Applications of Sparse Representation and Compressive Sensing

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I. INTRODUCTION

In the past several years, there have been exciting breakthroughs in the study of high-dimensional *sparse signals*. A sparse signal is a signal that can be represented as a linear combination of relatively few base elements in a basis or an overcomplete dictionary. Much of the excitement centers around the discovery that under surprisingly broad conditions, a sufficiently sparse linear representation can be correctly and efficiently computed by greedy methods and convex optimization (i.e., the $\ell^1 - \ell^0$ equivalence), even though this problem is extremely difficult—NP-hard in the general case. Further studies have shown that such high-dimensional sparse signals can be accurately recovered from drastically smaller number of (even randomly selected) linear measurements, hence the catch phrase *compressive sensing*. If these are not surprising enough, more recently, the same analytical and computational tools have seen similarly remarkable successes in advancing the study of recovering high-dimensional *low-rank matrices* from highly incomplete, corrupted, and noisy measurements.

Sparse representation and compressive sensing establishes a more rigorous mathematical framework for studying high-dimensional data and ways to uncover the structures of the data, giving rise to a large repertoire of efficient algorithms.

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These results have already caused a small revolution in the community of statistical signal processing as they provide entirely new, or even somewhat paradoxical, perspectives to some of the fundamental principles and doctrines in signal processing such as the sampling bounds and the choice of bases for signal representation and reconstruction. A recent IEEE SIGNAL PROCESSING MAGAZINE special issue on compressive sampling has captured some of the most recent and exciting developments in this field.

We, the guest editors of this special issue, strongly believe that these new results and the general mathematical principles behind them are of great interest to scientific and engineering communities far beyond signal processing. These new results and revelations have forever changed our perspective and enhanced our ability in acquiring, processing, and analyzing massive high-dimensional data, regardless of their physical nature. Therefore, the theme of this new special issue for the Proceedings of IEEE is to introduce to the entire electrical engineering community highlights of these exciting new results, their likely future theoretical extensions, their rapid algorithmic developments, and their far-reaching impact on many engineering applications, including but no longer limited to conventional signal processing.

II. INTERPLAY OF THEORY, ALGORITHMS, AND APPLICATIONS

On the theoretical side, studies of sparse representation and compressive sensing have woven together results from multiple mathematical fields including high-dimensional statistics, harmonic analysis, measure concentration, polytope geometry, combinatorics, and convex optimization. In addition to signal and image processing, the new results have helped advance almost all theoretical foundations of electrical engineering and computer sciences at a time when all these fields move on to deal with ever more complex, higher dimensional systems, codes, and data, for example, the error-correction problem in coding and information theory, the rank minimization problem for system identification in control, the approximate embedding problem in data mining, the face recognition problem in computer vision, and the locality sensitive hashing in theoretical computer science. Great promises offered by these theoretical advances have also generated a resurrection of interest in developing more efficient and scalable algorithms for many important classes of convex optimization problems.

On the practical side, these new theoretical results and fast algorithms could not have come at a better time. Recent advances in information technologies have produced massive high-dimensional data such as audio, image, video, web documents, and bioinformatic data that all demand efficient processing and analysis whereas extent theoretical frameworks and computational tools have shown troubling signs of deficiency. Much of the deficiency was due to the lack of fundamental understanding about the geometry and statistics of high-dimensional spaces. The new theory of sparse representation and compressive sensing not only establishes a more rigorous mathematical framework for studying high-dimensional data, but also provides computationally feasible ways to uncover the structures of the data, giving rise to a large repertoire of efficient algorithms. The good scalability of these algorithms also means that they can be readily implemented on parallel and distributed computing platforms. This allows people to fully leverage on the rapidly growing massive parallel, distributed, and cloud computing technologies, and process data at unprecedented scale and speed with hundreds and thousands of machines. In the emerging era of data-intensive computing and data-driven discovery, this body of new knowledge has become much more relevant and useful for future scientists, engineers, and practitioners.

As we will see in this special issue, these new algorithms have been rapidly applied to ever diverse range of practical engineering problems and almost always lead to striking results that significantly advance the state-of-the-art. These applications range from conventional audio/image/video processing tasks (denoising, deblurring, inpainting, compression, and super-resolution) to speech and object recognition (source separation and classification); from multimedia data mining to bioinformatic data decoding; from correcting error for corrupted data (face recognition despite occlusion) to detecting activities and events through a large network of sensors and computers. One particular attractive feature of this new framework is that it encourages the use of dictionaries that are adaptive to specific classes of signals or data of interest, depending on the applications. Such dictionaries can be effectively learned from exemplars with sparse representation property ensured. In many applications mentioned above, the so-obtained dictionaries give superior performance to traditional general-purpose bases or dictionaries.

