instrumentationnotes

Andrzej Michalski

The Accuracy of the Global Positioning Systems

by Jacek Czajewski

In May 2000, the United States stopped encrypting the precise signal (which was previously available only to the U.S. military) received from a satellite navigation system named the global positioning system (GPS). A receiver could use the signal to compute geographical position. This made the precise signal available for all civil users. Before then, the only signal available was a much less precise, degraded one. This presents a great opportunity to discuss the accuracy of GPS. First, however, I should explain its principle of operation.

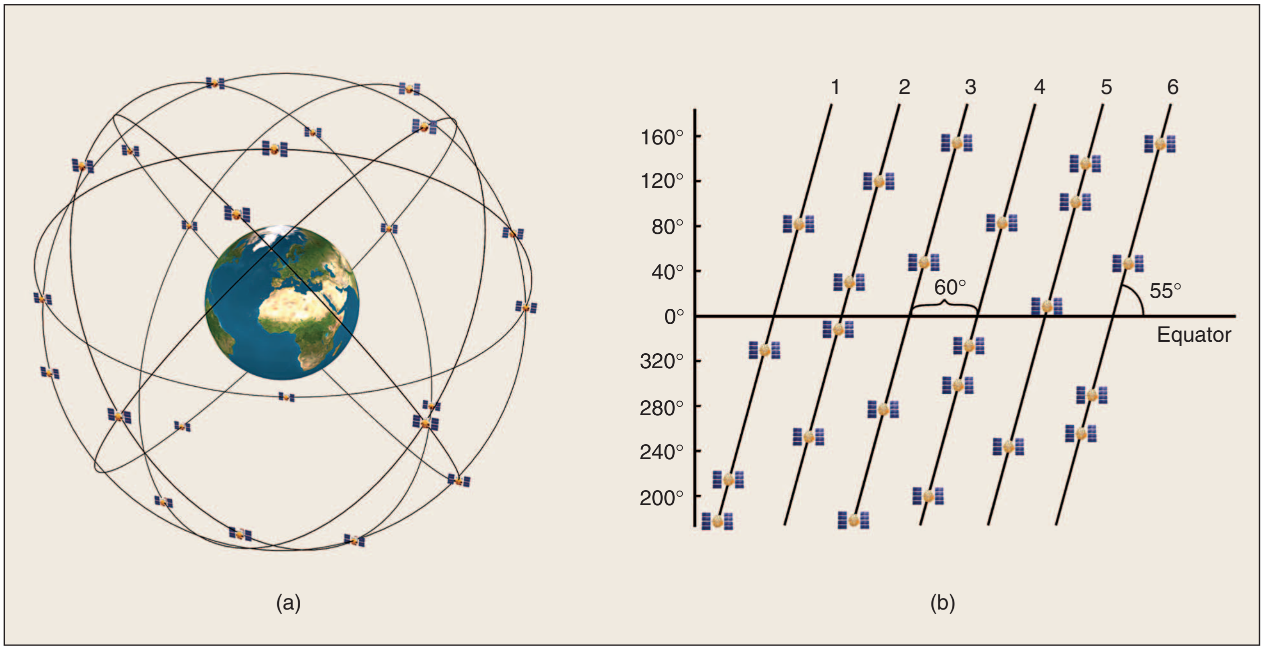
The GPS

The first satellite of the GPS was launched into orbit in June 1977. The aim of the GPS was to replace the old, inaccurate, and inconvenient system of satellite navigation called TRANSIT, which was put into operation in 1959 to control Polaris rockets. In March 1994, the U.S. Department of Defense announced preliminary permission to use the GPS, and it was declared fully operational on 17 July 1995.

The GPS constellation consists of 24 solar-powered satellites (including three spare satellites) that orbit the earth in 12 hours. They are equally spaced (60° apart) on six circular orbits about 20,183 km above the earth's surface, and inclined at about 55° with respect to the equatorial plane (Figure 1).

This constellation allows concurrent communication with at least four satellites at any one time, which enables the assessment of two coordinates (latitude and longitude—for sea vessels and land vehicles) or three coordinates (additionally altitude—in case of aircraft). It also assesses the accuracy of this estimation.

