IEEE Photonics Journal
An IEEE Photonics Society Publication

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Volume 4, Number 1, February 2012

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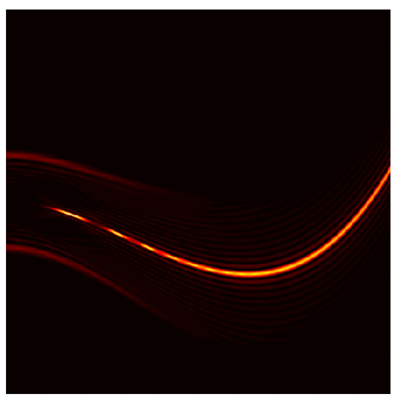

DOI: 10.1109/JPHOT.2011.2182338

Dynamic Control of Circular Airy Beams With Linear Optical Potentials

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DOI: 10.1109/JPHOT.2011.2182338

Manuscript received November 24, 2011; revised December 26, 2011; accepted December 28, 2011. Date of publication December 30, 2011; date of current version January 24, 2012. This work was supported by the National Research Foundation and the Ministry of Education, Science, and Technology of Korea through the Creative Research Initiatives Program (Active Plasmonics Application Systems). Corresponding author: B. Lee (e-mail: byoungho@snu.ac.kr).

Abstract: We investigate the propagation dynamics of radially symmetric, circular Airy beams through a medium having linear optical potentials. On the basis of analytic expressions and numerical calculations, we show that "off-axis" autofocusing can be implemented. We also show that we can shift the field distributions of Bessel-like circular Airy beams transversely via the introduction of appropriate linear potentials. Furthermore, by a suitable choice of initial launch condition, the focal length of the autofocusing beam can be modulated.

Index Terms: Beam propagation, nondiffracting beam.

1. Introduction

Using the concept of Airy wave packets in the context of quantum mechanics [1], nondiffracting Airy beams have been introduced in the area of optics [2], [3]. Airy beams have an intriguing property: Their main intensity lobes are accelerated transversely as they propagate in a homogeneous medium with no external potential [4]–[7]. This strange behavior comes from the fact that Airy beams are generated by the curved envelope of the rays [8]. Airy beams also have the so-called self-healing property [9], which indicates that Airy beams can reconstruct their inherent amplitude patterns even when they propagate in adverse conditions. Exploiting these unique characteristics, Airy beams have been used in various applications [10]–[15].

Up to date, most researches have considered Airy beam propagations in homogeneous media, i.e., without any optical potentials. Recently, several studies showed that if there is a linear optical potential, the trajectories of Airy beams can be modified, retaining their self-healing properties [16]–[19]. In this case, the trajectories of the beams are determined by two factors: One is the intrinsic acceleration of Airy beams, and the other is the index gradient that makes the beams follow curved paths. Combining these two factors, we can selectively launch Airy beams into various directions [16]–[18]. Furthermore, a recent study has shown that the Airy beam trajectory can be engineered to follow a predesigned path, although with some physical limitations [19], [20].

Recently, radially symmetric or circular Airy beams have been reported both theoretically and experimentally [21]–[24]. These beams show ring-shaped initial transverse amplitude patterns, and are accelerated in either inward or outward directions as they propagate, exhibiting abrupt autofocusing behavior [21], [22], [25] after they propagate a certain distance and then gradually becoming Bessel beams in the far-field [23], [24]. Like conventional Airy beams, circular Airy beams can also be used for many purposes, such as optical trapping or biomedical applications.

In this paper, we analyze the roles of linear potentials on the trajectories of circular Airy beams. Using both analytical and numerical results, we demonstrate that we can not only control the transversal position of the autofocusing point but also shift the transverse beam distributions of circular Airy beams by the introduction of appropriate optical potentials. Moreover, by combining the curved trajectories of Airy beams and appropriate initial launch angles, we can change the longitudinal focal distances of autofocusing beams.

This paper is structured as follows. In Section 2, we derive an analytical expression of circular Airy beams in a linear optical potential. In Section 3, we present the propagation features of circular Airy beams under several conditions using numerical calculations. We also study the effect of initial launch angles on the propagation dynamics of inwardly and outwardly accelerated circular Airy beams. In Section 4, we summarize our results and discuss some possible applications.

