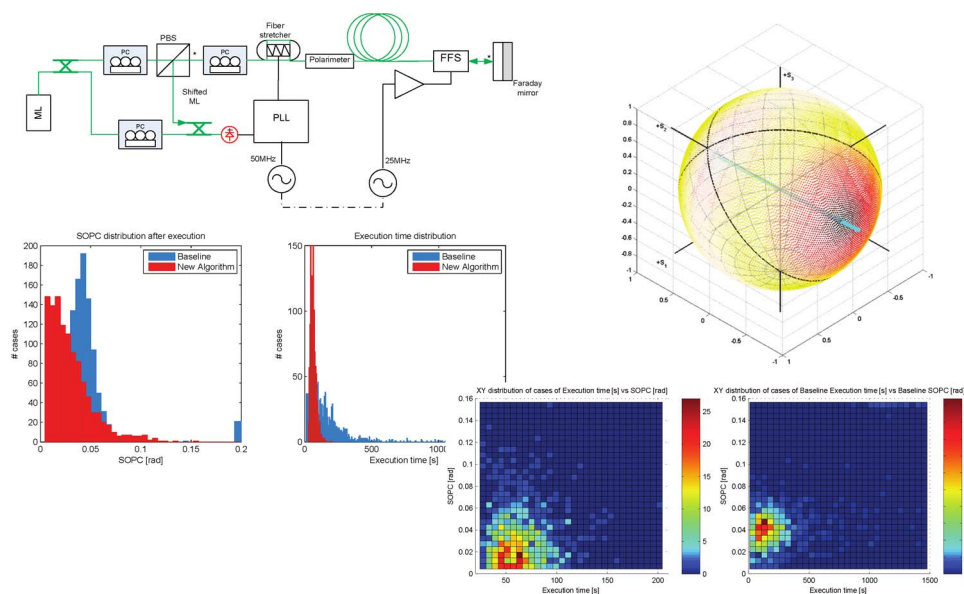


# Fast Algorithm to Find the Minimal Polarization Change Points on a Fiber-Optic Stretcher Using One Polarization Rotator and a Polarimeter

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# Fast Algorithm to Find the Minimal Polarization Change Points on a Fiber-Optic Stretcher Using One Polarization Rotator and a Polarimeter

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**Abstract:** Fiber stretchers are used to stabilize the effective length of optical fiber links, but if they are not carefully calibrated, they can produce polarization changes and instabilities, which interact with the polarization mode dispersion (PMD) of the fiber link producing phase changes on the transmitted signals. Those can be minimized if the input polarization is aligned to one of the two orthogonal polarization states of the fiber stretcher, which show minimum polarization change across the dynamic range of the stretcher. This paper presents a noniterative method to find these eigenstates, dubbed “sweet spots,” based on the Jones matrix Eigen-analysis known for measuring PMD of optical fibers, and its application to the line length correction system of the Atacama Large Millimeter/submillimeter Array (ALMA). The paper reviews the theoretical relationship between the sweet spots and the principal states of polarization of the fiber in the stretcher. The proposed method was found to be more accurate and faster than the iterative optimization method originally used for this system.

**Index Terms:** Optical fiber, analog signal, phase drift, phase reference, state of polarization change (SOPC), polarization mode dispersion (PMD), fiber stretcher, line length corrector.

## 1. Introduction

Finding the input state of polarization (SOP) that causes a minimal SOP change (SOPC) for a fiber-optic stretcher is a complex task. The position of these SOP points, dubbed “sweet spots,” changes with time and can be located at any point on the Poincaré sphere. Maintaining a stable SOP over the entire optical link is important in many applications, one of them being the line length correction (LLC) scheme in the ALMA Photonics Central Local Oscillator (PCLO) system [1], [2], which uses a fiber stretcher driven by a phase-lock loop (PLL) to stabilize the optical path length of a photonic signal that is sent to many antennas distributed over distances of up to 15 km (see Fig. 1). At the antennas, a photomixer is used to receive the mm-wave signal in order to lock the PLL of the local oscillator multiplier chain in the antenna front end. Any SOPC generated by the fiber stretcher is converted, through the polarization mode dispersion (PMD) of the buried long fiber and other optical components including the photomixer, into a phase change at the output of the photomixer. To avoid this undesired effect, a polarization controller (PC) is used to set the input polarization of the stretcher to

