Corrections to "Slanted-Wall Beam Propagation"

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***Abstract*—Recently, a new algorithm for wide-angle beam propagation was reported that allowed grid points to move in an arbitrary fashion between propagation planes and was thus capable of modeling waveguides whose widths or centerlines varied with propagation distance. That algorithm was accurate and stable for TE polarization but unstable for wide-angle TM propagation. This deficiency has been found to result from an omission in one of the wide-angle terms in the derivation of the finite-difference equation and is remedied here, resulting in a complete algorithm accurate for both polarizations.**

***Index Terms*—Beam propagation, diffractive optics, finite-difference method.**

I. Introduction

Due to the increase in complexity of waveguide geometries required in today's photonic circuitry, numerical algorithms capable of modeling a very general class of waveguides whose features change with propagation distance has long been an important goal. Furthermore, it is crucial that such algorithms allow nonrectangular grids so as to avoid "stair-stepping" errors that can be dominant for high-index-contrast waveguides or when a prediction of small reflections is important. To this end, we recently reported a wide-angle beam-propagation algorithm [1] capable of modeling arbitrary changes in slab waveguide geometries within the angular restrictions required by Pade(1,1) wide-angle propagation. This algorithm is tridiagonal and thus numerically efficient, generalizes immediately to 3-D for a wide class of structures, and admits a very simple method for adding or subtracting extra grid points whenever and wherever needed. Although both accurate and stable for TE polarization, it was found to be stable for TM polarization only when the wide-angle terms were omitted. This deficiency has been discovered to result from a problem in the way the wide-angle terms were treated in the derivation and is remedied in the more correct derivation presented below.

The same accuracy test reported previously [1] for propagation along a uniform waveguide tilted at an angle to the propagation direction now demonstrates TM accuracy using the new algorithm at a level comparable to that found before for TE polarization.

Thus the algorithm is now of general utility for either polarization.

II. New Propagation Equation

A corrected derivation of the finite-difference equations describing wide-angle beam propagation proceeds in an identical manner to that presented previously [1] until the evaluation of the first term on the right-hand side (RHS) of (18). As reported, this term is

$$PH|_{+}^{n+1} - PH|_{+}^{n} \approx PH|_{0+}^{n+1} - PH|_{0+}^{n}$$

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which, when combined with the corresponding expression on the opposite side of the interface, is easily evaluated [2]. However, when those terms were combined, the term

$$\varepsilon_{+}^{p}\left.\frac{\partial H}{\partial x}\right|_{0+} - \varepsilon_{-}^{p}\left.\frac{\partial H}{\partial x}\right|_{0-} = \frac{1}{2}\left(\varepsilon_{+}^{p} - \varepsilon_{-}^{p}\right) \times \sin 2\theta\left\{ikH + \frac{1}{\cos\theta}\frac{\partial H}{\partial z'}\right\}$$

was neglected, which is nonzero for TM polarization. When this term is included, the propagation coefficients become

$$A_i = -\frac{\varepsilon_{+}^{p}}{\Delta x_{+} + \Delta x_{-}} \times\left[-\frac{i\Delta z}{2k\Delta x_{+}} + \frac{w\eta_{+}}{2k^2\Delta x_{+}} + \gamma_{+}\Delta x_{+} \times\left(\frac{\eta_{+}}{2} - \frac{1}{6}\right) + 2i\eta_{+}k\xi^{*}\tan\theta_{+}\right] \tag{1}$$

$$C_i = -\frac{\varepsilon_{-}^{p}}{\Delta x_{+} + \Delta x_{-}} \times\left[-\frac{i\Delta z}{2k\Delta x_{-}} + \frac{w\eta_{-}}{2k^2\Delta x_{-}} + \gamma_{-}\Delta x_{-}\left(\frac{\eta_{-}}{2} - \frac{1}{6}\right) + 2i\eta_{-}k\xi^{*}\tan\theta_{-}\right] \tag{2}$$

$$B_i = \langle\eta\varepsilon^{p}\rangle + A_i + C_i + \xi k_0^2\langle\eta\varepsilon^{p}(\varepsilon - \overline{\varepsilon})\rangle + \frac{ik\xi^{*}\left(\varepsilon_{+}^{p} - \varepsilon_{-}^{p}\right)\sin 2\theta}{(\Delta x_{+} + \Delta x_{-})} \tag{3}$$

$$A_i' = -\frac{\varepsilon_{+}^{p}}{\Delta x_{+} + \Delta x_{-}} \times\left[\frac{i\Delta z}{2k\Delta x_{+}} + \frac{w\eta_{+}}{2k^2\Delta x_{+}} + \gamma_{+}^{*}\Delta x_{+} \times\left(\frac{\eta_{+}}{2} - \frac{1}{6}\right) + 2i\eta_{+}k\xi\tan\theta_{+}\right] \tag{4}$$

$$C_i' = -\frac{\varepsilon_{-}^{p}}{\Delta x_{+} + \Delta x_{-}} \times\left[\frac{i\Delta z}{2k\Delta x_{-}} + \frac{w\eta_{-}}{2k^2\Delta x_{-}} + \gamma_{-}^{*}\Delta x_{-} \times\left(\frac{\eta_{-}}{2} - \frac{1}{6}\right) + 2i\eta_{+}k\xi\tan\theta_{-}\right] \tag{5}$$

$$B_i' = \langle\eta\varepsilon^{p}\rangle + A_i' + C_i' + \xi^{*}k_0^2\langle\eta\varepsilon^{p}(\varepsilon - \overline{\varepsilon})\rangle + \frac{ik\xi\left(\varepsilon_{+}^{p} - \varepsilon_{-}^{p}\right)\sin 2\theta}{(\Delta x_{+} + \Delta x_{-})}. \tag{6}$$

