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Comments on "Fields at the Tip of an Elliptic Cone"

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Electromagnetic fields near the tip of an elliptic cone can be represented by a sum over a set of vector wave functions. Only the leading terms of this sum contribute to the near field. A useful classification of these terms based on symmetry and periodicity properties of the eigenfunctions of the problem is presented. This classification suggests that there must be a second magnetic field singularity which is not given in the above letter.[1] Data on the related eigenvalue are given.

I. INTRODUCTION

In the above letter,[1] singularities of electromagnetic fields at the tip of an elliptic cone are discussed. The authors give a brief summary of the theory of electromagnetic fields associated with this geometry and investigate the asymptotic behavior of the field in the vicinity of the tip. They point out that the singularity of the electric field is related to the first even Dirichlet solution of the boundary value problem and that of the magnetic field to the first odd Neumann solution. These considerations seem to be correct and the given eigenvalues are identical with those which we calculated using different methods [4]. In contrast to the above letter,[1] our method reveals that there is a further singularity of the magnetic field which belongs to an even Neumann solution. The corresponding eigenvalue is given below.

II. A SECOND MAGNETIC FIELD SINGULARITY

The diffraction of scalar waves at an elliptic cone had been investigated by Kirchner [1]. Starting with an analytical field expansion he found the eigenvalues given in the above letter[1] by using a numerical technique to solve the eigenvalue problem. Subsequent research ([2], [3]) in antenna problems led to series expansions of the eigenfunctions which were also used to find the solution of the diffraction problem [4]. In these papers, a representation of the eigenfunctions is used, which is quite different from that given by Vafiadis and Sahalos [5].

We used a coordinate system similar to that used in the above letter.[1] The differences are trivial. The solution of the problem is built up by products of Lamé functions in ϑ and φ. The one in φ is periodical and symmetrical. The function in ϑ is nonperiodic and fulfills the Dirichlet or Neumann condition at the cone $\vartheta = \vartheta_0$.

There are four classes of periodic Lamé functions[2]

1) π-periodic and even
2) 2π-periodic and odd[3]
3) π-periodic and odd
4) 2π-periodic and even.[3]

The symmetry is related to the plane of the largest flare angle of the cone. These functions are calculated with the aid of Fourier series.

Each periodic Lamé function is associated with one regular nonperiodic function, which is represented by series of associated Legendre functions ([2], [3]). The eigenvalue v occurs as the lower index of the Legendre functions and the natural upper index is the summation variable.

The eigenvalue is determined by the fact that the nonperiodic function underlies certain boundary conditions. Eigenvalues smaller than 1 cause a field singularity. An eigenvalue v_{e_1} is given in the above letter[1] which yields a singularity of the electric field and is related to function type 1.

The second eigenvalue v_{o_2} given there yields a singularity of the magnetic field and belongs to an eigenfunction of type 2. We found out that there is a second eigenvalue $v_{e_2} < 1$ which is related to the Neumann condition (Fig. 1). It belongs to a function of type

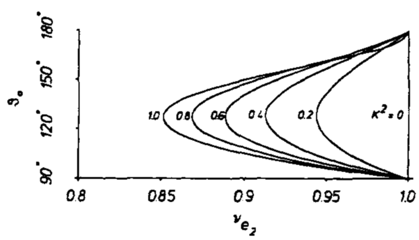

Fig. 1. The eigenvalue v_{e_2} as a function of ϑ_0 for various ellipticity parameters k^2.

Manuscript received December 27, 1984.

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[1] E. Vafiadis and J.S. Sahalos, *Proc. IEEE*, vol. 72, no. 8, pp. 1089–1091, Aug. 1984.

[2] This classification was suggested by Ince [6].

[3] 2π-periodic means not π- but 2π-periodic

0018-9219/85/1200-1857$01.00

4 and yields a second field singularity. This eigenvalue approaches unity as the cone degenerates to a sector ($\vartheta_0 = 180°$), a halfspace ($\vartheta_0 = 90°$), or a wedge ($k^2 = 0$). In these cases it does not lead to an additional field singularity. For the case of a circular cone, ν_{e_2} takes the same value as ν_{o_2}. This degeneracy vanishes in the more general case of an elliptic cone. A second electric field singularity does not exist, because the associated eigenfunction would be identical to 0.

