# Comment on "Decomposition of a Data Base and the Theory of Boolean Switching Functions"

Two of the authors of this letter have presented a theorem (Appendix C of [1], hereinafter called "Theorem C") relating Codd's third normal form for a data collection [2] to the minimum cover for an associated Boolean function. Actually this theorem is incorrect as stated. The third author (Bernstein) in his dissertation [3] has produced a counterexample to the theorem and has developed techniques for decomposing a data base into third normal form under general conditions.

The major results of [1] concerned the mathematical similarity between functional dependencies in a data base and a class of Boolean functions. These results are unaffected by the correction given here. Indeed, the fundamental equivalence has been redemonstrated in a different manner by Armstrong [4] and by Fagin [5]. Theorem C actually concerns a property somewhat stronger than Third Normal Form, since the latter allows a transitive dependency $A \rightarrow B \rightarrow C$ to exist in a relation $R$ if attribute $C$ belongs to a minimal key for $R$.

Bernstein's work may also be used to correct Theorem C. If his Property P is included in the hypothesis, then the theorem becomes true. Here we present an alternative way of correcting it.

Bernstein's counterexample and a graphical representation of his functional relations (FR's) are shown in Fig. 1 (see [1] for the definition of terms such as "functional relation"). The graph of the counterexample is seen to contain a cycle along the path $R_3$-$X$-$R_4$-$C$-$R_2$-$B$-$R_3$. This occurrence is related to the failure of Theorem C of [1], as is evidenced by the proof we have included at the end of this letter. If one adds to the hypothesis of the original Theorem C the condition that the graph of the minimum cover be acyclic, then the theorem is true. The property of acyclicity is a stronger condition for third normal form than Bernstein's Property P, although the former is probably easier to check algorithmically for a given set of FR's.

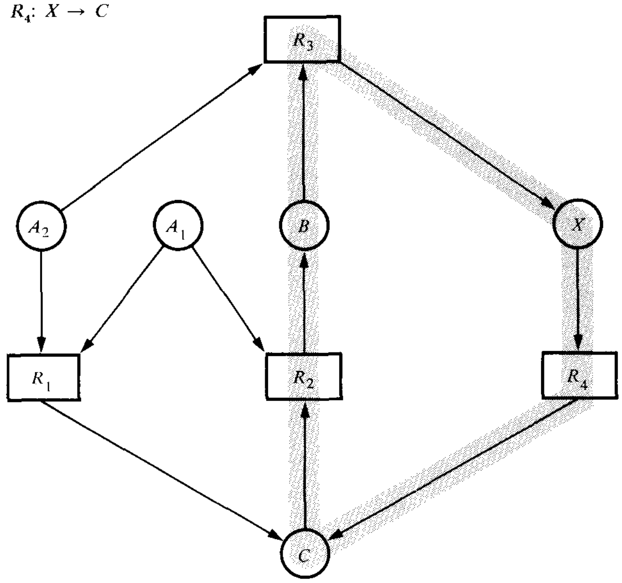
**Figure 1** A counterexample. The FR's given in [2], and a graphical representation of them, are shown. Data base attributes are represented by circles, the given functional relations by boxes, and connecting arrows denote the left and right sides of the respective FR's. The existence of a circuit in the graph is indicated by shading.

$R_1$: $A_1A_2 \rightarrow C$

$R_2$: $A_1C \rightarrow B$

$R_3$: $A_2B \rightarrow X$

$R_4$: $X \rightarrow C$




*Theorem* If $MC$ is a minimum cover of a set $F$ of functional relations, and if the graph of $MC$ is acyclic, then there is no attribute $X \neq \phi$ such that the following statements are true:

$S_1$: $A_1 \rightarrow X \in F$;

$S_2$: $A_2X \rightarrow C \in F$;

$S_3$: $A_1A_2 \rightarrow C \in MC$,

where $A_1$, $A_2$, and $C$ are attributes (or attribute sets) such that

$A_1 \cap A_2 = \phi$;

$X \cap A_2 = \phi$.

*Proof* Assume the contrary, i.e., there is such an $X$. We shall show that $S_1$, $S_2$, $S_3$ then imply either that the graph contains a cycle or else that $MC$ is not a minimum cover. This constitutes proof by contradiction.

The following four cases are examined:

1. $S_1, S_2 \in MC$;
2. $S_1 \not\subset MC$, $S_2 \in MC$,
3. $S_1 \in MC$, $S_2 \not\subset MC$,
4. $S_1, S_2 \not\subset MC$.

*Case 1* As developed in [1] the FR's $S_1$-$S_3$ are not independent. That is, $S_1$ and $S_2$ imply $S_3$. Thus all three cannot belong to the same minimum cover, so this case leads to an immediate contradiction.

*Case 2* Since $S_1$ belongs to $F$, it must be derivable from the FR's of the minimum cover:

$S_1 = f(V_1, V_2, \cdots, V_n)$, where $V_i \in MC$.

Let the symbol $\oplus$ represent the pseudotransitivity operation as defined in [1]. That is, if we have three functional relations $R_1$, $R_2$, and $R_3$ such that

$R_1$: $E \to F$,

$R_2$: $F, G \to H$,

$R_3$: $E, G \to H$,

then we shall write

$R_1 \oplus R_2 = R_3$.

It is important to note that the closure of a given set of FR's (i.e., the set of all FR's derivable from them) can be obtained by successive application of $\oplus$ to members of the given set. The function $f$ defined above is an instance of such an application.

From the definition of $\oplus$ it follows that

$S_3 = S_1 \oplus S_2$.

Suppose that $S_3 \neq V_i$ for any $i$. Then

$S_3 = f(V_1, V_2, \cdots, V_n) \oplus S_2$,

which shows that $S_3$ can be derived from members of $MC$ and thus cannot belong to $MC$ itself, a contradiction.

On the other hand, consider $S_3 = V_i$ for some $i$. Since $S_2 \in MC$, there is a path from $X$ to $C$ in the graph of $MC$. But since

$S_1 = f(V_1, V_2, \cdots, V_{i-1}, S_3, V_{i+1}, \cdots, V_n)$,

then there is a path from $C$ to $X$, i.e., a cycle, contradicting our hypothesis. Cases 3 and 4 are proved the same way.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