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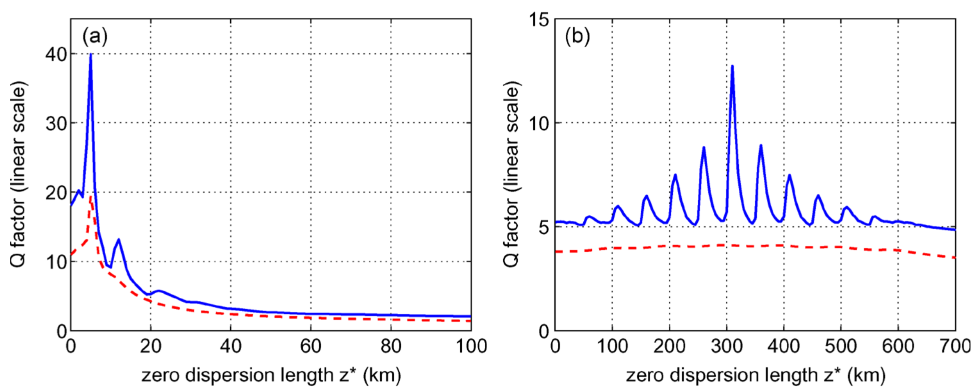

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A Unified Theory of Intrachannel Nonlinearity in Pseudolinear Phase-Modulated Transmission

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Abstract: It is shown that the phase distribution of the field and the detection scheme strongly affect the strength of the nonlinear impairments in phase-modulated transmission systems. It is found that precompensation is always a useful tool for minimizing the nonlinear impairments in differential phase-shift keying, where the mechanism for minimization is the reduction of the in-phase component of the nonlinear displacement. For differential quadrature phase-shift keying (DQPSK), the nonlinear impairments are instead minimized by a dispersion profile that maximizes the correlation of phase fluctuations of two consecutive pulses. When the number of interacting pulses is large, the performance of a DQPSK system is only weakly dependent on the dispersion profile.

Index Terms: Coherent transmission, intrachannel four-wave mixing, pseudolinear transmission.

1. Introduction

A few years ago, in a series of three papers [1]–[3], it was shown using first-order perturbation theory that a careful design of the dispersion profile may significantly improve the performance of high bit-rate transmission in systems in which the nonlinearity is minimized by a quick spreading of the pulses (pseudolinear transmission). The theory was originally developed for the only practical scheme at the time, namely, on–off keying (OOK) intensity-modulation direct-detection (IMDD) transmission. In this paper, this theory is extended to systems based on differential phase-shift keying (DPSK) and differential quadrature phase-shift keying (DQPSK), which are nowadays becoming increasingly popular [4].

The phase distribution of the signal and the detection scheme is found to strongly affect the system nonlinear dynamics. In particular, it will be shown that nonlinear impairments are minimized in DPSK transmission when the in-phase component of the nonlinear displacement is minimum, whereas they are minimized in DQPSK when the correlation of the phase fluctuations of two consecutive pulses is maximum. The optimal dispersion profile is virtually the same in both systems. While dispersion profile always strongly affects DPSK transmission, it does not have a significant effect on DQPSK performance when the interval between dispersion compensating stations is large.

2. First-Order Perturbation Theory

Let us start with the nonlinear Schrödinger equation for the scalar electric field amplitude u, averaged to account for the small-scale polarization evolution (no polarization-dependent effects will be considered throughout this paper) and rescaled to account for the linear fiber attenuation

$$\frac{\partial u}{\partial z} = -i\frac{\beta''}{2}\frac{\partial^2 u}{\partial t^2} + i\gamma f(z)|u|^2 u \tag{1}$$