2. Theoretical Formulation

Let us first derive analytic representations of circular Airy beams that propagate through a medium having linear optical potentials. The 3-D [(2 + 1)D] normalized paraxial equation under a linear potential is described as

$$\frac{\partial^2 u}{\partial s_x^2}+\frac{\partial^2 u}{\partial s_y^2}+i2\frac{\partial u}{\partial \xi}=\left[g_x(\xi)s_x+g_y(\xi)s_y\right]u \tag{1}$$

where u represents the complex envelope of the optical field, $g_x(\xi)s_x$ and $g_y(\xi)s_y$ denote linear potential terms (linear along the transverse direction) which are dependent on the longitudinal coordinate as well. $s_x=x/r_0$, $s_y=y/r_0$, and $\xi=z/kr_0^2$ are the normalized coordinates, with r_0 an arbitrary scaling constant and k being the wavenumber. By solving (1) with an initial field distribution of $u(s_x,s_y,0)=\Pi_{m=x,y}Ai(s_m+d)\exp[(a+i\psi)(s_m+d)]$, we can obtain the following finite-energy Airy beam solution [20], [26]:

$$u(s_x,s_y,\xi)=\prod_{m=x,y}u_m(s_m,\xi) \tag{2}$$

where

$$u_m(s_m,\xi)=Ai[s_m+p_m(\xi)]\exp\left[\left\{i\frac{1}{2}\xi-i\frac{1}{2}G_m(\xi)+(a+i\psi)\right\}s_m+q_m(\xi)\right] \tag{3}$$

$$p_m(\xi)=-\frac{1}{4}\xi^2+i(a+i\psi)\xi+d+\frac{F_{1,m}(\xi)}{2} \tag{4a}$$

$$\begin{aligned}q_m(\xi)=&-i\frac{1}{12}\xi^3-\frac{1}{2}(a+i\psi)\xi^2+i\frac{1}{2}\left\{(a+i\psi)^2+d+\frac{F_{1,m}(\xi)}{2}\right\}\xi\\&-i\frac{1}{8}F_{2,m}(\xi)+\frac{1}{2}(a+i\psi)F_{1,m}(\xi)+(a+i\psi)d\end{aligned} \tag{4b}$$

$$G_m(\xi)=\int_0^\xi g_m(s)\,ds \tag{5a}$$

$$F_{1,m}(\xi)=\int_0^\xi G_m(s)\,ds \tag{5b}$$

$$F_{2,m}(\xi)=\int_0^\xi G_m^2(s)\,ds \tag{5c}$$

where *Ai* indicates the Airy function, ψ and d are related to the initial launch angle [4] and the transverse displacement, respectively, and a denotes a positive exponential truncation constant. Equation (3) shows that the Airy beam solution is given by the product of the Airy function which maintains its original profile during propagation, a phase term, and an exponentially increasing or decreasing term over the propagation distance. To find the analytic expression of circular Airy beams in an optical potential, we first have to obtain a rotated version of (2) by the substitution $s_x \rightarrow s_x \cos\theta + s_y \sin\theta$ and $s_y \rightarrow -s_x \sin\theta + s_y \cos\theta$ (see [24] for details). Here, it is crucial to note that (1) should be premodified as (6) in order to maintain the directions of the potential gradient under rotation in the transverse plane

$$\begin{aligned}\frac{\partial^2 u}{\partial s_x^2} + \frac{\partial^2 u}{\partial s_y^2} + i2\frac{\partial u}{\partial \xi} &= \left[g_x(\xi)(s_x \cos\theta - s_y \sin\theta) + g_y(\xi)(s_x \sin\theta + s_y \cos\theta)\right] u \\ &= \left[s_x\{g_x(\xi)\cos\theta + g_y(\xi)\sin\theta\} + s_y\{-g_x(\xi)\sin\theta + g_y(\xi)\cos\theta\}\right] u. \end{aligned} \tag{6}$$

Equation (6) suggests that the terms related to optical potentials should also be transformed as $g_x(\xi) \rightarrow g_x(\xi)\cos\theta + g_y(\xi)\sin\theta$ and $g_y(\xi) \rightarrow -g_x(\xi)\sin\theta + g_y(\xi)\cos\theta$. That is, the form of transformation of $(g_x(\xi), g_y(\xi))$ is identical to that of (s_x, s_y). In the cylindrical coordinate, the resulting "rotated" Airy beam solution is given as

