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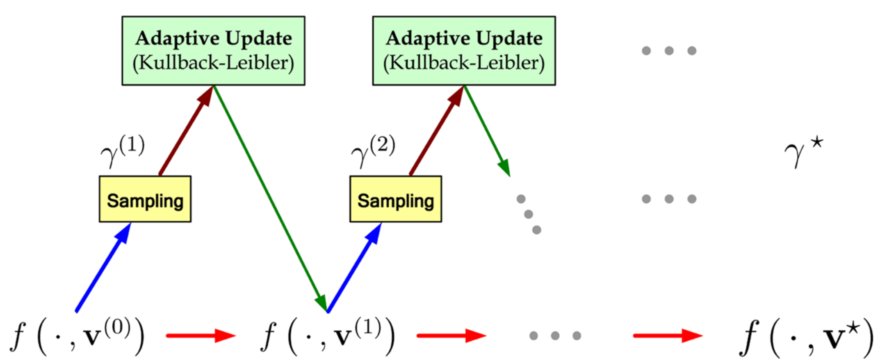

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Cross-Entropy Optimization for the Design of Fiber Bragg Gratings

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Abstract: This paper considers the synthesis problem of a low-dispersion fiber Bragg grating (FBG) filter. This problem can be formulated as a nonlinear objective problem in practice. However, solving this problem with traditional methods is difficult because of its multimodal and ill-conditioned character. Inspired by the efficient ability of cross-entropy optimization (CEO) method to find near-optimal solutions in large search spaces, we propose the application of CEO method to search for design variables to satisfy the target design criteria. Computer simulation results show that the proposed CEO method can successfully achieve the target values of the design specifications. It also provides a reliable convergence to the near-optimum solution. These findings indicate that the CEO algorithm is more appropriate for the FBG filter design compared with conventional particle swarm optimization method.

Index Terms: Cross-entropy optimization (CEO), fiber Bragg grating (FBG), filter design.

1. Introduction

Fiber Bragg gratings (FBGs) have attracted considerable attention because of their potential to perform various functions for optical signal processing applications [1]–[4]. A powerful FBG filter design method should determine a promising fiber grating index modulation profile corresponding to a given spectrum in order for the method to fit different applications. Various methods for the synthesis of FBG-based filters have been proposed in the literature [5]–[12] to achieve this goal.

Existing synthesis approaches for FBG design can be classified into two categories according to the design principle involved: inverse scattering method and optimization method. The former directly calculates the required index modulation profiles from the targeted reflection spectrum by solving the mathematical inverse problem. The latter employs stochastic optimization techniques such as genetic algorithm [9] and particle swarm optimization (PSO) [12] to search for index modulation profiles that minimize the difference between the synthesized and the targeted spectrum. Optimization method allows weighting mechanisms to be integrated into the prescribed FBG-based filter specifications for synthesis. Therefore, the index modulation profiles of the synthesized FBGs obtained by optimization method can be implemented practically, easily, and flexibly. This work concentrates on optimization method as applied to the FBG filter design problem.

The first step in optimization method is to define the objective function of the FBG synthesis problem. This definition will guide stochastic optimization technique in obtaining an optimum index modulation profile of the FBG structure. To design dispersionless FBG filters that have a flat group delay profile, Baskar *et al.* in [12] first proposed a novel fitness function to characterize the spectral features of the FBG filter through the parameters of interest. PSO is applied to optimize these parameters and to design a low-dispersion FBG filter. The design in [12] can search for the index modulation profile to achieve a reasonable reflectivity spectrum and a smooth in-band group delay response, but there is still room for improvement. With the success of cross-entropy optimization (CEO) method [14] in solving challenging continuous multiextremal optimization problems, we propose the use of CEO method to solve FBG synthesis problems. Simulation results show that the proposed CEO-based FBG filter design algorithm provides better performance than the PSO-based FBG filter design algorithm [12] for three desired values of maximum reflective power.

