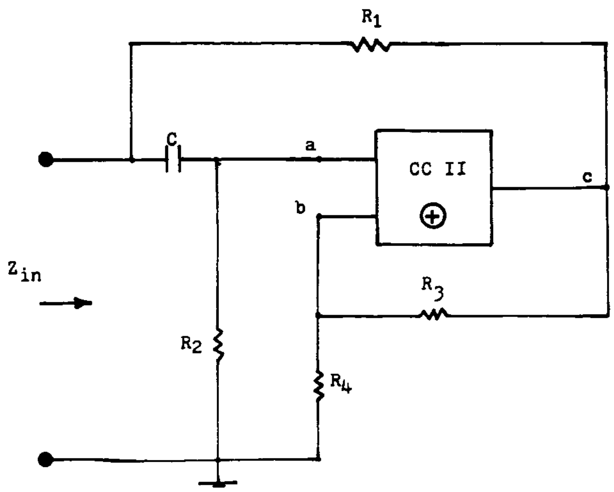

Fig. 1.

TABLE I

The realizable impedance function	Series RL	Parallel RL	Ideal L
Conditions on Z_{in} coefficients	$a_2 = 0$	$b_1 = 0$	$a_2 = b_1 = 0$
Zero position	$-b_1/a_1$	origin	origin
Pole position	$-\infty$	$-b_2/a_2$	$-\infty$
Realizability conditions	$K < \frac{1}{2}$ $\frac{1}{R_2} = \frac{2}{R_1} \cdot \frac{R_3}{R_4} + \frac{1}{R_4}$	$K > \frac{1}{2}$ $\frac{1}{R_2} = \frac{1}{R_1}(1+\frac{R_3}{R_4}) + \frac{1}{R_4}$	$K = \frac{1}{2}$ $\frac{1}{R_2} = \frac{2}{R_1} + \frac{1}{R_4}$
Equivalent circuit parameters	$L = \frac{CKR_1^2}{2(1-K)}$ $R_s = R_1 \cdot \frac{1-2K}{2(1-K)}$	$L = CKR_1^2$ $R_p = \frac{KR_1}{2K-1}$	$L = \frac{1}{2} CR_1^2$

where

$$a_1 = CK \quad b_1 = \frac{K}{R_2} - \frac{1}{R_1} - \frac{1}{R_3 + R_4}$$

$$a_2 = C\left[\frac{K}{R_2} - \frac{2(1-K)}{R_1} - \frac{1}{R_3 + R_4}\right]$$

$$b_2 = \frac{1}{R_1}\left[\frac{K}{R_2} - \frac{1}{R_3 + R_4}\right] \quad K = \frac{R_4}{R_3 + R_4}. \tag{3}$$

The circuit is capable of realizing an ideal inductance or a lossy inductance in the form of a series RL or a parallel RL by simple circuit tuning. The results for these three cases are summarized in Table I.

It is noted that the parameter K is adjusted by tuning the grounded resistor R_4. The capacitor C controls the magnitude of L without affecting the realizability conditions. For the realization of an ideal inductor, two degrees of freedom are available, thus R_1 and R_3 may be taken arbitrarily. The design equations for the remaining circuit components are:

$$R_4 = R_3 \tag{4}$$

$$\frac{1}{R_2} = \frac{2}{R_1} + \frac{1}{R_4} \tag{5}$$

$$C = \frac{2L}{R_1^2}. \tag{6}$$

It is worth noting that this inductance circuit is generated and generalized from the oscillator circuit described recently by the author [5]. The simulated inductance may be employed in realizing tunable filters using element replacement techniques.

III. Conclusions

It is proved that an active-gyrator circuit is realizable with four resistors and a single CC II along similar steps to the Orchard-Willson active-gyrator which requires six resistors and a single operational amplifier [6].

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Comments on the Performance of Maximum Entropy Algorithms

N. ANDERSEN

***Abstract*—Two points recently brought up in this journal concerning the performance of maximum entropy spectral analysis are discussed. First, an additional recursive formula is presented which simplifies and reduces the computational load of the Burg algorithm. Second, attention is drawn to some recent results in the geophysics literature related to the proper selection of prediction filter length.**

The maximum entropy method developed by Burg [1], [2] has become increasingly used for spectral analysis of a large variety of data records in physics, geophysics, astronomy, and engineering. Algorithms have been developed by Andersen [3] for real data and Smylie *et al.* [4] for complex valued records. The principal mathematical characteristics have been explored, though unsolved problems still remain [5], [6]. This letter briefly comments upon two points of a more technical character which become important when maximum entropy estimation is carried out in practice, as recently brought up in this journal [7]–[9].

