Comments on "Computer Simulation of Frequency Swept Imaging"

D. N. SWINGLER AND C. S. NILSEN

Abstract–It is demonstrated that the swept frequency hologram obtained in the above paper by Farhat, Dzekov, and Le Det exhibits anamalous behavior and that, in general, their 2-D imaging process seems unacceptable.

In the above letter,[1] Farhat *et al.* provided a computer simulation of a 2-D imaging process in which one orthogonal axis of the recording aperture is synthesised on sweeping the illuminating frequency. It is the intention of this note to demonstrate that this imaging technique is, in general, not acceptable. The well defined rectangular image shown in Fig. 2(b) of the above paper can best be described as fortuitous.

There are four major points of interest.

a) The far-field distribution due to the rectangular object used as an example by Farhat *et al.* is described therein simply by a sinc (X) sinc (Y) function. Unfortunately, this does not take cognizance of the nonnegligible curvature of the object wavefront which produces a significant phase gradient over each sidelobe and which must be taken into account when forming a hologram via a linear receiver array as in Farhat *et al.*

b) Notwithstanding a), the internal form of the actual sinc (X) sinc (Y) function used is also of interest. It is rewritten here for convenience as

$$U_{m,n} = \text{sinc}\left[\frac{m}{5}\left(1+\frac{n}{25}\right)\right]\text{sinc}\left[10\left(1+\frac{n}{25}\right)\right] \quad \text{(A)}$$

where m and n represent the Cartesian coordinates of the aperture. The mn product in the first sinc $(\cdot)$ is of concern as it has no equivalent in the conventional sinc $(\cdot)$ sinc $(\cdot)$ distribution obtained from a "straight" far-field recording. It thus seems unreasonable to expect that the modified function should, in general, produce the same type of rectangular image observable on Fourier transforming the conventional expression.

c) It is easy to demonstrate the peculiarities of the "hologram" due to (A) (see Fig. 2(a) in Farhat *et al.*, page 1453) on the optical bench. If, for instance, a segment at the "bottom" [relative to Fig. 2(a)] of the hologram is blocked off, then a gap appears in the central region of the original well defined rectangular image (see Fig. 2(b) in Farhat *et al.*, p. 1453), leaving two rectangles, one to each side of the dc point. The aspect ratio of these rectangles can be changed at will by varying the depth of blockage (their height remains more or less that of the original rectangle). Similarly, if one side of the hologram is obscured, then again areas of the original rectangle disappear this time leaving a set of four rectangles arranged checker-board fashion.

Now blocking of the hologram simply implies a reduction in the recording aperture, yet the changes in the image are very much of a gross kind. This is obviously an unacceptable characteristic of a practical imaging system.

d) As a final check on the performance of this type of simulation, an equivalent binary hologram was fabricated for the case of a circular disk object. (The diameter of the disk equalled the longer side of Farhat's rectangle, the offset (B) was reduced by a factor of 5 to provide sufficient hologram fringes; all the other parameters were as in Farhat *et al.* The phase curvature was ignored, and the hologram closely resembled a set of nested V's). The desired circular distribution could not be observed in the optical Fourier transform plane, even on changing the hologram's aspect ratio.

In conclusion, it seems that the 2-D image processing technique described by Farhat *et al.* is somewhat suspect.

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[1] N. H. Farhat, T. Dzekov, and E. Le Det, *Proc. IEEE* (Lett.), vol. 64, pp. 1453–1454, Sept. 1976.

Reply[2] *by N. H. Farhat*[3]

The anomalous behavior of the hologram referred to in Swingler and Nilsen's comments has already been observed in our work and a note to this effect explaining this behavior and giving a somewhat modified hologram that gives rise to a proper image is being published in these PROCEEDINGS [1], thus establishing the viability of frequency swept imaging.