In practice, GPS receivers located between 80 °N and 80 °S that are not occluded by cliffs, mountains, skyscrapers, etc. can simultaneously receive signals from five to nine satellites (at least six satellites were always seen in the field of view of the receiver's antenna during this research). The signal transmitted by a satellite comprises its identification code, a GPS time stamp, and location information (including location of other satellites). The receiver ana-



(a)

(b)

Fig. 1. GPS constellations.

1094-6969/04/$20.00©2004IEEE

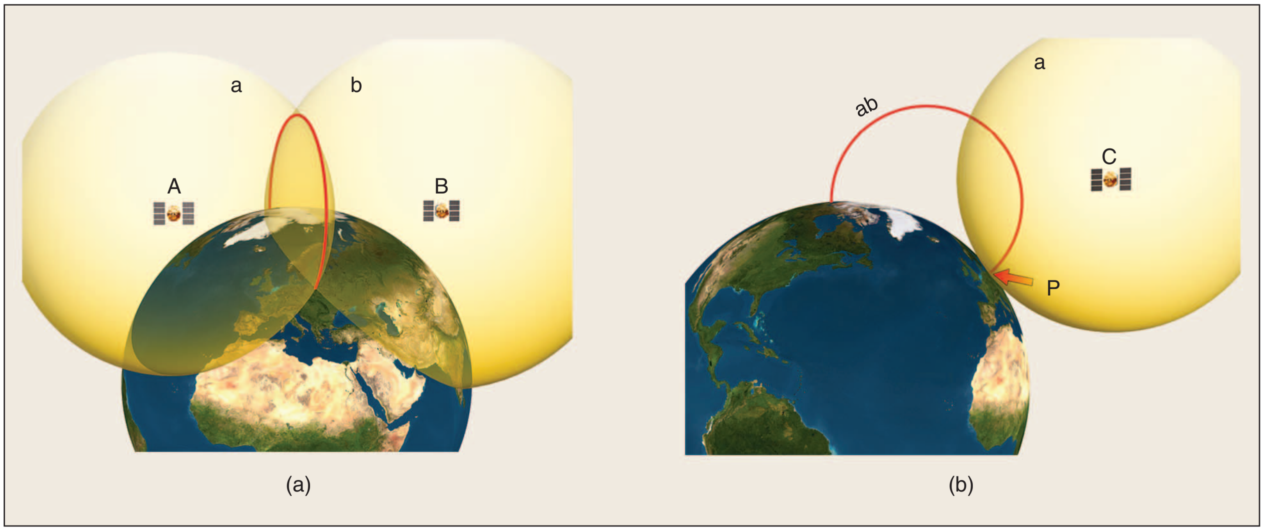


Fig. 2. An illustration of geographical position calculation using the GPS system (a) the first step uses two satellites, *A* and *B*, (b) the second step uses satellite *C*.

lyzes the data. It rejects records from satellites that are less than 10 to 15° above the horizon and then uses the transmission from the satellites in the optimal geometric configuration to calculate the maximum positioning accuracy. The receiver then displays the result as geographical coordinates.

The geographical position calculation is based on the precise measurement of a distance between a GPS receiver and the satellites. The locations of the satellites are constantly tracked by ground stations and are known exactly at all times. The distance measurement is based upon the accurate measurement of intervals between the signal transmission from a satellite and its receipt by a receiver, which is installed, for example, on a ship or a car whose position is being calculated. Each satellite of the system is equipped with four atomic clocks, which automatically correct the quartz clocks of GPS receivers.

Measuring the distance from one satellite [for example, *A* in Figure 2(a)] practically equals finding a spherical position surface [*a* in Figure 2(a)], which contains points equidistant from the satellite (the satellite becomes the center of the sphere). It is obvious that the GPS receiver and the ship equipped with it are somewhere on that sphere, but their precise location is not known. Concurrent measurement of a distance from the second satellite [for example, *B* in Figure 2(a)] determines a second position sphere.

The intersection of these two spheres yields a circular position line [*ab* in Figure 2(b)], but still a single position point on this line is unknown. The measurement of the distance from the third satellite [for example, *C* in Figure 2(b)] and the intersection of the circular position line *ab* and the third sphere [for example, *c* in Figure 2(b)] produces two points. One of these points [for example, *P* in Figure 2(b)] is a geographical position of the ship. The other one is so far away from the previously calculated position (say, on another hemisphere) that it is obvious which point should be eliminated from further calculations. A fourth distance measurement from yet another satellite could be used to eliminate the above ambiguity, but its primary role is to assess the precision of the position calculation.