2. System Model and Problem Definition

We consider a simple FBG model with N-equal length piecewise uniform sections [11]. To obtain the spectral and phase responses of the FBG filters, we use transfer matrix method to solve the FBGs because of the high computational efficiency and the high reliability of the method [13]. Each section can be represented by a 2×2 analytic transfer matrix in transfer matrix method. Once all matrices for individual sections are known, the transfer matrix for the *entire* grating structure can be obtained via multiplication of individual transfer matrices.

To achieve a desired reflectivity spectrum and a smooth in-band group delay response for a low-dispersion FBG design, Baskar *et al.* in [12] proposed a novel fitness function to characterize the reflective spectrum and the group delay of the FBG filter through six design parameters: 3-dB bandwidth (BW), sidelobe level (SLL), first null BW (FNBW), in-band ripple (Rrip), maximum reflective power (Rmax), and in-band group delay ripple (Trip). With these design specifications defined, the "error" between the calculated specification and the target specification can be evaluated with the following fitness function [12]:

$$\mathcal{F} = (\mathrm{FNBW}_d - \mathrm{FNBW})^2 + (\mathrm{BW}_d - \mathrm{BW})^2 + (\mathrm{Rmax}_d - \mathrm{Rmax})^2 + \mathcal{G}(\mathrm{Rrip}_d - \mathrm{Rrip}) + \mathcal{G}(\mathrm{SLL}_d - \mathrm{SLL}) + \varpi \times \mathcal{G}(\mathrm{Trip}_d - \mathrm{Trip}) \quad (1)$$

where the subscript d denotes the desired target values of the design specifications, $\varpi \in [0, 1]$ is the nonnegative weighting coefficient that is given according to the importance of the corresponding error term, and

$$\mathcal{G}(x) \overset{\mathrm{def}}{=} \begin{cases} 0, & \text{if } x \geq 0 \\ x^2, & \text{otherwise.} \end{cases} \quad (2)$$

Note that the first five terms in (1) will contribute to dissatisfying the desired reflective spectrum, whereas the last term corresponds to the minimization of in-band group delay ripple. Therefore, the objective of this FBG filter design problem is to use optimization techniques such as PSO [12] to find the optimized solution of the FBG design through a search for the minimum of (1). Based on a number of trials, the PSO can possibly search for design variables to satisfy the target design. However, the search results of the PSO-based FBG filter design algorithm in [12] are almost changed with trials, and the obtained solutions are generally not optimal. Therefore, a novel FBG filter design based on CEO algorithm is proposed in the next section.

3. FBG Design Using CEO Method

CEO method was first proposed by Rubinstein [14] to solve rare-event estimation problems within complex networks and was afterward successfully extended to solve both combinatorial and

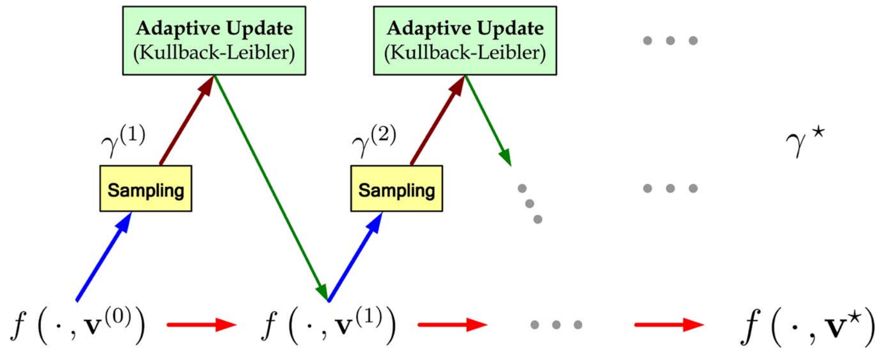

Fig. 1. The CEO adaptive approach.

continuous optimization problems such as [15]–[17]. In principle, CEO is an iterative population-based search method similar to most stochastic optimization techniques, such as genetic algorithm and PSO. However, unlike the conventional search methods that operate directly on the samples in the candidate population, CEO maintains a *distribution* of possible solutions and updates this distribution accordingly. Therefore, the goal of CEO is to find a sample distribution that generates an optimal solution. To achieve this objective, CEO follows the following two iterative phases for the optimization problem:

1) randomly generates candidate solutions by sampling from a predefined probability density function (pdf).
2) updates the parameters of the pdf based on the selected elite samples using cross-entropy minimization.