III. Conclusion

An additional singularity of the magnetic field at the tip of an elliptic cone is given. The corresponding field has a symmetry different from that given in the above letter.[1] Although this second magnetic field singularity is a weak one it must be taken into consideration, because it is the only one for certain conditions of excitation.

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[6] E.L. Ince, "Further investigations into the periodic Lamé functions," *Proc. Roy. Soc. Edinburgh*, vol. 60, pp. 83–99, 1939/40.

Exact Sensitivities for Nonreciprocal Two-Port Power Elements

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A novel method is presented to calculate exact first-order sensitivity formulas for nonreciprocal two-port power elements using the short-circuit admittance description. The method exploits an augmented Tellegen's theorem and is applied to an elegant adjoint transformer model to evaluate sensitivities with respect to control parameters of phase-shifting transformers.

I. The Augmented Tellegen Theorem

Consider a two-port element connecting buses p and q. Let $\boldsymbol{y}$ be the 2×2 short-circuit admittance matrix of this element. Then, the current–voltage relations associated with the element are expressed in a compact form

$$\boldsymbol{I} = \boldsymbol{y}\boldsymbol{V} \tag{1}$$

where $\boldsymbol{I} \triangleq [I_p \; I_q]^T$, $\boldsymbol{V} \triangleq [V_p \; V_q]^T$, I_p and I_q are the line currents, and V_p and V_q are the corresponding line voltages [1]. We write the first-order variation of $\boldsymbol{I}$ of (1) as

$$\delta\boldsymbol{I} = \boldsymbol{y}\delta\boldsymbol{V} + \delta\boldsymbol{y}\boldsymbol{V}. \tag{2}$$

Manuscript received November 13, 1984. This work was supported by the Natural Sciences and Engineering Research Council of Canada under Grants A7239 and A1708.

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The augmented Tellegen's theorem adopted by Bandler and El-Kady [2] applied to the network branches of a system, includes terms related to the two-port element under investigation as

$$\cdots + \hat{\boldsymbol{I}}^T\delta\boldsymbol{V} + \hat{\boldsymbol{I}}^{*T}\delta\boldsymbol{V}^* - \hat{\boldsymbol{V}}^T\delta\boldsymbol{I} - \hat{\boldsymbol{V}}^{*T}\delta\boldsymbol{I}^* + \cdots = 0 \tag{3}$$

where $\hat{}$ distinguishes the two-component vectors associated with a topologically similar adjoint system, and * denotes the complex conjugate. Furthermore, we express the first-order change of a real or complex network function as [3]

$$\delta f = \cdots + \left(\frac{\partial f}{\partial \boldsymbol{I}}\right)^T \delta\boldsymbol{I} + \left(\frac{\partial f}{\partial \boldsymbol{I}^*}\right)^T \delta\boldsymbol{I}^* + \left(\frac{\partial f}{\partial \boldsymbol{V}}\right)^T \delta\boldsymbol{V} + \left(\frac{\partial f}{\partial \boldsymbol{V}^*}\right)^T \delta\boldsymbol{V}^* + \cdots. \tag{4}$$

Subtracting (3) from (4), we rearrange the resulting expression in the form

$$\delta f = \cdots + \left(\hat{\boldsymbol{V}} + \frac{\partial f}{\partial \boldsymbol{I}}\right)^T \delta\boldsymbol{I} + \left(\hat{\boldsymbol{V}}^* + \frac{\partial f}{\partial \boldsymbol{I}^*}\right)^T \delta\boldsymbol{I}^* - \left(\hat{\boldsymbol{I}} - \frac{\partial f}{\partial \boldsymbol{V}}\right)^T \delta\boldsymbol{V} - \left(\hat{\boldsymbol{I}}^* - \frac{\partial f}{\partial \boldsymbol{V}^*}\right)^T \delta\boldsymbol{V}^* + \cdots. \tag{5}$$

