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On the Measurement of Impurity Atom Distributions by the Differential Capacitance Technique*

In a recent paper,[1] it was shown that the profile inferred from differential capacitance measurements[2-5] of semiconductor junctions is not that of the impurity atom distribution but, instead, that of the majority carrier distribution. For this reason, conventional differential capacitance measurements can be used to evaluate the impurity atom distribution only in charge neutral semiconductor material (where the majority carrier density equals the density of ionized impurity atoms). This requirement of charge neutrality limits the applicability of this measurement technique to semiconductor material containing a minimum impurity atom density of about 10^{16} atoms/cm^3.

This letter describes a method whereby the requirement of charge neutrality is eliminated. Equations are developed that rigorously relate the majority carrier distribution (as established from differential capacitance measurements) to the associated impurity atom distribution. Thus, in conjunction with differential capacitance measurements, the equations presented here provide a means to establish the impurity atom distribution in a semiconductor of homogeneous conductivity type, regardless of the electrostatic charge produced by this impurity distribution.

To begin this analysis, we repeat Eq. (8) of Ref. 1 as Eq. (1) below, to mathematically relate the measured differential capacitance C of the test junction and the majority carrier distribution $n(x)$:

$$n(x) = -\frac{C^3}{q\kappa\epsilon_0}\left(\frac{dC}{dV}\right)^{-1}, \tag{1}$$

where $\kappa\epsilon_0$ is the permittivity of the semiconductor material, q is the electron charge and x is the test junction space-charge layer width at the applied biasing voltage V. Throughout this discussion, the semiconductor material under consideration is assumed to be n-type; thus, the majority carriers are electrons.

The electric current within this material due to both drift and diffusion of majority carriers is given by

$$J_n = qD_n\frac{dn}{dx} - q\mu_n n\frac{d\Psi}{dx}. \tag{2}$$

An electric current of zero implies that the diffusion and drift terms in Eq. (2) are of equal magnitude, but in the opposite direction; hence, from (2) we obtain an electric field of magnitude

$$E(x) = -\frac{d\Psi}{dx} = \frac{kT}{q}\frac{1}{n(x)}\frac{dn(x)}{dx}. \tag{3}$$

Equation (3) establishes the electric field distribution necessary to maintain an electric current of zero in n-type material containing local variations of electron density.

Assuming extrinsic semiconductor material (the minority carrier density has negligible influence upon the structure under consideration), we have from Poisson's equation

$$\frac{dE}{dx} = \frac{q}{\kappa\epsilon_0}[N(x) - n(x)]. \tag{4}$$

[The divergence of the electric field (3) is determined by both the impurity atom distribution $N(x)$ and the majority

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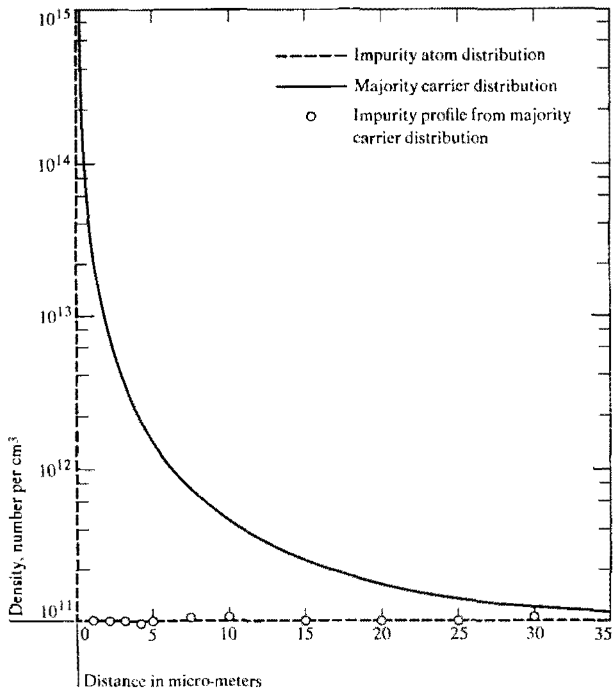


Figure 1 Illustration of the impurity profile that would be inferred from differential capacitance measurements on the low-doped side of an abrupt high-low junction.

carrier distribution $n(x)$.] By combining (3) and (4) we obtain

$$\frac{kT}{q}\frac{d}{dx}\left\{\frac{1}{n(x)}\frac{dn(x)}{dx}\right\} = \frac{q}{\kappa\epsilon_0}\left[N(x) - n(x)\right] \qquad (5)$$

and therefore

$$N(x) = n(x) + \left(\frac{kT}{q}\right)\left(\frac{\kappa\epsilon_0}{q}\right)\frac{d}{dx}\left[\frac{1}{n(x)}\frac{dn(x)}{dx}\right]. \qquad (6)$$

Equation (6) rigorously relates the desired impurity atom distribution $N(x)$ to the measured majority carrier distribution $n(x)$.