The fourth position sphere (*d* in Figure 3) should intersect the other spheres exactly at the same point *P*. Due to inaccuracy of measurements, however, it will actually set another intersection point close to the previously calculated position. Thus, four surfaces yield three position lines that do not cross at one point, but form a triangle of error (*PQR* triangle in Figure 3). The observed geographical position is usually (but not always) assumed at the center of the triangle, while its size corresponds to the accuracy of the assessment. In maritime navigation, one could actually replace one of the position surfaces with the earth's surface, if it would be the true ball. Nevertheless, it is impossible to do so for aircraft, where the altitude is also important. Similarly, for land vehicles moving at different sea levels, reducing the number of satellites would cause additional errors.

Until May 2000, two GPS positioning services of different precision were

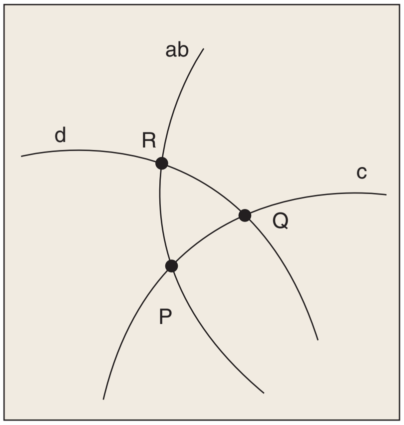


Fig. 3. GPS—triangle of error.

available. The more-accurate one, called Precise Positioning Service (PPS), was based on an encrypted signal called P-Code ("protected" or "precise"), transmitted at a carrier frequency of 1,227.60 MHz. The position estimation error was not greater than ±18 m horizontally and ±23 m vertically [3] (or even 5–10 m, according to [6]). PPS was available only for the U.S. military. The less-precise service, called Standard Positioning Service (SPS), was available for all civil users worldwide without charge or restrictions. A C/A Code ("coarse acquisition") was transmitted from a satellite at 1,575.42 MHz. The accuracy of PPS (estimated differently in various studies) was intentionally degraded by the U.S. Department of Defense.

An abundance of error estimation methods for satellite navigation systems, in general, makes their comparison and evaluation difficult. Some sources cite root-mean-squared (RMS) error of consecutive measured positions; others rely only on selective measurements in a given period (e.g., executed twice every hour within two days). Moreover, this error is often calculated with a different confidence level. It may refer to the real receiver's position (from a chart) or to the average position of all the conducted measurements. The error values are given as inaccuracy in latitude and longitude or as a distance between real and determined position. These results are also blurred by discrepancies concerning different receivers and regions of measurement.

According to the U.S. Department of Defense, the SPS horizontal accuracy is ±100 m at 99.5% confidence level and ±300 m at 99.9% [6].

The main component of GPS positioning error results from time- and space-varying conditions of radiowave propagation, which depend on atmospherics, disturbances in the satellite constellation, and orbit stability, among other things. The error caused by the transmitter and receiver operation precision or time-measurement accuracy is negligible. For instance, in [7] the error of distance measurement from a motionless receiver to a satellite, verified by a laser method, was as small as ±1 mm. Therefore, a statistical approach to stationary receiver position measurement gives very promising results. A very large number of measurements minimize the error and the mean value converges to the real one. As shown in [6], the mean position of 100 measurements performed within a week was less than 9 m apart from the real position of the receiver, while another 100 measurements conducted in 60-s intervals yielded practically perfect results. Obviously, such high precision is not achievable for moving receivers, and one should be prepared for errors of hundreds of meters when using classical GPS. In most cases, this accuracy is more than sufficient; there are many real-life situations (excluding military applications), however, where higher precision is required (e.g., navigation in a confined space at poor visibility, hydrographical tasks, or even finding a car in a large supermarket parking lot).

Observations of position estimation precision conducted by the author in August 1999 on the research vessel r/s Oceania (property of the Institute of Oceanography of the Polish Academy of Sciences) at a landing pier in Gdańsk showed that the maximum position inaccuracy was ±0.1 nautical mile (Nm) latitude and ±0.2 Nm longitude, which amounts to ±185 m and ±216 m, respectively.