Formally, we let $\mathcal{F}(\mathbf{X})$ denote the fitness function of the considered optimization problem. CEO is then applied to *minimize* the fitness function. In the first phase, we randomly generate a set of samples $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_K$ with respect to pdf $f(\cdot; \mathbf{v})$, where $\mathbf{v}$ is the parameter vector to be optimized. In the second phase, we select a threshold value of the objective function γ and only focus on those samples whose performance is better than this threshold, i.e., samples in which $\mathcal{F}(\mathbf{X}_k) \leq \gamma$ are selected. Those good samples are referred to as the "elite" samples. Then, the new parameterized distribution $f(\cdot; \mathbf{v}')$ is updated to close the target distribution $f(\cdot; \mathbf{v}^\star)$ by minimizing the Kullback–Leibler divergence, i.e., cross entropy. This process completes one iteration. Based on the above procedure, CEO iteratively updates $f(\cdot; \mathbf{v})$ to produce a family of pdf's $f(\cdot; \mathbf{v}^{(1)}), f(\cdot; \mathbf{v}^{(2)}), \ldots, f(\cdot; \mathbf{v}^\star)$, which are directed by $\gamma^{(1)}$, $\gamma^{(2)}, \ldots, \gamma^\star$ toward the neighborhood of the optimal density function $f(\cdot; \mathbf{v}^\star)$, as shown in Fig. 1. Only a few parameters, such as the size of the sample and the percentage of elite samples, need to be determined using the CEO algorithm. This procedure indicates that the CEO algorithm is easy to implement.

Next, we describe our detailed implementation of the CEO method for determining the optimal index modulation profile of the FBG filter design problem. As mentioned earlier, CEO involves an iterative procedure where each iteration can be broken down into two phases. In the first phase, we generate K random candidate solutions $\{\mathbf{X}_k\}_{k=1}^K$ for the objective function in (1), where $\mathbf{X}_k = [X_{k1}, X_{k2}, \ldots, X_{kN}] \in \mathbb{R}^N$ represents the index modulation values of N-uniform sections, and each individual component X_{kn} is drawn *independently* from the specified pdf $f(\cdot; \mathbf{v}_n)$. A normal distribution function $\mathcal{N}(\mu_n, \sigma_n)$ with its mean μ_n and standard deviation (STD) σ_n in most applications is selected as the pdf of $f(\cdot; \mathbf{v}_n)$. In this case, the sampling distribution for $\mathbf{X}_k$ can be characterized by a normal $\mathbb{R}^N$-dimensional distribution with independent components with means $\mu = [\mu_1, \mu_2, \ldots, \mu_N]$ and STDs $\sigma = [\sigma_1, \sigma_2, \ldots, \sigma_N]$, which is denoted by $\mathcal{N}(\mu, \sigma)$.

In the second phase, parameter vectors μ and σ are updated using only the elite samples to steer the search toward the global optimum in subsequent iterations. Based on the available K random samples $\{\mathbf{X}_k\}_{k=1}^K$ generated in the first phase, the elite samples are selected according to the performance criterion. These elite samples form an elite set Ω, represented by $\Omega = \{\mathbf{X}_k : \mathcal{F}(\mathbf{X}_k) \leq \gamma\}$, where $\mathcal{F}(\mathbf{X}_k)$ is the fitness value of the index modulation profiles $\mathbf{X}_k$ and γ is the threshold that determines the elite set. In practice, the most common method of determining the elite set is to choose $\lceil \rho K \rceil$ *best* samples in the elite set Ω, where $0 < \rho < 1$ is called the elite ratio and $\lceil \cdot \rceil$ is the ceiling operation used to return the smallest integer that is not less than the argument. In this case, γ is chosen as