Burg [2] outlined an iteration scheme for estimation of the prediction filter coefficients $(a_{1m}, \cdots, a_{mm})$ for a given data record $(X_1, \cdots, X_N)$, here assumed for simplicity to be real. The reader is referred to Andersen [3] for details on the recursive technique. First we shall present an additional recursion formula which apparently has been overlooked in the literature, and which simplifies the Burg algorithm. It involves the denominator in the expression for evaluation of a_{mm}:

$$a_{mm} = \frac{2 \sum_{n=1}^{N-m} b_{m-1\,n+1} b_{m-1\,n}^{1}}{\sum_{n=1}^{N-m} (b_{m-1\,n+1}^{2} + b_{m-1\,n}^{1\,2})} = \frac{2 \sum_{n=1}^{N-m} b_{m-1\,n+1} b_{m-1\,n}^{1}}{\text{den}\,(m)}. \tag{1}$$

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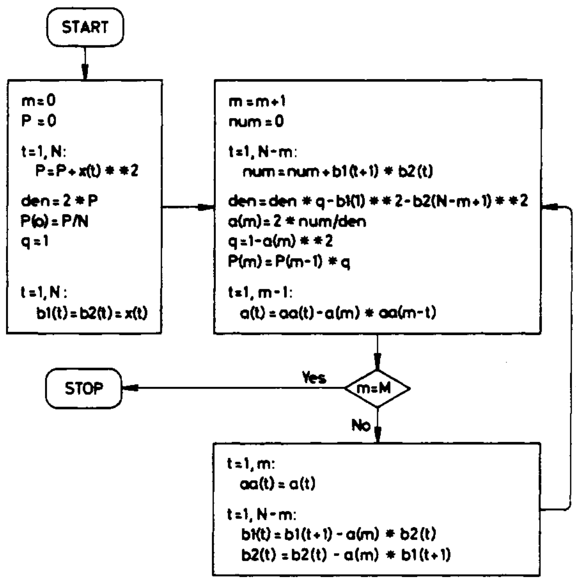

Fig. 1. Flow diagram of the improved Burg algorithm. The last statement in the second box should be omitted in the first step when $m - 1 = 0$.

The two b-arrays are the prediction errors when the data series is passed through the filter $(1, -a_{m1}, \cdots, -a_{mm})$ in the forward and backward direction, respectively:

$$b_{mt} = X_{t+m} - \hat{X}_{t+m}, \qquad \hat{X}_t = \sum_{i=1}^{m} a_{mi} X_{t-i} \tag{2a}$$

$$b^1_{mt} = X_t - \hat{X}^1_t, \qquad \hat{X}^1_t = \sum_{i=1}^{m} a_{mi} X_{t+i}. \tag{2b}$$

They obey simple recursive equations [3] by means of which it is found that

$$\text{den}\,(m) = \text{den}\,(m-1) \cdot (1 - a^2_{m-1\,m-1}) - b^2_{m-1\,1} - b^{1\,2}_{m-1\,N-m+1}. \tag{3}$$

This formula reduces the number of operations needed for evaluation of the denominator from $4(N - m)$ to five, in each step. (The quantity $(1 - a_{mm}{}^2)$ has already been evaluated in the previous iteration step.) The simplified Burg algorithm is summarized in Fig. 1. Generalization to complex valued data is straightforward. The computational load of the Burg algorithm is thus reduced compared to the estimate of Jarrott [9] based on the old version [3] of the algorithm. Experience over the years with the improved algorithm has shown consistently better performance than the old version, especially when long data series or low-noise signals are analyzed. Double precision numbers are recommended. However, for extremely low-noise signals, accumulation of round-off errors may occasionally become important in the numerator, causing the computed value of a_{mm} to become slightly larger than unity, though this is not algebraically possible, cf (1). The problem may be avoided in these cases by returning to the defining equation (1) for den (m).

A second problem raised by Jarrott [9] concerns the deviation between the prediction error calculated from the algorithm and the prediction error calculated from the actual data record, using (2), a problem which becomes of special importance in connection with the selection of an appropriate filter length m. It might have escaped notice that this problem has been treated and at least partly solved in recent geophysics literature by introduction of the so-called Final Prediction Error (FPE) criterion by Akaike [10]. Space does not allow a presentation of the FPE criterion here, but algorithms incorporating this feature can be found in [5] which also gives a good discussion of the concepts. It has, however, recently been shown that for the Burg estimate the FPE criterion tends to overestimate the filter length [11]. Furthermore, the FPE criterion is not always a sufficient condition for optimal spectral resolution [6].

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Noninvasive Measurement of Biological Information by Nuclear Magnetic Resonance: Measurement of Relaxation Time of a Particular Target by Magnetic Focusing Method

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Abstract—In order to clarify the possibility of detecting cancer noninvasively by the measurement of the relaxation time, model experiments were performed by applying our proposed magnetic focusing method to conventional nuclear magnetic resonance (NMR) techniques. It was made clear that the relaxation time of a particular target was selectively estimated from the proximity of the target by this method.

Biological applications of the nuclear magnetic resonance (NMR) phenomenon have been proposed as a method for cancer detection [1] or image formation of two-dimensional cross-sectional area related to the distribution of protons in the body [2], [3], [4]. Recently, Mansfield produced cross sectional images of a finger *in vivo* by NMR [5].

We have conducted a study related to a noninvasive method applying NMR technique. We proposed a new method in which a specified magnetic field is focused on a particular target within the body (we call this a magnetic focusing method) [6]. Fundamental problems such as the generation of the focusing magnetic field and the attainable resolution

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