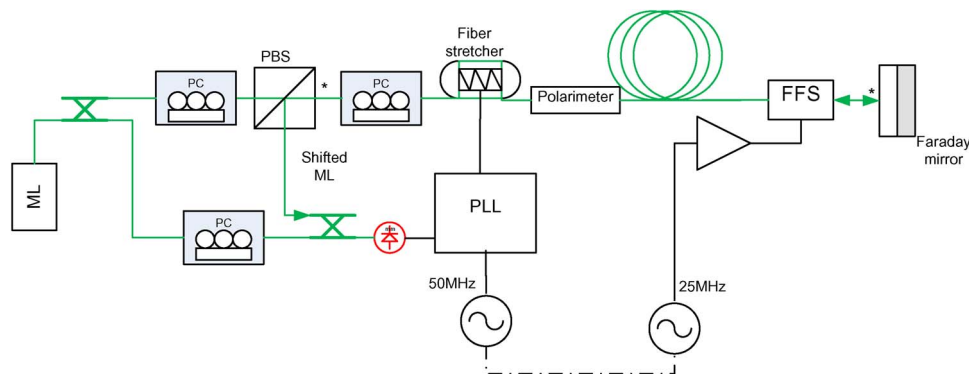


Fig. 1. Simplified diagram of the ALMA line-length correction (LLC) system. A signal generated at the ultra phase-stable master laser (ML) is transported to the Faraday rotator mirror, where it is reflected back, thus crossing the fiber frequency shifter (FFS) twice, producing a total frequency shift of 50 MHz, and furthermore undergoing a Poincaré sphere inversion of its polarization state. The signal returning to the polarizing beam splitter is then mixed with a direct ML signal to produce a beat note at 50 MHz. This signal is used to lock the PLL that drives the fiber stretcher, effectively stabilizing the link total length between the asterisks. If a SOPC of the optical signal is produced by the stretcher, due to the PMD of the rest of the fiber line and other optical components such as the photomixer (not shown in this figure), a phase change of the signal at the antenna will be produced, thus possibly rendering the antenna phase out of the phase stability specification. Therefore, just before the input of the fiber stretcher, a PC is inserted which is used together with the polarimeter to find the fiber stretcher sweet spots.

one of the minimal-SOPC spots [3], [4]. In the current ALMA system, the calibration of the input SOP is performed using an algorithm that iteratively searches for the right input polarization to the stretcher. This algorithm is effective, but its convergence time is typically a few minutes and depends on the initial set point of the fiber stretcher [5]. Since ALMA is a complex system with expensive observing time, every opportunity to speed up and simplify operations is welcome.

We propose that the position of these sweet spots can be calculated in a fixed number of SOP measurements, using a method developed by Heffner [7], [8] and generalized by Shieh [4], for measuring PMD using Jones Matrix Eigen-analysis (JME). While Heffner's original method was intended to use measurements of the transition matrix at two different wavelengths, we use the identical algorithm with transition matrices from two different stretching positions of the fiber stretcher. This is justified because Heffner's original method was derived not only for the wavelength regime but also more generally for the phase regime, and a phase change can be realized either by a wavelength change or an optical path length change.

Besides being faster than the previously used method, our proposed method is also implementable using the existing LLC hardware, which already includes a nondeterministic PC and, after the stretcher, a polarimeter. This is achieved through a small relaxation of the original requirements of the Jones method for experimental determination of the transmission matrix, supposing that this uncertainty, resulting from the absence of a polarimeter before the stretcher, is represented by a constant transformation matrix.

In Section 2, the theoretical background is reviewed, as well as the applicability of the algorithm to the ALMA LLC system. In Section 3, we explain the preliminary implementation into the ALMA photonic system, a preliminary measurement, and the experimental methodology. This is followed by a description of the technical implementation (see Section 4), the presentation of the field test results, and the conclusions.