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The peculiar, but in itself interesting, behavior of the hologram, described in c) of Swingler and Nilsen's comments and also observed in our work, is traced in [1] to the deviation of the frequency synthesized scan lines used in displaying the computed hologram from their theoretically predicted length and orientations. The deviation was deliberately introduced at the time to simplify computer printout. When the precise orientation and length of the frequency synthesized scan lines produced by the postulated linear array of receivers is utilized in displaying the computed hologram, the anomalous behavior disappears. As shown in [1], a trapezoidally shaped hologram, instead of the original rectangular hologram, is obtained and an edge enhanced image of the rectangular object is recovered from it. Such an edge enhanced image is what would be expected in conventional Fourier transform holography when the object diffraction pattern is recorded over a two-dimensional aperture that is not centered *in front* of the object and therefore records a nonsymmetric portion of the spatial frequency spectrum of the object.

The quadratic phase factor produced by the curvature of the object wavefield over the receiver array, referred to in a) of Swingler and Nilsen's comments, was left out since it can be removed ultimately with a conical lens and its inclusion in [1] would have complicated the discussion unnecessarily. To see this we refer to [1, Fig. 1] and write the expression for the far-field amplitude produced in the $x_h - y_h$ plane by the object wave

$$0(x_h, y_h) = \frac{j}{\lambda Z_0} e^{-jkZ_0} e^{-j(k/2Z_0)(x_h^2+y_h^2)} \cdot \iint_{-\infty}^{\infty} D(x_0, y_0)\, e^{j(k/Z_0)(x_h x_0 + y_h y_0)}\, dx_0\, dy_0$$

$$= \frac{j}{\lambda Z_0} e^{-jkZ_0} e^{-j(k/2Z_0)(x_h^2+y_h^2)}\, \tilde{D}\left(\frac{k}{Z_0}x_h, \frac{k}{Z_0}y_h\right) \quad (1)$$

where $D(x_0, y_0)$ is the object transmittance (or reflection) function assumed to be nondispersive (independent of the wavenumber k) and where $\tilde{D}$ is the Fourier transform of D.

It is instructive to recall at this point that in conventional Fourier transform holography, the quadratic phase term in x_h and y_h appearing in (1) is eliminated by recording a hologram of the object wavefield in the back focal plane of a convergent lens, while in lensless Fourier transform holography the same is achieved through the use of a reference point source in the object plane, located suitably close to the object. In either case, a hologram containing a record of 0 (and its conjugate) is recorded in which the quadratic phase appearing in (1) is not present, thus permitting image reconstruction through a direct Fourier transform operation.

In frequency swept imaging, the removal of the quadratic phase term is effected as follows.

The field amplitude at the mth receiver in Fig. 1 of our original letter, on page 1453, located at $x_h = B$, $y_h = m\Delta y$, $m = 0, \pm 1, \pm 2, \cdots, \pm(N/2)$ [where $(N+1)$ is the total number of elements in the array and Δy is the spacing between adjacent receivers] can be expressed using (1) in a form that shows the explicit dependence on wavenumber k

$$0(k, m\Delta y) = \frac{jk}{2\pi Z_0} e^{-jZ_0[1+(B^2/2Z_0^2)]k} e^{-j(k/2Z_0)(m\Delta y)^2} \cdot \tilde{D}\left(\frac{B}{Z_0}k, \frac{km\Delta y}{Z_0}\right) \quad (2)$$

where in practice $(k_1 = k_0 - (\Delta k/2)) < k < (k_2 = k_0 + (\Delta k/2))$, $\Delta k = k_2 - k_1$ being the wavenumber sweep width, and $k_0 = (k_1 + k_2)/2$ is the mean wavenumber of the sweep. Note that the quadratic phase term in x_h has been changed in (2) into a linear phase dependence on k because $x_h = B$. The quadratic phase term in y_h (now $m\Delta y$) remains. The effect of this term can now be determined by considering the display of the field $0(k, m\Delta y)$ observed by the mth receiver along its equivalent frequency synthesized scan vector in the $x_h - y_h$ plane. The coordinates (x_m, y_m) of a point on the scan vector of the mth receiver can be expressed as