$$\gamma = \widetilde{\mathcal{F}}_{(\lceil \rho K \rceil)} \tag{3}$$

where $\widetilde{\mathcal{F}}_{(k)}$ represents the kth *order statistic* of the sequence $\mathcal{F}(\mathbf{X}_1), \mathcal{F}(\mathbf{X}_2), \ldots, \mathcal{F}(\mathbf{X}_K)$. The order statistics of $\mathcal{F}(\mathbf{X}_1), \mathcal{F}(\mathbf{X}_2), \ldots, \mathcal{F}(\mathbf{X}_K)$ is the ordered sample given by

$$\widetilde{\mathcal{F}}_{(1)} < \widetilde{\mathcal{F}}_{(2)} < \cdots < \widetilde{\mathcal{F}}_{(K)} \tag{4}$$

where

$$\widetilde{\mathcal{F}}_{(1)} = \min\{\mathcal{F}(\mathbf{X}_1), \mathcal{F}(\mathbf{X}_2), \ldots, \mathcal{F}(\mathbf{X}_K)\} \tag{5}$$

$$\widetilde{\mathcal{F}}_{(K)} = \max\{\mathcal{F}(\mathbf{X}_1), \mathcal{F}(\mathbf{X}_2), \ldots, \mathcal{F}(\mathbf{X}_K)\} \tag{6}$$

and $\widetilde{\mathcal{F}}_{(k)}$ is the kth smallest among $\mathcal{F}(\mathbf{X}_1), \mathcal{F}(\mathbf{X}_2), \ldots, \mathcal{F}(\mathbf{X}_K)$.

After the elite set Ω is determined, $\widetilde{\mu} = [\widetilde{\mu}_1, \widetilde{\mu}_2, \ldots, \widetilde{\mu}_N]$ and $\widetilde{\sigma} = [\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_N]$ can be updated via

$$\widetilde{\mu}_n = \frac{1}{\lceil \rho K \rceil} \sum_{k \in \Omega} X_{kn} \tag{7}$$

$$\widetilde{\sigma}_n^2 = \frac{1}{\lceil \rho K \rceil} \sum_{k \in \Omega} (X_{kn} - \mu_n)^2 \tag{8}$$

respectively, for the next parameter vectors μ' and σ'. Note that the update criterion in (7) and (8) are obtained through minimization of the cross entropy (or Kullback–Leibler distance) between the updated random mechanism and the probability distribution of the selected elite samples. The algorithm might converge to a degenerate distribution in typical applications, and this phenomenon might result in an algorithm stuck in a suboptimal solution. Therefore, we use the linear parameter smoothing procedure provided by [14] to update the parameter vectors as

$$\mu' = \alpha \times \widetilde{\mu} + (1 - \alpha) \times \mu \tag{9}$$

$$\sigma' = \alpha \times \widetilde{\sigma} + (1 - \alpha) \times \sigma \tag{10}$$

where $\alpha \in (0, 1]$ is the smoothing parameter. The steps for the proposed algorithm are summarized as follows:

Step 1: Set the iteration counter $t := 1$, and initialize the mean vector $\mu^{(0)}$ and STD vector $\sigma^{(0)}$, where the super-index of μ and σ denotes the iteration index.
Step 2: Draw K random samples $\{\mathbf{X}_k^{(t)}\}_{k=1}^K$ from density function $\mathcal{N}(\mu^{(t-1)}, \sigma^{(t-1)})$ and calculate their objective values $\{\mathcal{F}(\mathbf{X}_k^{(t)})\}_{k=1}^K$.
Step 3: Use (3) in order to obtain $\gamma^{(t)}$ to determine the elite set $\Omega^{(t)}$.
Step 4: Update $\mu^{(t)}$ and $\sigma^{(t)}$ via (7) and (8), respectively.
Step 5: Obtain the smoothed $\mu^{(t)}$ and $\sigma^{(t)}$ by (9) and (10), respectively.
Step 6: Repeat Steps 2 to 5 for $t := t + 1$ until the predefined number of iterations is met.