## 2. Theoretical Background

### 2.1. Description of the Method

The method presented here, as well as the old algorithm, is based on the empirical observation that the SOP output of the stretcher for a constant SOP input rotates around the axis through both

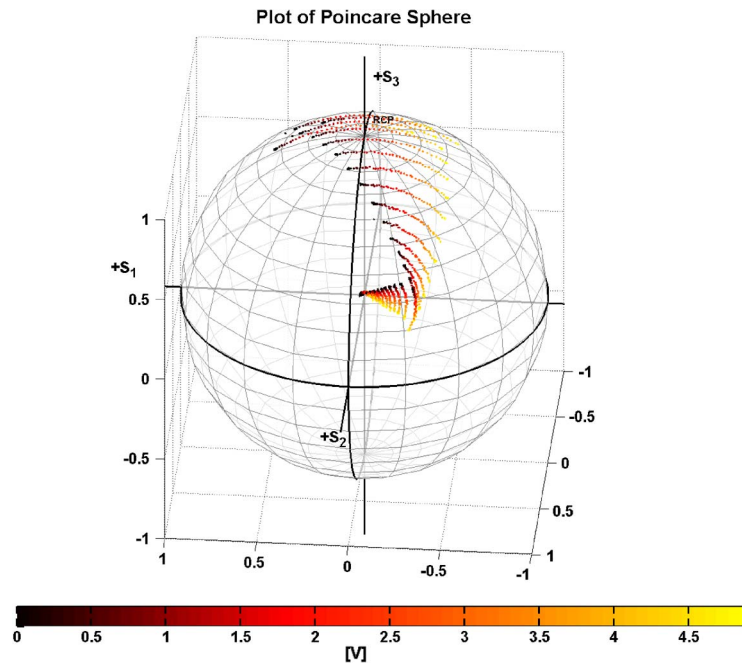


Fig. 2. Poincaré sphere showing the polarization trajectory of a stretcher while being optimized using the iterative baseline algorithm. It can be seen that the polarization tends to rotate around the finally found “sweet spot” after many sweeps of the stretcher. In the plot the color bar indicates the piezoelectric actuator voltage of the stretcher in a given moment. In this view, the sweet spot is hovering over the origin of the Cartesian axes at the center of the view.

sweet spots (see Fig. 2) when the fiber stretcher is actuated. This is the same effect found in the PMD, where the SOP rotates around the axis through the principal SOP (PSP) when a monochromatic wave is swept in frequency through a birefringent fiber [9].

The explanation is that a wavelength change is equivalent to a change of the fiber length by mechanical stretching. In both cases, the ratio of the optical path length to the wavelength is changing the same way, and therefore, the effect on the output phase is the same. This is only true if the stretching range is small enough (linear approximation), so that the fiber cross section and its elliptical distortions in the bends of the stretcher windings are assumed not to change. Under this assumption, we would see the SOP describing a noncircular path over the Poincaré sphere.

Furthermore, measured PSPs (eigenstates with respect to wavelength) and sweet spots (eigenstates with respect to length change) should be identical if the fiber length affected by the stretching is the complete fiber between the PC and the polarimeter. However, due to instrumental reasons, the stretched section is always shorter than the complete fiber, preventing the experimental verification of the similitude between sweet spots and PSP. Just for clarity, we will keep calling the PSPs “sweet spots” when they are measured using a length change instead of a wavelength change, as in the new algorithm.

The calculation required to measure the PMD vector using this scheme was formulated by Heffner [7], [8] and is based on the determination of the rotation of the SOP around the axis through the PSPs.

As previously explained, we propose that this algorithm can be used analogously to calculate the position of the sweet spots by replacing the parameter of the frequency change by the stretcher state change, a principle which was proposed already earlier in a more generalized way [4].