The Differential Global Positioning System

A method for GPS precision enhancement commonly used by civil users is the Differential Global Positioning System (DGPS.) It increases the precision of position estimation locally in harbor approach areas, for example. It is based on additional GPS measurements of a precise position from a nearby correcting station. The error, which is characteristic for a large region around the station, is radio transmitted and taken into account by DGPS receivers as a position correction (it is automatically added with the opposite sign to the current position). It is transmitted at the radio beacon frequency band 285 to 315 kHz, at radio frequencies of 1.6 to 4 MHz, at UHF above 30 MHz, and from the Inmarsat telecommunication satellite at 1.5 GHz.

Precision estimation research was conducted in 1994 in the Gulf of Gdańsk in Poland with various receivers [8]. According to the results of DGPS on land, at sea, in motion, and motionless, the error oscillated between 1 and 10 m. Irish Internet navigation services report that they attempt to keep this error below 10 m on Irish coastal waters, while British services assess it at around 5 m.

Observations of position estimation precision conducted by the author in August 1999 on Oceania at a landing pier in Gdańsk showed that the differences in position assessment did not exceed $\Delta\phi \cong \pm 13$ m of latitude and $\Delta\lambda \cong \pm 6.5$ Nm of longitude within a few hours. The real vessel's position was within this error margin. It's worth mentioning that differences of latitude and longitude estimation between GPS and DGPS (for the same position and time) were in the range of ±0.1 Nm and ±0.2 Nm, or around 185 m and 216 m, respectively.

Other Satellite Navigation Systems and Their Accuracy

The former U.S.S.R. developed a GPS-like system called GLONASS (Global Navigation Satellite System). They launched the first of its 24 satellites into orbit in 1982 and expanded the system for a dozen years. GLONASS was not meant for civil users and, thus, it featured only one precision level almost as good as the PPS of GPS. Maximum errors of 559 position measurements performed in October 1995, when only 16 satellites were in service, did not exceed 30 m meridian-wise and 60 m parallel-wise. This high precision makes GLONASS particularly interesting for civil users; however certain technical differences (e.g., operating frequencies and time measurement method) make the double-system receivers complicated and expensive.

There are also quasinavigation satellite systems designed for Search and Rescue (SAR) operations. They receive signals from emergency radio buoys, determine their position, and transmit collected information to ground SAR services. Radio buoys are activated automatically when put into water and transmit signals to satellites at frequencies of 406 to 406.1 MHz and 121.5 MHz to aircraft. The SAR system consists of two satellite systems: COSPAS, started by the former U.S.S.R. and the western (USA, Canada, Great Britain, France) SARSAT. COSPAS has two satellites that orbit the earth in 105 minutes ca. 999 km above its surface. Their orbits are perpendicular to each other and are inclined at about 83° with respect to the equatorial plane. The constellation of SARSAT is quite similar, but the satellites' altitude is 850 km, inclination 99°, and orbiting period 100 min.

Sometimes similar quasinavigation satellite systems are set up for short-term military or civil purposes. A good example is ARGOS, which was used in 1980 during transatlantic single-hand regattas for tracking yachts in order to find them easily in case of an emergency. The yachts were equipped with devices that automatically transmitted signals every minute for a period of one second, which were received by two satellites of the system. Signal travel time from the transmitter to the satellite in any given moment and time of emission was recorded by the satellite and transferred to ground computing centers during a pass over them. The yachts' positions were calculated there 14 times a day with accuracy of ca. 0.5 Nm.

The European Union is planning to set up its own satellite navigation system called "Galileo."

Actual Accuracy of GPS and DGPS

The author carried out the first observations of the GPS PPS accuracy just two weeks after PPS was made available to civil users. The experiments were conducted with a Geonav LCD11 and a computerized GPS receiver equipped with electronic charts manufactured by Navionics, an Italian company.

Tests conducted during stopovers at harbors outside DGPS range, Tromso in Norway and Longyearbyen, Ny-Alesund on Spitsbergen, proved maximum errors (converted to meters) in the range of $\Delta\varphi = \pm 4.7$–10.2 m for latitude and $\Delta\lambda = \pm 3.0$–11.5 m for longitude. Measurements made with DGPS in Wilhelmshaven showed that the maximum inaccuracy is $\Delta\varphi = \pm 3.7$ m, $\Delta\lambda = \pm 4.8$ m. Subsequent tests, in other havens and using other receivers, confirmed these results.