4. Numerical Results

Computer simulations are conducted to compare the performance of the proposed CEO-based FBG filter design algorithm with that of the PSO-based FBG filter design algorithm in [12]. The

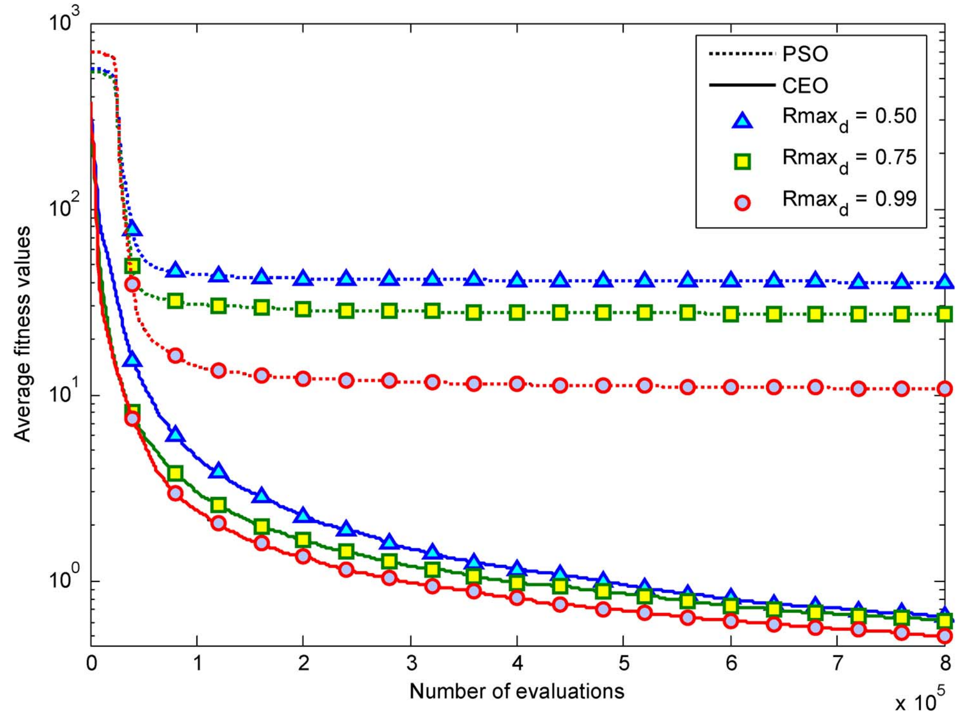

Fig. 2. Average fitness values versus the number of evaluations.

TABLE 1

Comparisons of the means and STDs of the six design parameters using various algorithms

Design Example	Rmax_d	Algorithms	BW (nm) mean	BW (nm) STD	FNBW (nm) mean	FNBW (nm) STD	SLL (dB) mean	SLL (dB) STD
Case 1	0.99	PSO	0.200	2.23×10^{-3}	0.251	1.78×10^{-3}	−34.330	10.240
		CEO	0.200	2.54×10^{-17}	0.250	1.52×10^{-16}	−41.466	0.823
Case 2	0.75	PSO	0.200	2.13×10^{-3}	0.250	2.61×10^{-3}	−21.378	16.122
		CEO	0.200	2.54×10^{-17}	0.250	1.52×10^{-16}	−41.751	0.889
Case 3	0.50	PSO	0.200	4.00×10^{-3}	0.249	3.83×10^{-3}	−16.489	17.017
		CEO	0.200	2.54×10^{-17}	0.250	1.52×10^{-16}	−44.375	2.144

Design Example	Rmax_d	Algorithms	Rrip mean	Rrip STD	Rmax mean	Rmax STD	Trip (ps) mean	Trip (ps) STD
Case 1	0.99	PSO	0.451	0.149	0.977	1.91×10^{-2}	12.275	16.228
		CEO	0.341	0.035	0.990	2.80×10^{-4}	0.473	0.118
Case 2	0.75	PSO	0.443	0.092	0.797	8.78×10^{-2}	12.855	16.426
		CEO	0.331	0.064	0.750	8.79×10^{-4}	0.551	0.153
Case 3	0.50	PSO	0.448	0.128	0.605	1.67×10^{-1}	19.725	23.692
		CEO	0.326	0.092	0.500	1.55×10^{-3}	0.556	0.156