The final method then consists of measuring the transmission matrix  $T$  at two different stretcher positions yielding  $T(L)$  and  $T' = T(L + \Delta L)$ , where  $L$  is the fiber length, and the sweet spots are given by the eigenvectors of the product matrix  $T'T^{-1}$ .

For an experimental determination of the two forward transmission matrices  $T$  of an unknown, but linear, time-invariant optical device, according to Heffner, the algorithm of Jones is used [10]: A

laser followed by a PC generates a linearly polarized wave parallel to the  $x$ -axis, and the output polarization is measured by a polarimeter, resulting in the Jones vector  $h$ . Then, signals polarized parallel to the  $y$ -axis and the bisector axis between  $x$  and  $y$  are transmitted, giving the vectors  $v$  and  $q$ , respectively.

The transmission matrix can then be calculated as

$$T = \begin{bmatrix} k_1 k_4 & k_2 \\ k_4 & 1 \end{bmatrix}; \quad k_1 = \frac{h_x}{h_y}; \quad k_2 = \frac{v_x}{v_y}; \quad k_3 = \frac{q_x}{q_y}; \quad k_4 = \frac{k_3 - k_2}{k_1 - k_3}. \quad (1)$$

## 2.2. Effect of PDL

Just as in Heffner's scheme, a nonorthogonality of the calculated sweet spots can be produced by a polarization-dependent loss (PDL). In the case of a fiber stretcher, the PDL should be kept very small by design [6], and in fact, this is valid for the whole ALMA photonic system. Therefore, such an effect can be neglected here.

## 2.3. Effect of Components Between the PC and the Stretcher

The proposed method requires the production of specific and precise input polarizations to the fiber stretcher. Unfortunately, the current hardware of the ALMA LLC system only has a piezoelectric polarization rotator based on fiber squeezers at the input and a polarimeter at the output of the stretcher, making it impossible to control precisely the polarization state delivered to the stretcher. However, there is a way around this problem.

To model this uncertainty, let us suppose that we have a precise way to set the required input polarization, but between this and the stretcher, there is an unknown polarization-changing device with a transfer matrix  $T_u$  that does not change with time or the stretcher position. Then, if the method is applied, two transmission matrices will be obtained  $T'_k$  and  $T_k$ ; each of them containing the matrix  $T_u$  as

$$T'_k = T' T_u; \quad T_k = T T_u. \quad (2)$$

Then

$$T'_k T_k^{-1} = (T' T_u)(T T_u)^{-1} = T' T_u T_u^{-1} T^{-1} = T' T^{-1}. \quad (3)$$

As shown in (3), the uncertainty on the input polarization is not important for the method, unless it changes during the measurement. This means that the method can be applied by only measuring the output polarization and keeping the relationship between the output polarization vectors consistent with the explained Jones method, which means two of them need to be mutually orthogonal and a third located half way between the formers. With this, from the point of view of the measurement and actuation, the current hardware is sufficient to apply the method to the ALMA LLC System.

## 2.4. First Experimental Validation Through Measurement and Simulation

As a preliminary validation of the concept before its full implementation into a large group of telescopes, the algorithm was run with a single fiber stretcher, and the results were tested using a simulation in MATLAB. This simulation was performed using a data set of transmission matrices obtained from a production fiber stretcher of the ALMA photonic system, using an Agilent optical component analyzer, an instrument that uses JME to measure PMD in optical components. One part of its output is the Jones transmission matrices at different wavelengths used to calculate the PMD of the component. The instrument has a deterministic PC to set the required input polarizations for the device under test (DUT) and a polarimeter to measure the output polarizations of the DUT. The light source was an Agilent tunable external cavity diode laser.