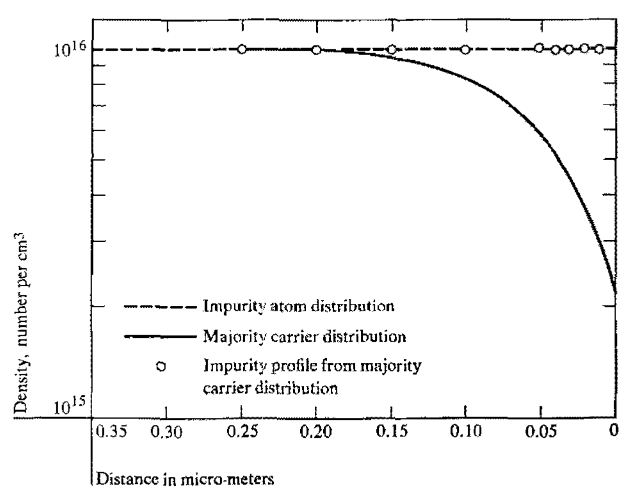
In this development, questions arise concerning the uniqueness of the majority carrier distribution $n(x)$, due to a given impurity atom distribution $N(x)$. Although there is little to gain by presenting here a complete uniqueness proof for (6), the uniqueness of this equation has been investigated. It can be shown that (6) satisfies a Lipschitz condition[6] of the first order throughout regions of the semiconductor where $n(x)$ has a non-zero magnitude. Therefore, any given impurity atom distribution will have associated with it a unique majority carrier distribution. Furthermore, from measured values of this majority carrier distribution (which are obtained from differential capacitance measurements), Eq. (6) establishes the associated impurity atom distribution.

To illustrate this proposed method for profiling semiconductor material, two mathematical models have been selected in which the assumed impurity atom distribution produces a substantial electrostatic charge.

For illustrative purposes, the majority carrier distributions within these models have been calculated using previously described computational techniques.[1] (In a laboratory experiment, these majority carrier distributions would not be established by calculations using a model, but by differential capacitance measurements upon semiconductor material containing the prescribed impurity atom distribution.) From these majority carrier distributions, graphical methods are used in conjunction with Eq. (6) to establish the associated impurity atom distribution. In this fashion, a comparison is obtained between the impurity atom distribution assumed within the models and the impurity atom distribution implied by this revised theory for the differential capacitance experiment.

The first example is an abrupt high-low junction containing an impurity atom density of 10^{16} atoms/cm^3 on the high-doped side, and an impurity atom density of 10^{11} atoms/cm^3 on the low-doped side. Because the space-charge layer widths are substantially different on each side of this junction, the results of these calculations are presented in two different illustrations: Fig. 1 and Fig. 2 show, respectively, the low-doped and high-doped side of the structure characterized by the model. Each of these illustrations shows the assumed impurity atom distribution, the calculated majority carrier distribution (which would be obtained from differential capacitance measurements upon such a structure[1]) and the impurity atom distribution established from this majority carrier distribution, using Eq. (6). This example demonstrates

Figure 2 Inferred impurity profile for the high-doped side of an abrupt high-low junction.



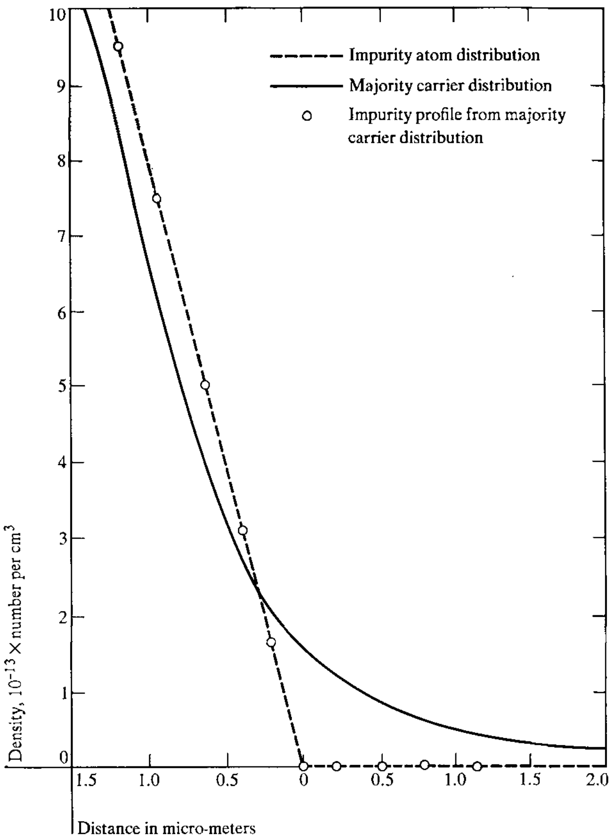


Figure 3 Inferred impurity profile for a linearly-graded high-low junction.

that differential capacitance measurements can be used to establish the impurity atom distribution in semiconductor material containing a substantial electrostatic charge.

Figure 3 presents the results of a similar series of calculations for a structure containing a linearly-graded impurity atom distribution that is discontinuously terminated into a region of constant doping density (10^{11} atoms/cm^3). Conventional differential capacitance measurements upon material of this type would yield only the illustrated majority carrier distribution. If, however, the results of these measurements are used in the manner prescribed by Eq. (6), the capacitance-inferred profile thus obtained is the impurity atom distribution throughout this semiconductor structure.

References

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