MATLAB codes for implementing the PSO-based FBG filter design algorithm are available from [12], and their parameter settings basically follow those in [12]. The considered FBG filter design problem is also the same as that in [12]. We consider an FBG filter with a BW of 0.2 nm and a grating length of 4 cm, where the center wavelength of the filter is set as $\lambda_c = 1550$ nm, and the grating length is divided into $N = 20$ uniform sections. The allowable index modulation profile values are limited between -3×10^{-4} and 3×10^{-4}, indicating that only 0 or π phase shifts are permitted in this design, as was also done in [12]. The target design specifications used in our simulations are as follows: SLL of 40 dB, BW of 0.20 nm, FNBW of 0.25 nm, Rrip of 0.5 dB, and Trip

TABLE 2

Obtained design parameters of the PSO-optimized FBG filters and the CEO-optimized FBG filters for the three design examples, where those values inside rectangular boxes fail to reach the desired target

Design Example	Rmax_d	Algorithms	BW (nm)	FNBW (nm)	SLL (dB)	Rrip (dB)	Rmax	Trip (ps)
Case 1	0.99	PSO	0.20	0.25	−40.256	0.358	0.990	0.705
		CEO	0.20	0.25	−43.049	0.368	0.990	0.244
Case 2	0.75	PSO	0.20	0.25	−39.958	0.345	0.750	0.654
		CEO	0.20	0.25	−41.799	0.330	0.750	0.289
Case 3	0.50	PSO	0.20	0.25	−40.617	0.429	0.500	1.534
		CEO	0.20	0.25	−45.829	0.395	0.500	0.320

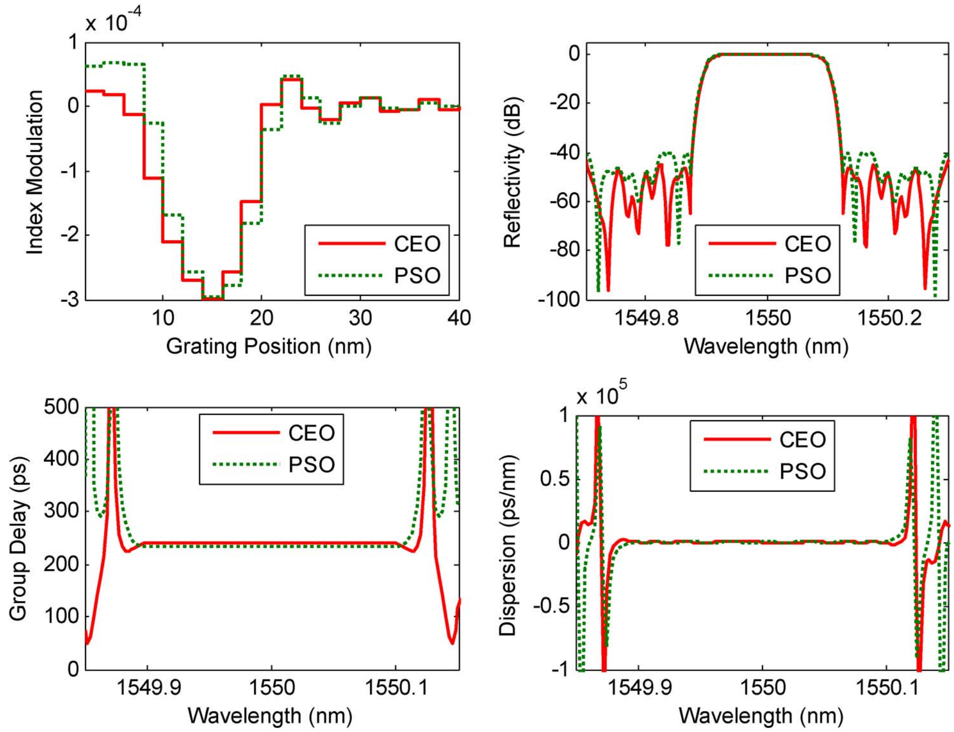

Fig. 3. Comparison of characteristics of optimized FBG filters by different methods with $\mathrm{Rmax}_d = 0.99$.

of 0.5 ps. To verify the effectiveness of the proposed method for different applications, we also test the proposed CEO algorithm and the conventional PSO for an FBG filter specification with three different, desired power reflection coefficients: Rmax $= 0.99$, Rmax $= 0.75$, and Rmax $= 0.50$.