$$\begin{aligned} u(\rho,\varphi,\xi,\theta) = Ai&\left[\rho\cos(\theta-\varphi) + \frac{1}{2}\{F_{1,x}(\xi)\cos\theta + F_{1,y}(\xi)\sin\theta\} + \alpha(\xi)\right] \\ &\times Ai\left[-\rho\sin(\theta-\varphi) + \frac{1}{2}\{-F_{1,x}(\xi)\sin\theta + F_{1,y}(\xi)\cos\theta\} + \alpha(\xi)\right] \\ &\times \exp\left[\rho\{\cos(\theta-\varphi) - \sin(\theta-\varphi)\}\left\{i\frac{1}{2}\xi + (a+i\psi)\right\}\right] \\ &\times \exp\left[\left\{i\frac{1}{4}\xi + \frac{1}{2}(a+i\psi)\right\}\{F_{1,x}(\xi)(\cos\theta - \sin\theta) + F_{1,y}(\xi)(\cos\theta + \sin\theta)\}\right] \\ &\times \exp\left[-i\frac{1}{2}\rho\{G_x(\xi)\cos\varphi + G_y(\xi)\sin\varphi\} - i\frac{1}{8}\{F_{2,x}(\xi) + F_{2,y}(\xi)\} + \beta(\xi)\right] \end{aligned} \tag{7}$$

where

$$\alpha(\xi) = -\frac{1}{4}\xi^2 + i(a+i\psi)\xi + d \tag{8a}$$

$$\beta(\xi) = -i\frac{1}{6}\xi^3 - (a+i\psi)\xi^2 + i\left\{(a+i\psi)^2 + d\right\}\xi + (a+i\psi)d \tag{8b}$$

where $\rho = (s_x^2 + s_y^2)^{1/2}$, $\tan\varphi = s_y/s_x$, and θ denotes the angle of rotation. Equation (7) describes the propagation of rotated (2 + 1)D Airy beams with linear index potentials along the transverse directions. The properties of circular Airy beams in the presence of linear potentials can be studied by carrying out numerical integrations of (7) over θ.

The parameter d plays an important role in the propagation trajectory of circular Airy beams: the sign of d determines the direction of transverse acceleration, either inward $(d > 0)$ or outward $(d < 0)$ self-bending, and the magnitude of d is associated with the size of the initial primary ring. The inward-type circular Airy beams exhibit abrupt autofocusing phenomena, while the outward-type ones are gradually changed into Bessel beams as they propagate (Therefore, we can refer to this outward-type circular Airy beam as a Bessel-like beam.). In both cases, the aforementioned autofocusing and Bessel-like beaming characteristics are observed at the off-axis positions in the presence of linear optical potentials.

3. Numerical Results

In this section, we investigate numerically the effects of linear potentials on the propagation characteristics of circular Airy beams. The common parameters adopted here are $\lambda = 532$ nm,

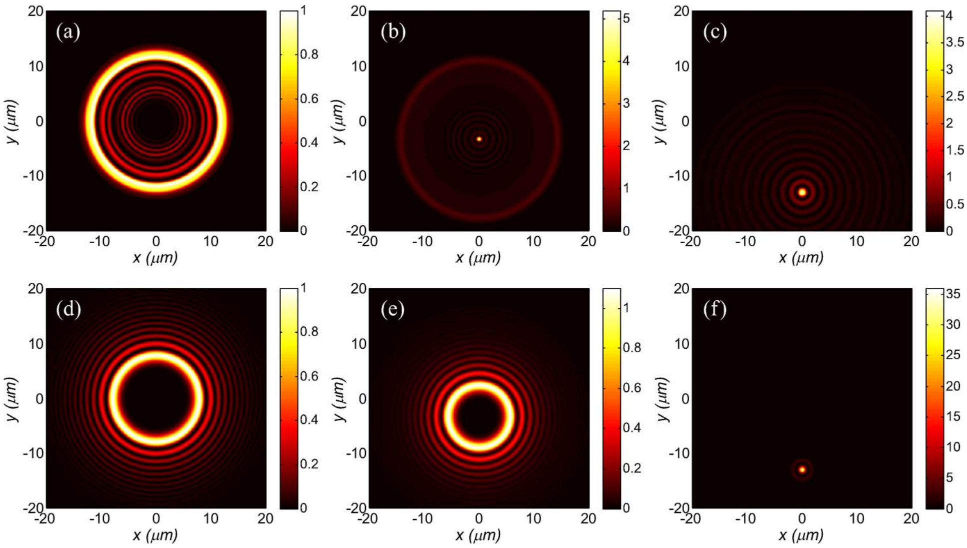

Fig. 1. Transverse intensity profiles of an outward-type circular Airy beam at (a) $z = 0$ μm, (b) $z = 30$ μm, and (c) $z = 60$ μm. Results for the inward-type one at the same distances are shown in (d)–(f). We assumed $\psi = 0$ and $g_y(\xi) = 1$. In each column of plots, intensity values are normalized with respect to the peak intensity at $z = 0$.