Error Estimation

Since there was no possibility of automatic calculation of the mean value of latitude and longitude, a series of positions acquired during several (up to a few dozen) hours was recorded, as points on the electronic chart. They formed a typical Gaussian distribution as they were symmetrically distributed around a central point (the farther from that point, the sparser the distribution and no points outside of a clearly marked outline). Thus, maximum latitude and longitude differences in this area were determined and, assuming that the real position falls in the center of the area, the error, with the ± sign, was defined as half of these differences converted to meters.

Conclusions

In certain cases, due to such high precision of geographical position estimation, there problems connected with the accuracy of systems for the projection of the globe convex surface to the two-dimensional surface of a chart emerge. When using DGPS, one should pay closer attention if the chart datum is compliant with GPS. Otherwise, the differences between GPS and the chart can amount to 0.01 Nm or 18.5 m (according to other sources up to 100 m). This small difference is more than enough to run a ship onto rocks or shoals.

Although DGPS can increase the precision of the SPS several times, the accuracy improvement of the PPS is very limited. Thus, further support of DGPS stations may be economically unjustified, except for hydrographical tasks. This may lead to their cancellation, as it is gradually happening to maritime radio beacons and had already happen to such radio-navigation systems as Consol, Omega, and Decca.

Satellite navigation systems used for nonmilitary purposes found their application mainly in maritime and aerial navigation; however, recently they started to invade inland applications. Expensive cars are being equipped with such systems, which transmit signals from remotely controlled transmitters hidden in vehicles in case of theft. Today, a GPS receiver's display can replace standard road or city maps as is already happening in maritime navigation, where classic, paper nautical charts, light and navigation sign lists, and tide tables are replaced by computers plotting the vessel's position on an electronic chart with user-selected scale, level of detail, and enabling precise distance and course measurement. Today, one can buy an inexpensive GPS or DGPS receiver capable of displaying position and traveled distance on a map of any selected region, in selected scale and with a variety of useful information such as location of hotels, restaurants, museums, or gas stations. Many developed countries are beginning to implement GPS receivers equipped with an interactive vehicle communication system that connects them with traffic control centers.

Acknowledgment

The author would like to thank Dr. Witold Czajewski, who created the figures used in this article.

References

[1] J. Czajewski, *Nawigacja dla ˙zeglarzy* (in Polish). Almapress., wyd. III, Warszawa 2002.

[2] J. Czajewski, *Radionawigacja* (in Polish). W.S. i T., wyd. II, Warszawa, 1987.

[3] J. Czajewski, *Systemy nawigacji satelitarnej i ich dokladność* (in Polish). Pomiary, Automatyka, Kontrola nr 9/1993.

[4] J. Czajewski, "GPS—Global Positioning System," (in Polish). Materialy XXXII Międzyucze-lnianej Konferencji Metrologów t. 1., Oficyna Wydawnicza Politechniki Rzeszowskiej, Rzeszów 2000.

[5] J. Czajewski, "GPS—Global Positioning System," presented at IV Symposium on Navigation, Wy˙zsza Szkola Morska, Gdynia, 2001.

[6] W. Kumm, *GPS Global Positioning System*. Bielefeld, Germany: Klasing, 1995.

[7] S. Schillak, "Wykonanie satelitarnych pomiarów laserowych z dokladnością milimetrową," in *Konferencja Grantowa "Metrologia na Uslugach Przemyslu,"* (in Polish), Bialystok, Poland, 1999.

[8] W. Baran i inni, "Analiza dokladno´sci wyznacze´n DGPS na obszarze Zatoki Gda´nskiej," in *Wy ˙zsza Szkola Morska, I Sympozjum Nawigacyjne*, (in Polish), Gdynia, Poland, 1995, pp. 127–135.

Jacek Czajewski is a professor of electrical engineering at Warsaw University of Technology, Warsaw, Poland, where he teaches courses and conducts research in electrical measurements and nonelectrical measurements using electrical methods. He was vice-dean (1986–1992) and dean (1992–1998) of the Electrical Engineering faculty. Since 1995, he has been the Electrical Measurement chair. He has published 19 books and handbooks and approximately 60 journal and conference papers. His current research interests are in radio navigation.