Regarding the parameter settings for CEO method, we set the fraction of samples selected for updating $\rho = 0.1$ and the smoothing factor $\alpha = 0.8$, and the algorithm is stopped when the iteration number t exceeds the predetermined value. The CEO and the PSO are both population-based search methods, so the computational complexity of population-based algorithms is typically analyzed in terms of the number of evaluations of the objective function. The number of evaluations for the PSO and the CEO is the number of samples K times the maximum number of iterations t, that is, $K \times t$.

The first simulation is targeted to determine the convergence behavior of the various algorithms with the use of the fitness functions, along with the fitness converged to. Fig. 2 shows the average fitness values versus the number of evaluations for three different desired power reflection coefficients, where 100 independent trials are conducted for each desired power reflection

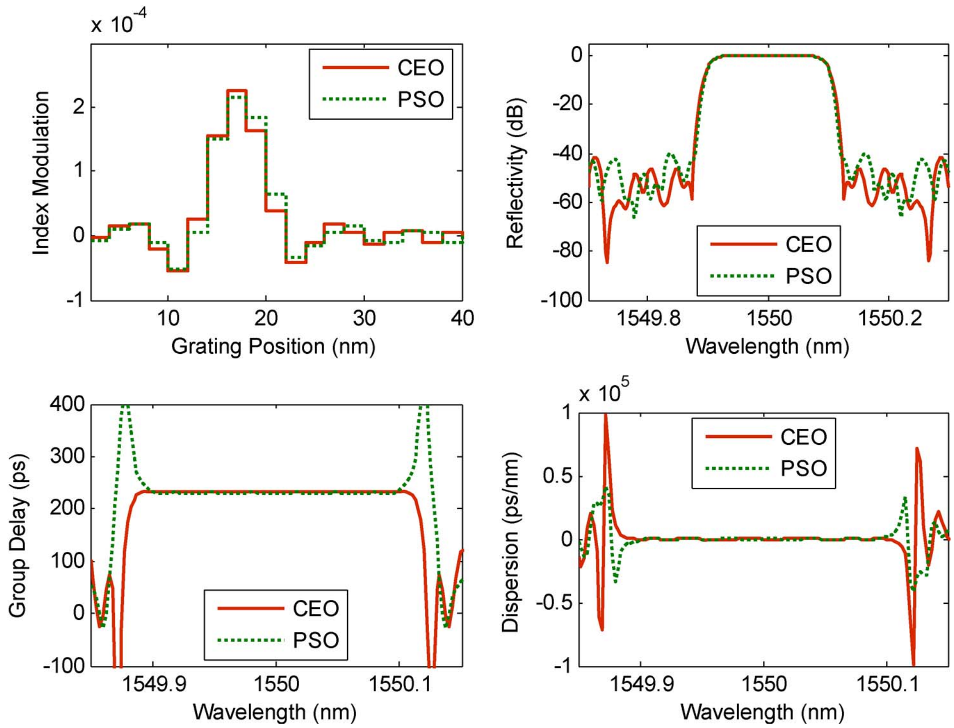

Fig. 4. Comparison of characteristics of optimized FBG filters by different methods with $Rmax_d = 0.75$.