To generate the plot shown in Fig. 3, the transmission matrices obtained with this instrument at 1556 nm were processed with a MATLAB script that calculated the SOPC between two respective stretcher positions with input polarizations covering the whole Poincaré sphere. A color code is



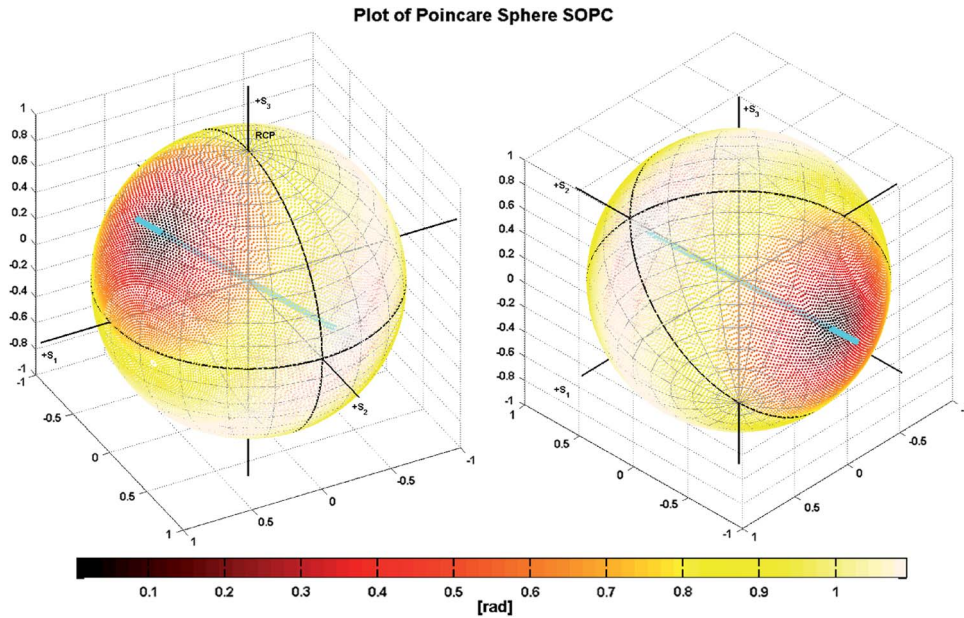


Fig. 3. Poincaré spheres of the calculated SOPC produced by a fiber stretcher actuation, showing two orthogonal sweets spots of a fiber stretcher (the dark areas, scale in radians). It also shows the calculation of sweet spots with the proposed method, as thick vectors. There is a good agreement between the exhaustive method and the positions calculated by the proposed method.

used to indicate the magnitude of the SOPC as a function of the input polarization. This plot clearly shows two orthogonal low-SOPC areas. Superimposed are the two sweet-spot vectors calculated by the proposed algorithm. The SOPC is defined by

$$\text{SOPC} = \arccos(\vec{S}_1 \cdot \vec{S}_2) \quad (4)$$

where  $S_1$  and  $S_2$  are the intensity-normalized Stokes vectors representing the SOP of the output of the respective measured matrices for two different stretcher positions.

### 3. Implementation of the New Algorithm

#### 3.1. Technical Aspects

The algorithm was implemented as a python script running in the ALMA control computers. Since it uses only 41 multiplications and 20 additions, it is possible to implement it at the firmware level on any microcontroller. The final implementation will depend on the current usage of processor resources in the LLC microcontroller.

A point that requires special attention is how to precisely control the SOP at the output of the stretcher, using the PC at the input. This PC has as control input of 4 V used to drive four squeezers that effectively rotate the output SOP around an axis. The control strategy for this PC is simple. One volt at a time is changed using a fixed step size to find the minimum angular distance between the current and the desired SOP. When the distance stops decreasing and starts increasing, the step size is multiplied by  $-0.5$ . If the distance change between two voltage steps is smaller than an absolute fixed threshold, or if it is smaller than a certain fraction of the current distance to the desired SOP, the method will move on to the next squeezer. It was found that  $1/40$  of the current SOPC works well for the second threshold. In this way, the method will give only a coarse optimization to squeezers that, at best, can change the SOP only a little towards the desired one and give a more precise and time-consuming optimization to those that can change the SOP more closely to the required SOP, for any starting SOP. The algorithm cycles through the four squeezers

in quick succession until the distance from the desired SOP is smaller than a second threshold, which is chosen as a compromise between short execution time and the precision that polarization control is capable of achieving, which, in ALMA, was set to 0.009 rad. It was found that this method always converges regardless of the starting SOP.

