together with a different approach for the infinite dimensional case by Chang and Pearson [37]. The Ball-Helton-Krein-Space approach is much easier to compute than it is to explain. It was used extensively in Francis' monograph [38].

Feedback sensor noise complicates the H^∞ optimization problem. If one is willing to make the simplifying assumption that disturbances and sensor noise are orthogonal in L^2, the result is a so-called two-block problem involving a mix of Hankel and

Toeplitz operators. Scalar case solutions to the two- block problem were given by Jonkheere and Verma in [39], based on Verma's Ph.D. thesis, and by Kwakernaak [40].

Robust Stabilization and Related Metrics

An early control application of Pick-Nevanlinna interpolation was Allen Tannenbaum's solution [41] to the problem of optimizing gain margin, which measures the largest plant perturbations that can be tolerated without destroying stability. Maintaining stability is a weaker measure of feedback performance than weighted sensitivity (to perturbations.) Optimization of gain margin or "robust stabilization" leads to correspondingly weaker conditions and can be achieved without recourse to weighting. An elegant solution to the problem of robust stabilization in terms of noneuclidean metrics was provided by Khargonekar and Tannenbaum [42]. Vidyasagar and Kimura [43], building on an earlier result of Kimura's, succeeded in calculating the exact radius of coprime perturbations that would not destroy stability, using H^∞ optimization.

The operator norm is well defined for stable systems and therefore for stabilized closed-loops, but something else is needed for unstable plants. Francis and Vidyasagar [44] proposed a metric on possibly unstable but stabilizable plants. They defined it in terms of the operator norms of the factors appearing in a coprime factorization of the plant. Zames and El Sakkary [45] in 1980 observed that a metric based on the concept of gap between systems could be defined directly in terms of the input-output graph of the system. This had the advantage of not depending on the particular coprime factorization used.

(The development of unstable system metrics accelerated after 1985 and is outside the scope of this article. We briefly note that Georgiou and Smith later succeeded in optimizing robustness in the gap metric by H^∞ methods; Vidyasagar introduced an alternative metric based on the graph which, however, turned out to be difficult to compute; and Vinnicombe found a frequency-domain metric equivalent to the gap metric which is advantageous in optimization problems posed in that domain.)

Links with Identification and Complexity

Feedback makes it possible to control a system without having a good plant model, but it is impossible to design the feedback without a model. How much of a model is needed? This is a conundrum which neither classical nor state space control could shed much light on. It provided a large part of the initial motivation to construct an H^∞ feedback theory.

The 1976 brief [30] proposed to resolve this perplexing issue by representing the model uncertainty as an H^∞ ball, calculating feedback performance as a function of the radius of the ball, and showing the function to be monotone: the smaller the uncertainty the better the performance. The optimal reduction of uncertainty by feedback can be viewed as a feedback performance measure, and would itself be a function of the initial uncertainty.

These ideas can be stated in another way. When feedback reduces model uncertainty, it reduces the difficulty of plant identification needed to control to a given tolerance. It was pointed out [30] that the difficulty of identification could be measured by a measure of metric complexity or entropy. Thus, feedback is an agent for the reduction of metric entropy! Furthermore, such a reduction of entropy could be viewed as the purpose of a large class of hierarchical feedback organizations, and could provide a powerful tool for understanding their structure.

However, all these goodies depended on being able to compute performance as a function of uncertainty. This could not be done without filling many gaps in optimization theory. There was little progress in filling these gaps until recently.

Epilogue

After 1985, a new generation of bright, mathematically proficient researchers entered the field. There was an explosion of feedback research, too great to attempt to summarize in this short article. The history of that period is best left for a sequel. However, we would like to comment on where matters stand in relation to the research problems started in the 1970s and 1980s. Most have remained unsolved.

The avowed purpose of H^∞ control was to find accurate ways of minimizing the effects of plant uncertainty. The early results outlined in 1979-81 [31, 32] were restricted to the limiting cases of small uncertainty. It was initially thought that the two-block results might give approximate solutions for large uncertainty. Francis [46] suggested that under certain conditions these approximations might be accurate to within a factor of $\sqrt{2}$. It turns out, however, that the requisite conditions are often violated in practice, in which case the approximations may be infinitely poor. Indeed, problems of large uncertainty usually lead to nonstandard H^∞ problems, such as the two-disc problem. These remained largely untouched until very recently.

In fact, most of the H^∞ efforts after 1985 were devoted to finding optimal controllers of least dimension for state space represented plants. Such controllers succeed in minimizing sensitivity to additive noise when the noise generator is uncertain to within an H^∞ tolerance. However, (bearing in mind the limitations of the two-block approach) they do not incisively treat the basic issues underlying the minimization of sensitivity to large plant perturbations. Their minimality has little practical significance, as a nominal plant of a given order may, in an arbitrarily small neighborhood, contain plant models of much lower order. To overcome this shortcoming, various "order reduction" methods have been proposed, usually relying on elimination of small modes. However, these are invalid unless the plant response consists mostly of dominant modes to begin with. Empirical or approximate methods of dealing with large plant uncertainty have been suggested which purport to be "practical" even if not fully supported by theory. But the elaborate machinery and complex solutions of H^∞ are hardly justified by more rules of thumb; classical control gave us enough of these, and far more simply.

Very recently there have been efforts to address the outstanding issues of plant uncertainty, such as the two-disc or model matching problems. After a two-decades long pause, there is renewed interest in the links between identification and complexity, which turn out to fit very neatly with H^∞. The beginnings of a complexity-based theory of adaptive control are on the horizon.

But we would conclude by arguing that, for the most part, the large volume of research conducted since 1985 has skirted the difficult questions that H^∞ was supposed to answer. Most remain to be answered. There is still much life in the area!

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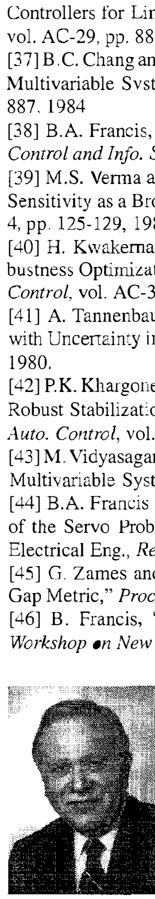

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