$r_0 = 1$ μm, and $a = 0.1$. In all figures, optical potentials change linearly only along the y direction [i.e., $g_x(\xi) = 0$]. The changes of gradients along the propagation direction are designed for our specific purposes. It should be noted that the autofocusing feature is shown only in the propagation of an inward-type circular Airy beam. The outward-type circular Airy beam shows very similar characteristics to those of a Bessel beam [23], [24].

Fig. 1 shows the intensity distributions of circular Airy beams undergoing outward [$d = -9$; (a)–(c)] and inward [$d = 5$; (d)–(f)] accelerations, with the conditions $\psi = 0$ and $g_y(\xi) = 2$. In both cases, we can observe that the intensity patterns are shifted along the y direction as the beams propagate. The transverse shifts after a propagation distance ξ can be written as

$$s_{0,x}(\xi) = -F_{1,x}(\xi)/2 \tag{9a}$$

$$s_{0,y}(\xi) = -F_{1,y}(\xi)/2. \tag{9b}$$

Using (9), we can predict the trajectory of the beam center. By adjusting the applied potential profiles appropriately, we can make the circular Airy beams follow a chosen path (e.g., the focusing position can also be specified).

To understand better the effects of linear potentials, we have plotted the intensity distributions in the $y - z$ plane ($x = 0$) in Fig. 2. The optical potentials adopted are $g_y(\xi) = 1$ [see Fig. 2(a) and (c)] and $g_y(\xi) = 8 - 3\xi$ [see Fig. 2(b) and (d)]. Fig. 2(a) and (b) correspond to the outward-type Bessel-like beam (with $d = -9$ and $\psi = 0$) [see Fig. 1(a)–(c)]. We can see that the propagating directions of these Bessel-like beams can be changed drastically by introducing appropriate $g_y(\xi)$. Off-axis focusing property can be found in Fig. 2(c) and (d), where we plotted the propagation dynamics of an inward-type autofocusing beam (with $d = 5$ and $\psi = 0$).

Now, we investigate in more detail the effect of the initial launch angle ψ on the propagation characteristics of inward-type circular Airy beams. By considering the parabolic trajectory of the Airy beams in free space, the longitudinal focal length of the inward-type beam can be approximately estimated from

$$\frac{1}{4}\xi_f^2 + \psi\xi_f = d \tag{10}$$

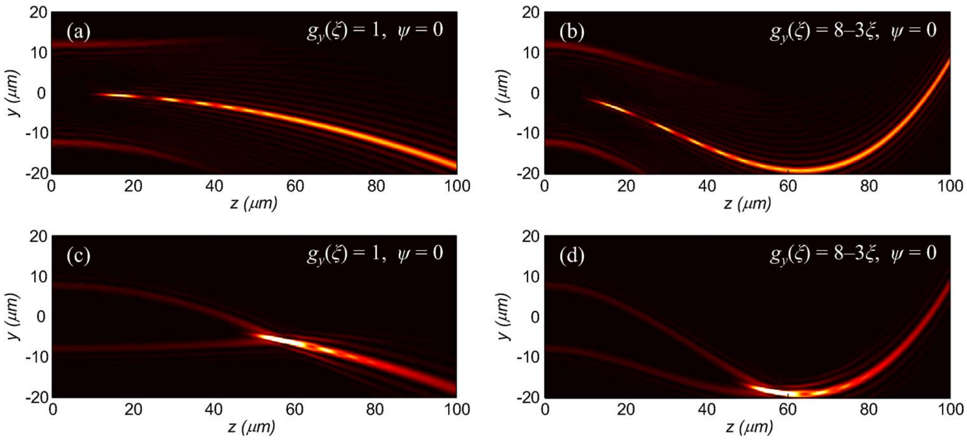

Fig. 2. Cross-sectional view of intensity distributions in the $y-z$ plane $(x = 0)$. (a) and (b) show the dynamics of the outward-type circular Airy (or Bessel-like) beam. (c) and (d) correspond to the case of the inward-type one, showing autofocusing features.