In order to reduce the execution time of the algorithm, the SOPs to be set for the measurement are selected as follows.

Whatever is the SOP at the beginning of the algorithm, it will be the first of the three SOP  $h$ , given in Stokes space using the Cartesian coordinates of the Poincaré sphere. The second SOP will be  $v = -h$  and the last one, i.e.,  $q$  will be selected by replacing the smallest Cartesian coordinate of  $h$  by minus the sum of the squares of the other two coordinates, divided by the coordinate to be replaced. For example, if  $S_1$  is the smallest coordinate, then it shall be replaced by  $S'_1$

$$S'_1 = \frac{-(S_3^2 + S_2^2)}{S_1}. \quad (5)$$

This is repeated for the other cases; if the smallest coordinate is 0, it is replaced by 1, and the other two are set to 0.

In addition, we must make sure that none of the vectors  $h$ ,  $v$ ,  $q$  is parallel to the axis  $x$  because, if that were the case, the calculation of the transmission matrix  $T$  would produce a division by 0 as can be seen in (1). It is also recommended that none of them be close to parallel to the  $x$  axis in order to reduce the impact of the measurement noise, how far away will depend on the noise of the SOP measurements. If the previous condition is not accomplished, then the vectors to use will be defined by

$$h = [1, 0, 0, 1]; \quad q = [1, 0, 1, 0]; \quad v = [1, 0, 0, -1]. \quad (6)$$

Another solution is to change the reference frame to avoid the division by 0, but this would increase the complexity of the method.

Although the proposed algorithm has no dependence on the starting point, this particular implementation has some degree of dependence, because of the design decision of using the current SOP as a seed to choose the SOPs required by the algorithm for the calculations. This decision reduces, on average, the time required to perform the calibration by 25%.

### 3.2. Experimental Methodology

For the algorithm performance test, two python scripts were written to perform recurrent runs of the baseline algorithm and the proposed one, using the maximum number of LLCs that was possible each time. Before each run, the starting SOP was randomized. The LLCs used for the test were located at the array operations site of ALMA located on the Chajnantor plateau at 5000 m above sea level. The tests were conducted during engineering time; for this reason the number of runs was limited to 50. For each data set, the test was run using every available LLC. However, it was not always possible to have the same set of LLCs on different days or for sufficient times in sequence to ensure a valid comparison. The presented analysis was made using only the LLCs that were present and had completed 50 runs in both data sets. For these comparisons of the performance of both algorithms, the SOPC is measured using the internal measurement routine of the LLC. This routine sweeps the stretcher position, while the SOPC is monitored. The reported values are the maximum SOPCs detected during this process.

### 3.3. Experimental Results

Following the methodology explained in the previous section, Figs. 4 and 5 were produced. They show the performance of both the base line and the proposed algorithm, as a function of the execution time and the final SOPC achieved after execution of each algorithm. In both cases, a smaller number indicates better performance. The performance indicators for each of the methods are presented in Table 1.

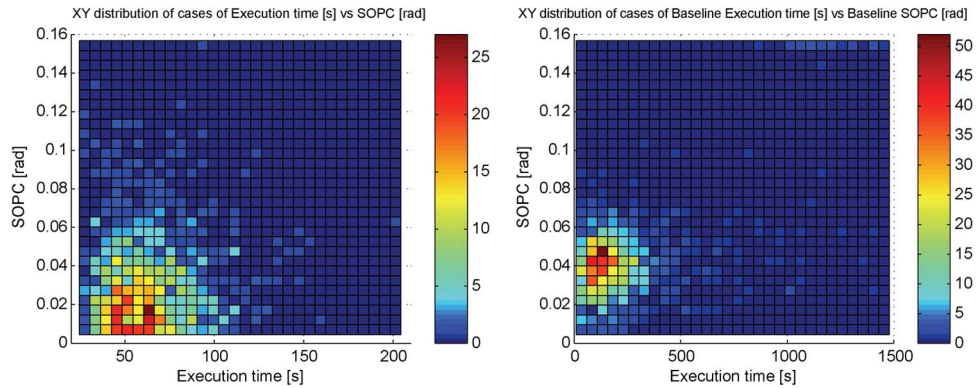


Fig. 4. Color XY distributions of case counts, which shows the interdependence between execution time and final SOPC. The SOPC axis is limited to 0.16 rad to allow easier comparison between the baseline and the proposed algorithm. A consequence of this is that all data points  $> 0.16$  rad are accumulated in this grid row, mostly seen in the plot for the baseline algorithm (right panel).