III. OVERVIEW OF THE PAPERS IN THIS ISSUE

In the past few years, thousands of papers have been published on sparse representation and compressive sensing, especially in the signal processing community. While we marvel at this phenomenal growth and impact, we must not forget that some of the basic ideas have been germinated for more than a century, in many fields across mathematics, science, and engineering. In the first "scanning the technology" article dedicated to this special issue, Prof. D. Donoho has provided us an excellent survey of this topic with a broader historical and scientific perspective.

To provide the readers a good exposition of all the modern developments on this topic, we guest editors have selectively invited leading researchers in each important subarea to contribute a paper about their favorite topics and results. The papers aim to provide good survey or review of past achievements in the field, or feature some new exciting developments by the authors, or discuss promising new directions and extensions. We hope that through the topics and examples featured in this special issue, the readers will be able to grasp the essence of this new framework so that they can apply it to solve existing or new problems in their own field. As the old Chinese proverb goes, this special issue is meant to "*lay bricks to attract jade*."

A total of 15 papers in this special issues are roughly grouped into three main clusters—five papers in each cluster. The first cluster of five papers survey theory and algorithms of compressed sensing and sparse representation.

- The paper by Donoho and Tanner provides an elegant geometric interpretation of the classical compressed sensing theory and results, revealing a strong connection to high-dimensional combinatorial geometry and providing a precise characterization of undersampling bounds for compressed sensing.

- The paper by Candès and Plan introduces a more recent trend of extending the study from the recovery of sparse signals to the completion of low-rank matrices, in which the sparsity-prompting ℓ_1 norm is replaced with the low-rank promoting nuclear norm of a matrix. This paper proves the stability of matrix completion under noise.
- While most compressed sensing methods harness the incoherence of dense random matrices, the paper by Gilbert and Indyk surveys a parallel line of research of using sparse measurement matrices for recovering sparse signals. One important advantage of using sparse matrices is their computational efficiency, in many cases allowing linear or sublinear time recovery algorithms.
- The paper by Tropp and Wright gives a comprehensive exposition of the many algorithms that have been developed in the past few years for sparse signal recovery.
- The paper by Cevher *et al.* demonstrates how the basic sparse signal model can be extended or generalized to much broader classes of low-dimensional models that encourage similar efficient and accurate signal reconstruction. These models are applicable to much wider range of applications.

The second cluster of five papers highlight some of the conventional applications of compressive sensing in signal processing, including images, audio, music, radar, and astronomical data. The editors believe that the readers can more easily grasp the essence of sparsity-promoting techniques by witnessing how they are used in these more classical settings.

- The survey paper by Elad *et al.* provides both conceptual and empirical justifications why sparse and redundant representation are crucial for image processing. Sparsity-promoting techniques have achieved state-of-the-art performance for many classical tasks such as image denoising, inpainting, super-resolution, etc.
- The paper by Fadili *et al.* applies sparse representation to a more specific image processing problem of decomposing an image into multiple unknown components, the so-called "cocktail party" problem for multiple source separation.
- The next three papers demonstrate how sparse representation and compressive sensing have been widely and successfully applied to processing and analyzing other types of signals, including audio and music data (by Plumbley *et al.*), radar imaging (by Potter *et al.*), and astronomical data (by Starck and Bobin).

The final cluster of five papers show how the sparsity promoting and compressive sensing techniques have started to create tremendous impact on a much broader range of engineering fields, including but not limited to pattern recognition, machine learning, communications, sensor networks, and imaging sensors.

- The paper by Wright *et al.* surveys some of the early successful applications of sparse representations in computer vision, especially in face recognition. In this paper, we will also see how special settings of the practical problems lead to new mathematical models and results that enrich the basic theory of sparse representation.
- The paper by Rubinstein *et al.* addresses the important problem how to automatically learn a dictionary from raw samples that best sparsifies the signals or data of interest. The paper gives a comprehensive survey of some of the most popular and effective algorithms for dictionary learning.
- The paper by Bajwa *et al.* shows how compressed sensing can be used to estimate the parameters of communication channels.
- The paper by Yang *et al.* demonstrates how sparse representation can be used for event detection and classification with sensor networks.
- The paper by Romberg *et al.* shows how the principles of compressive sensing can be used to design next generation of cameras and imaging sensors that achieve much higher resolution at a lower cost and power consumption. ■

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Richard G. Baraniuk (Fellow, IEEE) is the Victor E. Cameron Professor of Electrical and Computer Engineering at Rice University, Houston, TX. His current activities revolve around sparsity-based signal and image processing and open education. In 1999, he founded Connexions (cnx.org), a nonprofit publishing project that invites authors, educators, and learners worldwide to "create, rip, mix, and burn" free textbooks, courses, and learning materials from a global open-access repository.

