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Propagation in a Decoupled Twin-Core Waveguide: A Frequency-Domain Analysis

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Abstract: The behavior of two waveguides, which are decoupled at a single frequency when a broadband pulse is launched into one of them, was studied recently in the time domain. We reinvestigate it in the frequency domain. This contributes to the clarification of the scope of the validity of previous results and allows extending them to dispersive waveguides. New results include a power spectral density conservation law and a revised calculation of the pulse breakup distance.

Index Terms: Waveguides, pulse shaping, slow and fast light (SFL).

1. Introduction

The question of whether twin waveguide cores at finite distance can be decoupled or not was studied 20 years ago in theoretical terms [1] but became of practical interest more than a decade later, thanks to the advent of photonic band gap (PBG) waveguides [2]–[8]. Recent contributions (e.g., [9]) have shown that closely spaced waveguides are still of interest, regardless of the fabrication technology. Also, theoretical interest in the question has been revitalized by Liu and Chiang [10]. They pointed out that the spectrum of short optical pulses can extend far out of the frequency range where the decoupling recipe [1]—which, in principle, works at a single antiresonant frequency—is an acceptable approximation. Working in the time domain, they reached the conclusion that an input pulse, of completely arbitrary shape $a(t)$, launched into one of the cores breaks into four output pulses, namely, a pair in each core, separated by a time gap that grows linearly with distance, as the pulses travel down the waveguides. This result looked surprising at first sight, but was nicely explained [10] in terms of differential group velocities of the even and odd supermodes of the twin waveguides and has been demonstrated experimentally under various circumstances [13], [14].

In our opinion, those results deserve further investigation. To explain why, let us focus on the group velocities of the subpulses. If we take properly into account that, in the notation used in [10], $t = 0$ at a distance z at the time of arrival of a pulse traveling in the individual waveguide $t' = z/v_g$, where v_g is the waveguide group velocity, we find the propagation velocities of the subpulses

$$v'_{g\pm} = v_g/(1 \pm C' v_g) \tag{1}$$

where C' is the derivative with respect to ω (at the decoupling frequency ω_0) of the coupling coefficient. Therefore, the statement [10] that the subpulses "propagate at *slightly* different group velocities" applies only for $|C'|v_g \ll 1$. There is no indication in [10] of any restriction of this kind. If this restriction is violated, then the decoupled twin-core waveguides become a suitable candidate for slow and fast light (SFL). Given that, as we said before, decoupling is due to an antiresonant behavior, this SFL device would belong to the class of cascaded resonators (see, e.g., [11]).

Aiming at clarifying this issue, in the next Section, we will rephrase the problem in the frequency domain. We will reach the conclusion that the results of [10] are mathematically correct but imply a behavior of the coupler versus frequency which cannot be realistic over an arbitrarily broad frequency range. Clearly, we restrict ourselves to the linear regime, while [10] could take into account SPM. What we gain, on the other hand, is—in addition to our main task, which is to shed some light on the scope of validity of the results—to

1) show that the results of [10] can be extended to dispersive waveguides;
2) derive a simple conservation law for power spectral density, which is physically obvious but not easy to grasp from the time-domain formulation.

Incidentally, we will also recalculate the distance the subpulses must travel in order to be resolved in time. It was rather strange that the result found in [10] was independent of the group velocity. In fact, we will show that this is an acceptable approximation as long as $|C'|v_g \ll 1$.

2. Frequency-Domain Equations and Their Solution

Consider two infinitely long parallel waveguides, at finite distance from each other in the transverse plane. Suppose that the region between them has been tailored [1] so that the cores are decoupled at the angular frequency ω_0. Then, taking the Fourier transforms of [10, eqs. (1) and (2)] and using the same notation, dropping the SPM terms, we get

$$j\frac{d\psi_1}{dz} - \Delta\omega C'\psi_2 + \frac{\beta_2}{2}\Delta\omega^2\psi_1 = 0 \quad (2)$$

$$j\frac{d\psi_2}{dz} - \Delta\omega C'\psi_1 + \frac{\beta_2}{2}\Delta\omega^2\psi_2 = 0 \quad (3)$$

where $\Delta\omega = \omega - \omega_0$, C' was defined in the introduction, β_2 is the group velocity dispersion (GVD), and ψ_i $(i = 1, 2)$ are the Fourier transforms of the time-domain envelopes a_i of the waves which travel in the $+z$ direction.