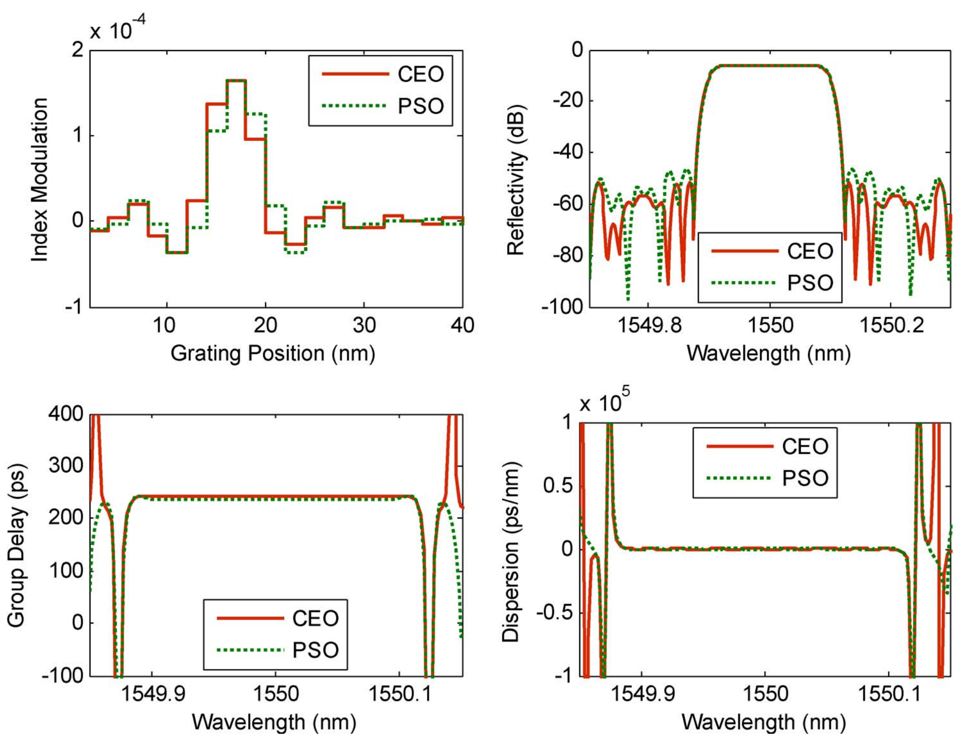

Fig. 5. Comparison of characteristics of optimized FBG filters by different methods with $Rmax_d = 0.50$.

coefficient with the use of the PSO and the CEO, respectively. The solutions optimized by the CEO algorithm have far better average fitness values than those optimized by the PSO regardless of the desired power reflection coefficients used. The mean and the STD of the obtained design specifications after 8×10^5 evaluations over 100 runs are also given in Table 1. From the simulation results, the obtained design specifications with the proposed CEO method can provide both better means and STDs of the values compared with those obtained with the PSO method for the three design examples.

Based on 100 independent trials in the first simulation, the solutions with the best fitness values are chosen as the design parameters of the PSO-optimized FBG filters and the CEO-optimized FBG filters. Table 2 shows the desired target parameters obtained with the PSO method and the CEO method for $\text{Rmax}_d = 0.99$, $\text{Rmax}_d = 0.75$, and $\text{Rmax}_d = 0.50$. Table 2 shows that the desired target specifications obtained by the proposed CEO algorithm are achieved successfully in every case. However, PSO algorithm may likely be trapped in local solutions. As a result, a few design parameters obtained by PSO algorithm cannot meet the desired target specifications. The index modulation profiles, reflection magnitude responses, group delays, and dispersion characteristics corresponding to the best solutions obtained in 100 runs for $\text{Rmax}_d = 0.99$, $\text{Rmax}_d = 0.75$, and $\text{Rmax}_d = 0.50$ are shown in Figs. 3–5, respectively. From the in-band group delay and in-band dispersion profiles of the three designs, both PSO and CEO not only have flat in-band group delay ripples but also achieve a nearly ideal, dispersionless, spectral profile within the in-band spectrum. However, the sidelobe suppression levels achieved by the PSO are lower than those by the CEO. Therefore, the proposed CEO method is a more suitable technique than the PSO method to design FBG filters, as it can well meet target specifications.

5. Conclusion and Future Work

A CEO-based algorithm was presented to synthesize a low-dispersion FBG-based filter with a specified reflective spectrum and group delay response. Simulation results showed that the proposed CEO method obtained better grating structures than the PSO method, and these structures are well suited for the design of low-dispersion FBG filters. Motivated by the efficient ability of the CEO method to design a low-dispersion FBG-based filter, we plan to apply further the CEO to other FBG design examples in our future work for different applications, such as the design of triangular, bandpass, and multichannel FBG filters, and so on.

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