A comparison with the previous results shows that only the last terms in the B_i, B_i' expressions are different from before, but those differences have significant impact on the performance of the propagation algorithm. For example, the above scheme is now exactly energy conserving for both polarizations when $\theta_{+} = \theta_{-} = \theta$, corresponding to the case of meandering waveguides with (approximately) constant width. The derived expression for the energy remains unchanged, but we list it here to correct some minor errors in the previous manuscript. The corrected (7) is

$$\mathtt{E} = \sum_i\left[\frac{\langle\varepsilon^{p}\rangle_i}{6} + \frac{\langle\eta\varepsilon^{p}(\varepsilon + \overline{\varepsilon})\rangle_i}{4\overline{\varepsilon}} - \frac{w}{2k^2}\left\langle\frac{\eta\varepsilon^{p}}{\Delta x^2}\right\rangle\right] \times(\Delta x_{+} + \Delta x_{-})|H_i|^2 + \Re\sum_i\left[\frac{w\varepsilon_{+}^{p}\eta_{+}}{k^2\Delta x_{+}^2} + \varepsilon_{+}^{p}\left(\eta_{+} - \frac{1}{3}\right)\right]\Delta x_{+}H_i^{*}H_{i+1} \tag{7}$$

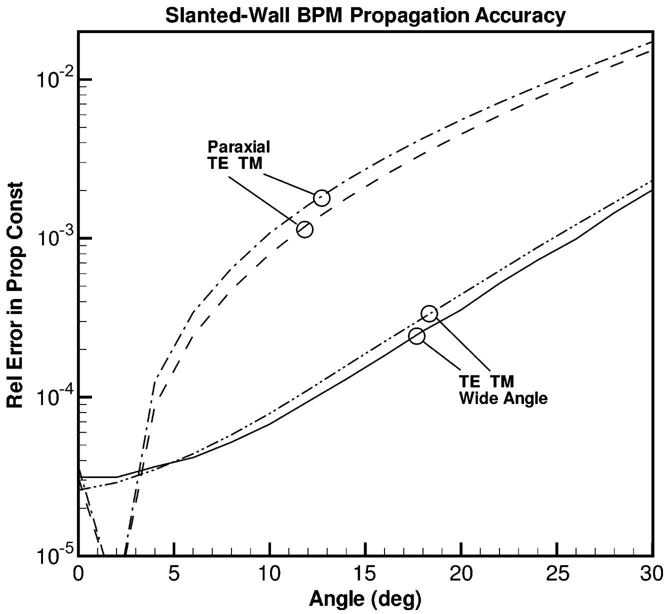

Fig. 1. Relative phase error for propagation of the fundamental mode of a waveguide tilted at various angles with respect to the propagation direction using the present slanted-wall algorithm. Generally, wide-angle propagation results in an order-of-magnitude improvement in accuracy.

where, as before, we have omitted the θ-dependent terms, because they are in general not energy conserving. However, the main difference is that now the algorithm is stable and accurate for TM propagation, as will be demonstrated below.

III. Benchmarking

A. Phase Accuracy

As before, we performed a series of propagation tests for a constant width waveguide of known propagation constant tilted at various (constant) angles. The results presented previously in Fig. 1 were presented without the wide-angle TM case since the algorithm would not propagate stably for that case. The present algorithm is identical to the previous one for paraxial and wide-angle TE, and nearly identical for paraxial TM, but has now been used to examine propagation accuracy for the wide-angle TM case, resulting in an additional curve for Fig. 1, as shown below. As can be seen, the calculated accuracies for wide-angle TE and TM propagation are nearly identical.

IV. Conclusion

Here, we have presented a modified general propagation algorithm for slab waveguide structures in which the dielectric interfaces may lie at a nonzero angle with respect to the propagation direction. The previous algorithm was unstable for wide-angle TM propagation due to an omission in the evaluation of certain terms in the finite-difference propagation equation. This error has been corrected here, with the result of a new algorithm that is not only stable for wide-angle TM propagation but is exactly energy conserving for both polarizations when all grid points move together so that grid point separations are unchanged (the case of meandering waveguides). The resulting algorithm is expected to find significant use in photonic design, particularly when extended to three dimensions.

References

[1] G. R. Hadley, "Slanted-wall beam propagation," *J. Lightw. Technol.*, vol. 25, no. 9, pp. 2367–2375, Sep., 2007.

[2] G. R. Hadley, "Low-truncation-error finite difference equations for photonics simulation I: Beam propagation," *J. Lightw. Technol.*, vol. 16, no. 1, pp. 134–141, Jan. 1998.