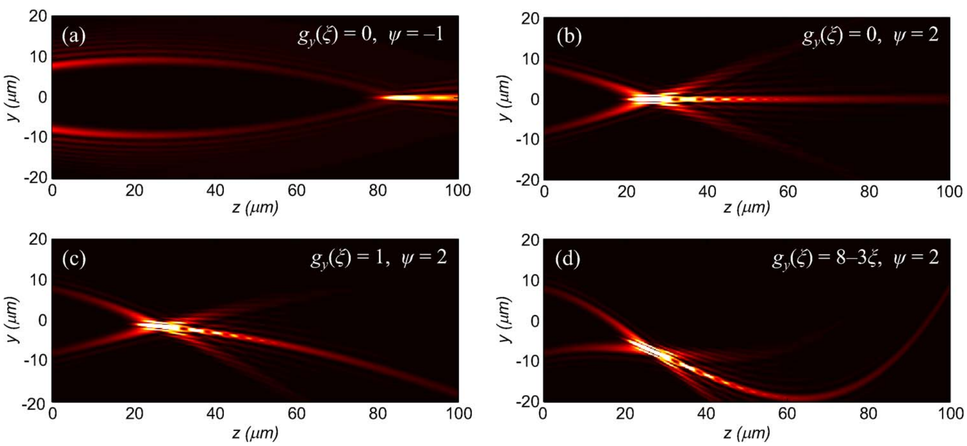

Fig. 3. Effects of the initial launch angle ψ. In (a) and (b), we assumed $\psi = -1$ and $\psi = 2$, respectively, without any optical potential. (c) and (d): Same as Fig. 2(c) and (d), except for $\psi = 2$.

where $\xi_f = z_f/kr_0^2$ indicates the normalized focal length. From (10), we can see that the focal distance can be designed theoretically by changing the initial condition ψ. This feature can be found in Fig. 3(a) and (b), where we plotted the intensity patterns of inward-type circular Airy beams with $\psi = -1$ and $\psi = 2$, respectively. We can clearly see that the change in ψ results in a change of the autofocusing point position. We can also see that negative ψ can make an inward-type beam look like an outward-type one near the launching position as is shown in Fig. 3(a).

Equation (10) is still valid when a linear potential is applied: the linear potential shifts the autofocusing point only transversely. Therefore, the change in optical potentials neither increase nor decrease the focal length along the propagation coordinate, and this property can be found in Fig. 2(c) and (d): Although the applied potentials are different, the corresponding longitudinal focusing distances are the same. When ψ is changed to $\psi = 2$, the corresponding plots of circular Airy beams are shown in Fig. 3(c) and (d), respectively. In these figures, we can confirm that the change in ψ shifts the position of the autofocusing point along the longitudinal direction. Since $\psi > 0$, the focal length is reduced compared with that in Fig. 2(c) and (d), as can be estimated by (10).

The previous discussions are also relevant to outward-type circular Airy beams, although the focal length is a meaningless concept in this case. However, negative ψ makes outward-type

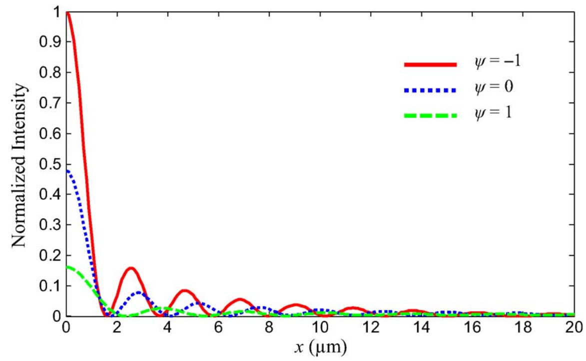

Fig. 4. Radial intensity distributions of the outward-type circular Airy (or Bessel-like) beams at $z = 100$ μm in the potential-free condition ($d = -9$) with $\psi = -1$, $\psi = 0$, and $\psi = 1$.

Bessel-like beams look like inward-type ones near their initial positions. In addition, ψ also determines the width of the central beam lobe in the far-field. This feature can be found in Fig. 4, where we plotted the calculated radial intensity distributions (at $z = 100$ μm) of outward-type Bessel-like beams for three different values of ψ. From the figure we can find that the width of central beam lobes is dependent on ψ, and it determines the beam shape and the effective width of the synthesized circular beam.

4. Conclusion

We have presented a numerical study for the propagation dynamics of circular Airy beams under linear optical potentials that vary along the propagation coordinate. Through numerical calculations, we have shown that the transversal positions of autofocusing point can be controlled by appropriately selected linear optical potentials. We have also shown that the focal length of the autofocusing circular Airy beam can be modulated as well. We believe this dynamic feature can be applied to various applications such as medical treatment, particle manipulation, and optical communication.

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