In the nondispersive case $(\beta_2 = 0)$, differentiating the first equation with respect to z, and replacing ψ_2 from the second one, we see immediately that both ψ_i satisfy a harmonic equation. Hence, if at $z = 0$ waveguide 1 is excited and waveguide 2 is at rest $(a_2(z = 0) = 0)$, we get

$$\psi_1 = A(\Delta\omega)\cos(\Delta\omega C' z) \quad (4)$$

$$\psi_2 = -jA(\Delta\omega)\sin(\Delta\omega C' z) \quad (5)$$

where $A(\Delta\omega)$ is the Fourier transform of the input pulse envelope $a(t)$. The inverse Fourier transforms of the cosine and sine factors are, respectively, half the sum and half the difference of two Dirac delta functions centered at $t = \pm C'z$. Hence, the convolution theorem tells us that, for *any* shape of the input pulse, the output consists indeed, as stated in [10], of four pulses, i.e., two in each waveguide, with a time shift of $\pm C'z$. The pulses in waveguide one have equal signs, and those in waveguide two opposite signs.

All this seems to confirm that the scope of validity of the results found in [10] is extremely wide. However, before drawing such a conclusion, notice that (4) and (5) indicate that any time-harmonic component of the input signal of angular frequency $\omega = \omega_0 + \Delta\omega$ undergoes—in space—a periodic transfer from one waveguide to the other and back, like in an ordinary directional coupler. The beat length is $L_B = 2\pi/(C'\Delta\omega)$. This indicates that there is an implicit assumption behind the equations that we just solved, as well as those that were solved in [10], namely, that *the coupling coefficient between the two waveguides varies linearly with* $\Delta\omega$, *from minus infinity to plus infinity*. This assumption does not look realistic. In fact, the physical explanation of the decoupling recipe is an antiresonance [1], indicating that a Lorentzian-type line shape would be far more realistic. As is very well known, the frequency range (normalized to the central frequency) over which the imaginary part of a Lorentzian can be approximated with a straight line is of the order of the Q-factor. This appears to set a limitation to the scope of validity of the results of [10]: The slope of the coupling coefficient versus frequency $|C'|$ must be small enough so that the whole spectrum of the input pulse fits into the frequency range where the linear approximation for C versus $\Delta\omega$ holds.

We will come back to $|C'|$, i.e., (4) and (5), for further comments, in the next Section. Let us first show that, in the frequency domain, results can be quickly extended to *dispersive* waveguides. With the following change of variables:

$$\Phi_i = \psi_i \exp\left(j\frac{\beta_2}{2}\Delta\omega^2 z\right), \qquad (i = 1, 2) \tag{6}$$

combined with (2) and (3), we see that the new unknowns Φ_i's satisfy two equations which are formally identical to those that the ψ_i's satisfy in the dispersionless case. Therefore, in the dispersive case, when, at $z = 0$, waveguide 1 is illuminated and waveguide 2 is not, the relevant solutions read

$$\psi_1 = A(\Delta\omega)\cos(\Delta\omega C' z)\exp\left(-j\frac{\beta_2}{2}\Delta\omega^2 z\right) \tag{7}$$

$$\psi_2 = -jA(\Delta\omega)\sin(\Delta\omega C' z)\exp\left(-j\frac{\beta_2}{2}\Delta\omega^2 z\right). \tag{8}$$

The inverse Fourier transform of the factor $\exp(-j\beta_2\Delta\omega^2 z/2)$, for $\beta_2 \neq 0$, is (see, e.g., [12]) $[\sqrt{2}/(1+j)]\exp(jt^2/2\beta_2 z)$. Then, applying twice the convolution theorem to the double product on the right-hand side in (7) and (8), we find that also in the dispersive case the output consists of four subpulses, whose envelope amplitudes undergo the same delays as in the dispersionless case, but which are chirped, with a phase delay proportional to the distance z through a coefficient $C'^2/(2\beta_2)$.

3. Further Comments

The elementary identity $\cos^2 + \sin^2 = 1$ tells us that, regardless of the waveguides being dispersive or not, power at any frequency (and hence, the overall power spectral density) of the input signal remains constant, while the pulses propagate through the waveguides. The device being linear and lossless, this result is physically obvious. Still, it is not easy to grasp it from the time domain results.