We utilize the conjugation property associated with a complex quantity ζ, i.e., $\partial f/\partial\zeta = (\partial f/\partial\zeta^*)^*$ for *real* network functions [3], and rewrite (5) as

$$\delta f = \cdots + \hat{\hat{\boldsymbol{V}}}^T\delta\boldsymbol{I} + \hat{\hat{\boldsymbol{V}}}^{*T}\delta\boldsymbol{I}^* - \hat{\hat{\boldsymbol{I}}}^T\delta\boldsymbol{V} - \hat{\hat{\boldsymbol{I}}}^{*T}\delta\boldsymbol{V}^* + \cdots \tag{6}$$

where

$$\hat{\hat{\boldsymbol{V}}} \triangleq \hat{\boldsymbol{V}} + \frac{\partial f}{\partial \boldsymbol{I}} \qquad \hat{\hat{\boldsymbol{I}}} \triangleq \hat{\boldsymbol{I}} - \frac{\partial f}{\partial \boldsymbol{V}}. \tag{7}$$

Substituting (2) and its conjugate into (6), we obtain

$$\delta f = \cdots + \left(\hat{\hat{\boldsymbol{V}}}^T\boldsymbol{y} - \hat{\hat{\boldsymbol{I}}}^T\right)\delta\boldsymbol{V} + \left(\hat{\hat{\boldsymbol{V}}}^{*T}\boldsymbol{y}^* - \hat{\hat{\boldsymbol{I}}}^{*T}\right)\delta\boldsymbol{V}^* + \hat{\hat{\boldsymbol{V}}}^T\delta\boldsymbol{y}\,\boldsymbol{V} + \hat{\hat{\boldsymbol{V}}}^{*T}\delta\boldsymbol{y}^*\boldsymbol{V}^* + \cdots. \tag{8}$$

Now the coefficient of $\delta\boldsymbol{V}$ (or $\delta\boldsymbol{V}^*$) in the above equation suggests an adjoint two-port element characterized by the current–voltage relations

$$\hat{\hat{\boldsymbol{I}}} = \boldsymbol{y}^T\hat{\hat{\boldsymbol{V}}} \tag{9}$$

which can be readily used in conjunction with the adjoint equations reported in the previous related work [2].

Sensitivity with Respect to a Complex Parameter

Substituting (9) and its complex conjugate into (8), we have

$$\frac{df}{d\zeta} = \hat{\hat{\boldsymbol{V}}}^T\frac{\partial\boldsymbol{y}}{\partial\zeta}\boldsymbol{V} + \left(\hat{\hat{\boldsymbol{V}}}^T\frac{\partial\boldsymbol{y}}{\partial\zeta^*}\boldsymbol{V}\right)^* \tag{10}$$

where ζ is complex and the conjugate manipulation follows Bandler and El-Kady [3]. The parameter ζ may be involved in reciprocal or nonreciprocal two-port models of power transmission network components. Examples of ζ associated with a phase-shifting transformer model [4], [5] are complex turns ratio and series impedance.

II. Sensitivity Evaluation

Consider a phase-shifting transformer having a complex turns ratio a_t and an equivalent impedance Z_t, both expressed in the per-unit system [4], [5]. The $\boldsymbol{y}$-matrix of this element is written in a convenient form

$$\boldsymbol{y} = \frac{1}{Z_t}\begin{bmatrix} \dfrac{1}{a_t^*} \\ -1 \end{bmatrix}\begin{bmatrix} \dfrac{1}{a_t} & -1 \end{bmatrix}. \tag{11}$$

The perturbed form of $\boldsymbol{y}$ of (11) is expressed compactly as

$$\delta\boldsymbol{y} = -\frac{1}{a_t}\boldsymbol{y}\bar{\boldsymbol{1}}\delta a_t - \frac{1}{a_t^*}\bar{\boldsymbol{1}}\boldsymbol{y}\delta a_t^* - \frac{1}{Z_t}\boldsymbol{y}\delta Z_t \tag{12}$$

where

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