$$y_m = \frac{m\Delta y}{B} x_m, \quad x_m = B\frac{k}{k_1} \quad (3)$$

which can be used in (2) to yield an expression for the linearly frequency swept hologram displayed in the (x_h, y_h) plane

$$0(x_h, y_h) = F(x_h, y_h)\,\tilde{D}\left(\frac{k_1}{Z_0}x_h, \frac{k_1}{Z_0}y_h\right)\cdot S(x_h, y_h) \qquad (4)$$

where

$$S(x_h, y_h) = \sum_{m=-N/2}^{N/2} \delta(x_h - x_m, y_n - y_m)$$

describes the form of the fanshaped, frequency synthesized sampling format produced by the linear array of receivers (see [1, Fig. 2])

$$F(x_h, y_h) = \frac{jk_1 x_h}{2\pi Z_0 B}\, e^{-jk_1[Z_0/B+B/2Z_0]x_h}\, e^{-j(k_1/2(Z_0/B)x_h)\,y_h^2}$$

Fourier transformation of the frequency swept hologram (4) with the aim of retrieving a diffraction limited image of the object will be hindered by the presence of the multiplying factor F. In particular, the presence in F of the multiplier x_h and the quadratic phase term in y_h complicate matters. On the other hand the linear phase term in x_h is not problematic and is in fact desirable since when retained it gives rise in the reconstruction of the hologram (4) to an off-axis image that is conveniently isolated from the zero order light in a manner similar to off-axis reference beam holography.

Equation (4) shows that the quadratic phase term in y_h can be removed through multiplication of the frequency swept hologram by the factor $\exp\{[jk_1/2f_{eq}(x_h)]\,y_h^2\}$ where $f_{eq}(x_h) = (Z_0/B)x_h$. This multiplying factor can be realized with a conical lens [2]. In this connection, the synthesis of an inverse conical lens from two cylindrical lenses of equal focal lengths in tandem one of which is tilted is particularly attractive because of simplicity and the ability to adjust the rate of linear dependence of the composite focal length on the x_h coordinate by merely changing the tilt angle of the one lens. Such an inverse conical lens is useful in frequency plane filtering of the quadratic phase term.

Since k is a variable under control, the multiplier x_h can be removed from the recorded data by either multiplying the output of each receiver [as in (2)] by $1/k$ or by overlaying the recorded hologram with a mask whose amplitude transmittance is uniform in the y_h direction and varies as $1/x_h$ in the x_h direction.

Removal of the quadratic phase term and the multiplier x_h in $F(x_h, y_h)$ permits now Fourier transforming the hologram (4) from the (x_h, y_h) domain into the image domain (ω_x, ω_x) to obtain

$$\mathcal{F}\left\{\frac{e^{j(k_1/f_{eq})y_h^2}}{x_h}\,0(x_h, y_h)\right\} = \frac{j}{2\pi Z_0}\,D((\omega_x + b), \omega_y) * \tilde{S}(\omega_k, \omega_y) \qquad (5)$$

where b, the off-axis position of the image, is proportional to the quantity $(Z_0/B + B/2Z_0)$, ω_x and ω_y are proportional to the image plane coordinates and $\mathcal{F}\{\ \}$ symbolizes the Fourier transform of the bracketed quantity. Note that since the object position b is a nonlinear function of B, the imaging operation described by (5) is not spatially invariant as far as object location is concerned. This is a direct consequence of the destruction of the spatial invariance of the diffraction integral under Fraunhofer (far-field) conditions. However, for the geometry of Fig. 1 [1], where an e^{-jkZ_0} reference signal is provided at the receivers, a linear dependence of off-axis object position on B is obtained.

Through the use of a sufficiently wide frequency sweep and a wide receiver array, the extent of the main lobe of $\tilde{S}$ in the ω_x, ω_y domain can be made quite narrow so that the convolution in the right hand side of (5) will yield a diffraction limited image of the object D. However, because of the nonsymmetric nature of the wavenumber aperture defining S (only positive values of k are realizable in practice), the transform $\tilde{S}$ will contain an exponential term linearly dependent on ω_x. The effect of this exponential term in the convolution of (5) can be shown to lead under certain conditions to the enhancement of the edges of the diffraction limited image that are parallel to the ω_y image coordinate.