Dr. Baraniuk received a NATO postdoctoral fellowship from NSERC in 1992, the National Young Investigator award from the National Science Foundation in 1994, a Young Investigator Award from the Office of Naval Research in 1995, the Rosenbaum Fellowship from the Isaac Newton Institute of Cambridge University in 1998, the C. Holmes MacDonald National Outstanding Teaching Award from Eta Kappa Nu in 1999, the University of Illinois ECE Young Alumni Achievement Award in 2000, the Tech Museum Laureate Award from the Tech Museum of Innovation in 2006, the Wavelet Pioneer Award from SPIE in 2008, the Internet Pioneer Award from the Berkman Center for Internet and Society at Harvard Law School in 2008, and the World Technology Network Education Award in 2009. In 2007, he was selected as one of *Edutopia Magazine*'s Daring Dozen educators, and the Rice single-pixel compressive camera was selected by *MIT Technology Review Magazine* as a TR10 Top 10 Emerging Technology. He was elected an IEEE Fellow in 2001 and an American Association for the Advancement of Science (AAAS) Fellow in 2009.

Emmanuel Candès received the Ph.D. degree in statistics from Stanford University, Stanford, CA, in 1998.

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Prof. Candès received numerous awards, most notably the 2006 Alan T. Waterman Medal, which is the highest honor bestowed by the National Science Foundation and which recognizes the achievements of scientists who are no older than 35, or not more than seven years beyond their doctorate. Other awards include the 2008 Information Theory Society Paper Award, the 2005 James H. Wilkinson Prize in Numerical Analysis and Scientific Computing awarded by the Society of Industrial and Applied Mathematics (SIAM) and the 2010 George Polya Prize awarded by SIAM. He has given over 40 plenary lectures at major international conferences.

Michael Elad (Senior Member, IEEE) received the B.Sc., M.Sc. (supervision by Prof. D. Malah), and D.Sc. (supervision by Prof. A. Feuer) degrees from the Department of Electrical Engineering, Technion—Israel Institute of Technology, Israel, in 1986, 1988, and 1997, respectively.

From 1988 to 1993, he served in the Israeli Air Force. From 1997 to 2000, he worked at Hewlett-Packard laboratories (Israel) as an R&D Engineer. From 2000 to 2001, he headed the research division at Jigami Corporation, Israel. During 2001 to 2003, he spent a postdoc period as a Research Associate with the Computer Science Department, Stanford University, Stanford, CA (SCCM program). In September 2003, he returned to the Technion, assuming a tenure-track assistant professorship position in the Department of Computer Science. In May 2007, he was tenured to an associate professorship. He works in the field of signal and image processing, specializing in particular on inverse problems, sparse representations, and overcomplete transforms.

Dr. Elad received the Technion's best lecturer award six times (1999, 2000, 2004, 2005, 2006, and 2009), he is the recipient of the Solomon Simon Mani award for excellence in teaching in 2007, and he is also the recipient of the Henri Taub Prize for academic excellence (2008). He is currently serving as an Associate Editor for both the IEEE Transactions on Image Processing and *SIAM Journal on Imaging Sciences* (SIIMS).

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Dr. Ma was the recipient of the David Marr Best Paper Prize at the International Conference on Computer Vision in 1999, the Longuet-Higgins Best Paper Award at the European Conference on Computer Vision in 2004, and the Sang Uk Lee Best Student Paper Award at the Asian Conference on Computer Vision in 2009. He received the CAREER Award from the National Science Foundation in 2004 and the Young Investigator Program Award from the Office of Naval Research in 2005. He has given several plenary lectures at international conferences. He currently serves as an Associate Editor for the IEEE Transactions on Pattern Analysis and Machine Intelligence. He is a member of the Association for Computing Machinery (ACM) and the Society of Industrial and Applied Mathematics (SIAM).