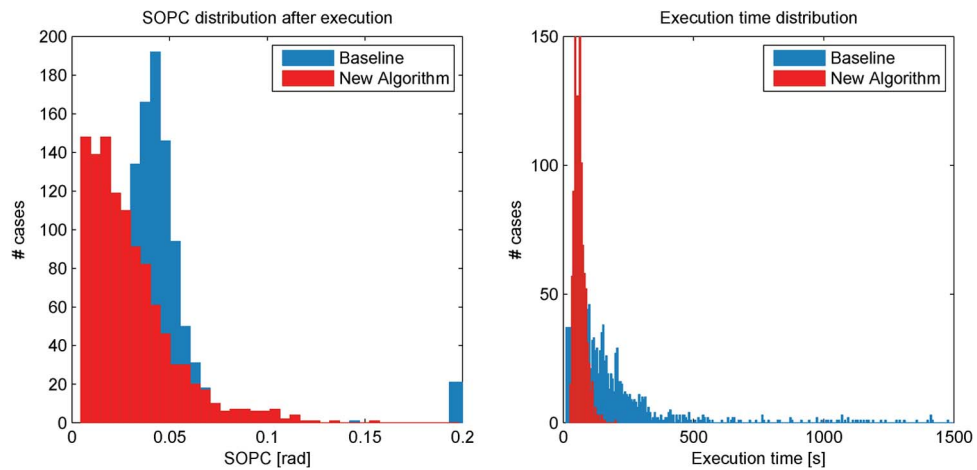


Fig. 5. Histograms showing the execution time and the SOPC after each execution. The left plot shows the SOPC after the execution of both algorithms, and the right plot shows execution time distribution of them. As mentioned in Section 2, the SOPC is measured by sweeping the fiber stretcher across its complete range and measuring the polarization change during this process; the reported value is the maximum measured change. The highest bin of the SOPC plot accumulates the values bigger than 0.2 rad, from this value onwards the calibration can be considered invalid.

TABLE 1

Compared performance statistics between baseline and the proposed algorithm

Algorithm	$\bar{x}$ Exec. time [s]	$\sigma$ Exec. time [s]	$\bar{x}$ Remaining SOPC [rad]	$\sigma$ Remaining SOPC [rad]
Proposed algorithm	62.5	21.1	0.0297	0.0218
Baseline	211.5	219.8	0.0583	0.1409

#### 4. Summary and Conclusion

In this paper, a method to calculate the minimal SOPC input polarization on fiber stretchers has been presented. We have demonstrated that characterizing the fiber stretcher at two positions is sufficient to find the position of the sweet spots. A good agreement of the sweet spots found in this way with a characterization of the SOPC through the whole Poincaré sphere has been shown using a simulation. The developed algorithm based on this has proven to be simple, fast, and accurate.



A field test was conducted using 50 LLCs, showing that, with the help of a simple polarization control algorithm, the proposed method reduces the average time to finish the LLC calibration process by 61% and the standard deviation of this time by 90%. It also reduced the remaining average SOPC after optimization by 50% and its standard deviation by 85%.

The equivalence between a wavelength step and a fiber length change produced by a well-designed stretcher has been explained, and we show that the sweet spots and the PSPs are conceptually the same.

The reported algorithm may be used in fiber stretcher applications where good polarization stability is required and time constraints exist, e.g., an environment that requires permanent recalibration of the stretcher polarization input, thereby producing a significant reduction in the accumulated calibration time.

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