The procedure of frequency swept imaging outlined in some detail here is quite general as far as the shape of the planar object is concerned. Therefore an image of the circular disc considered in d) of Swingler and Nilsen's comment can be obtained by sorting out the field data measured by the receiver array in accordance to the precise frequency synthesized scan format described by (3) as done in [1].

The one qualification on the object function made in the above analysis is that D be nondispersive, i.e., it is not a function of k. This automatically rules out complex phase objects that are dispersive by definition. For such dispersive objects, image detail will not be purely geometrical but will be modified by the spectral characteristics of the object. In this fashion an object "signature" related to both geometrical and spectral properties of the object can be obtained.

Finally it is worthwhile to point out that the treatment of frequency swept imaging given here is for planar objects. Its extention to three-dimensional perfectly reflecting objects provides the transition to the inverse scattering problem [3], [4].

References

[1] D. C. K. Chan, N. H. Farhat, M. S. Chang, and J. D. Blackwell, "New results in computer simulated frequency swept imaging," *Proc. IEEE* (Lett.), vol. 65, pp. 1214–1215, Aug. 1977.

[2] E. N. Leith, "Some microwave analogs of holography," in *Advances in Holography*, Vol. 2, N. Farhat, Ed. New York: Dekker, 1976, pp. 20–43.

[3] R. M. Lewis, "Physical optics inverse diffraction," *IEEE Trans. Antennas Propagat.*, vol. AP-17, pp. 308–314, May 1969.

[4] S. Rosenbaum-Raz, "On scatterer reconstruction from far-field data," *IEEE Trans. Antennas Propagat.*, pp. 66–70, Jan. 1976.

Power Factor Control of Synchronous Motors

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Abstract—**An adaptive control method is presented for controlling and maintaining the power factor of synchronous motors close to unity in the presence of load disturbances. Experimental results are provided.**

One of the features which makes a synchronous motor attractive for industrial applications is that its power factor, and hence armature current, can be controlled by adjusting its field excitation. The curve that shows the relation between a constant load is known as a *V*-curve because of its shape [1], [2]. For a constant power output, the armature current is a minimum at unity power factor and increases as the power factor decreases. Points to the right of the minimum of a *V*-curve correspond to overexcitation and leading current input, while points to the left correspond to underexcitation and lagging current input. As the load changes, the location and shape of the *V*-curve change. The purpose of this letter is to present a method for determining and maintaining the minimum of the *V*-curve, that is the unity power factor, in the presence of load disturbances.

The system considered is shown in Fig. 1, where K_1 is a constant, $i_f(t)$ is the rotor current, $i_a(t)$ is the stator current and $d(t)$ is a load disturbance. The current $i_a(t)$ is measured using an instrument $G_1(s)$, where

$$G_1(s) = \frac{K_2}{\tau s + 1}. \qquad (1)$$

K_2 and τ may vary slowly with time. i_a is related to i_f through the *V*-characteristics of the synchronous motor. $r(t)$ is the output of the controller and is taken to be

$$r(t) = \epsilon K \qquad (2)$$

where K is a constant and $\epsilon = \pm 1$. It is required to determine the decision variable $\epsilon(t)$ which brings and maintains $i_a(t)$ close to its minimum value even when the load on the motor is changed. $i_a(t)$ is not directly accessible, that is, $i_m(t)$ must be used.

If at any instant of time t_0, ϵ is changed say from ϵ_i to ϵ_f, then there will be a discontinuity in the second derivative of $i_m(t)$ given by

$$J_t = \frac{(\epsilon_f - \epsilon_i)K}{s}\,\frac{K_1}{s}K^*\,\frac{K_2}{\tau s + 1}\,s^2 s\,\Big|_{s\to\infty}$$

$$= \frac{KK_1K^*K_2}{\tau}(\epsilon_f - \epsilon_i) \qquad (3)$$

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