Behind the apparent invariance of the pulse shapes, the power of each spectral component shuttles back and forth from one core to the other. Hence, the statements [10] that "a pulse does not couple back and forth" and that "when the fiber is long enough ... mode coupling stops completely" must be taken with a grain of salt. To show that the two waveguides *do* interfere with each other also when the front and tail pulses do no longer overlap in time, suppose that one of the two cores ends abruptly. From there on, the time gap between the pulses in the surviving core will remain constant. Hence, as long as the time gap was growing linearly with z, there was an interplay with the other waveguide.

Incidentally, let us recalculate the distance z_w, after which, the two subpulses in the same waveguide are time resolved. As we said before, in the notation of [10], at a distance z, we have $t = 0$ at the arrival of a pulse in the isolated waveguide $t' = z/v_g$. The subpulses are resolved (in space) when the separation between their centers $\Delta v_g z/v_g$, (where Δv_g is the difference between the subpulses velocities) exceeds the spatial length of the input pulse, $v_g T_0$, T_0 being the input pulse length in time. Then, using (1), a straightforward calculation yields

$$z_w = T_0\left(1 - C'^2 v_g^2\right)/2|C'|. \tag{9}$$

The disagreement with [10, eq. (7)] seems to be due to taking into account the z-dependence of the instant $t = 0$. The disagreement is negligibly small for $|C'|v_g \ll 1$ so that the numerical examples presented in [10] are essentially correct, but it may become relevant in a device designed for slow-light applications.

Equation (9), in combination with what we said in the previous section about the spectral width of the antiresonance, indicates that the magnitude of $|C'|$ plays a crucial role. The smaller the $|C'|$, the broader the allowed spectral range, and therefore, the shorter the pulses that the device can handle without distortion. On the other hand, the smaller the $|C'|$, the longer the splitting distance (9), which might hinder the practical use of the device for SFL applications. Leaving apart multiple-step index-guiding structures—which were considered in the early paper [1] and in the references therein, but are now out of fashion—let us focus on PBG structures. We see from the literature (e.g., [2, Figs. 3 and 8]) that the slopes of the dispersion curves of the even and odd supermodes, and henceforth $|C'|$, depend very strongly on several parameters of the structure: at the least, the lattice type, the pitch, and the hole radius. The problem does not appear to lend itself to an analytical approach and must be tackled numerically. This is an open and promising subject for further investigation.

Let us now go back again to the frequency-domain results of the previous Section. The beat length $L_B = 2\pi/(\Delta\omega C')$, being inversely proportional to $\Delta\omega$ (assuming C' is constant over the whole band of interest), causes a nonmonochromatic signal not to be, in general, a periodic function of z. It becomes periodic in space, however (in the dispersionless case), if the input signal is time-periodic, like a sequence of identical pulses. In such a case, we find "copies" of the input pulse train at distances $z = nT_r/C'$, $n = 1, 2, \ldots$, where T_r is the period of the input sequence. In the dispersive case, the phase factors have another period in space $L_e = 4\pi/(\beta_2\Delta\omega^2)$. Hence, for the signal to be a periodic function of z, L_B/L_e must be a rational number. The meaning and the physical feasibility of this condition are not easy to grasp.

4. Conclusion

Propagation of a broadband pulse in twin cores, which are decoupled at the pulse central frequency, has been reinvestigated in the frequency domain. The analysis was motivated by the fact that the scope of validity of the previous time-domain results was not easy to assess. Our results reconfirm that the key point of [10]—i.e., that, irrespectively of its shape, the pulse breaks up into four subpulses, i.e., two in each core, with a time gap that grows linearly with distance—is mathematically correct. On the other hand, though, we have shown that these results are based upon a model of the frequency behavior of the device, which is tenable only under some restrictions. Namely, the pulse spectrum must stay within the range where the coupling coefficient between the guides varies linearly versus frequency deviation. Each spectral component of the pulse shuttles back and forth between the cores, and the total power spectral density of the entire signal (taking into account the fields in both cores) is preserved. The results have been extended to the case of dispersive waveguides, where the output signal is chirped, but there are no other major changes.

Another limitation has been shown to underlie some calculations and comments. It involves group velocity, namely, it was implicitly assumed in [10] that $v_g|C'| \ll 1|$. This restriction seems to be independent of the one concerning the pulse spectral width. However, this point deserves further investigation because, if the restriction can be removed, then the twin cores may become a candidate for SFL applications. In particular, what looks attractive is the fact that the two subpulses in the "slave" waveguide are of opposite sign, making it possible, at least in principle, to generate a zero-mean signal by means of a device which is passive and linear.

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