where β'' (negative in the anomalous dispersion region) is the group velocity dispersion, $\gamma = 2\pi n_2/(\lambda A_{\text{eff}})$ is the fiber nonlinear coefficient, n_2 is the nonlinear refractive index, and A_{eff} is the effective area of the fiber. Here, $f(z)$ rescales the fiber nonlinearity to include the effect of a nonuniform power profile caused by fiber loss. If equally spaced lumped Erbium amplifiers are used, $f(z) = \exp[-\alpha \bmod(z, z_s)]$ for $0 \le z < L$, where mod is the modulus function, α is the power attenuation coefficient, z_s is the span length, and L is the fiber length. The nonlinear term is treated using a perturbation approach inserting $\tilde{u}(z,\omega) = \tilde{u}_0(z,\omega) + \Delta u(z,\omega)$ into the Fourier transform of (1) and preserving terms up to first order in $\Delta u(z,\omega)$. The regime of operation where first-order perturbation theory is valid is known as quasi-linear transmission. Let us assume that the dispersion is always constant, with the exclusion of lumped locations where dispersion is added linearly to the field (dispersion compensating stations). We assume that at the line input, the field is linearly predispersed by some fixed amount of dispersion (usually opposite from that of the line so that predispersion is also a precompensation of the line dispersion) and then transmitted through the dispersive nonlinear fiber; then, the total accumulated dispersion of the field (predispersion + line dispersion) is fully compensated by a linear dispersion compensating device. In other words, we assume that the initial and final point of the first span between dispersion compensating stations are always points where the field experiences zero accumulated dispersion. Then, in the second span between dispersion compensating stations, the field is predispersed and transmitted again through the fiber, and the total accumulated dispersion is linearly compensated. The spans after the second are treated on equal footing. Using this procedure, the concatenation of more than one span between dispersion compensating stations is modeled as the concatenation of spans where the initial and final points have zero accumulated dispersion. Then, within linear perturbation theory, the perturbation at the end of the line will be the sum of the perturbations of sections between compensating stations.

Let us assume that the input field is made of a sequence of three un-chirped Gaussian pulses with the same pulse width $u_0(0,t) = \sum_{j=1}^{3} v_j(t - T_j)$ with $v_j(t) = A_j \exp[-t^2/(2\tau^2)]$. The interaction of n pulses can, to lowest order, always be decomposed as the sum of the interactions of all possible triplets of pulses. After some algebra, it is possible to give to the perturbation the expression [1]

$$\Delta u_{j,k,l}(t + T_{j,k,l}) = i\gamma A_j A_k^* A_l \mathcal{U}_{j,k,l}(L, t + T_{j,k,l}) \tag{2}$$

where $T_{j,k,l} = T_j - T_k + T_l$, and

$$\mathcal{U}_{j,k,l}(t + T_{j,k,l}) = \exp\left(-\frac{t^2}{6\tau^2}\right) \int_{-z^*}^{L-z^*} \frac{f(z' + z^*)dz'}{\sqrt{3q^*(q + 2i/3)}} \times \exp\left\{ i\frac{[2t/3 + (T_j - T_k)][2t/3 + (T_l - T_k)]}{\tau^2(q + 2i/3)} - \frac{(T_j - T_l)^2}{3\tau^2 q^*(q + 2i/3)} \right\} \tag{3}$$

and the complex parameter q is defined as $q = (z/z_d) - i$, where $z_d = \beta''/\tau^2$ the dispersion length that is either positive or negative, depending upon the sign of β''. Here, z^* is the zero accumulated dispersion point within the span. The amount of predispersion is $\beta_{\text{pre}} = -\beta'' z^*$ in picoseconds squared if β'' is in picoseconds squared per kilometer, and z^* is in kilometers. This expression can be easily generalized to the N span case, wherein there are N position where partial or total dispersion compensation is performed. The result will be the sum of N integrals of the kind given by (3) if we use for each span a local reference frame where the origin is set to the input of the span.

3. Analysis of DPSK and DQPSK Transmission

Let us now consider a sequence of equally spaced pulses. We will restrict ourselves to the case of a sequence of Gaussian pulses with the same pulse-width and complex amplitudes A_j with the same modulus A, spaced by the symbol time T_s (the inverse of the baud rate). The pulse phases are chosen within a set of N values φ_j, which are used to define the message. In DPSK, $N = 2$, and the phase can be either φ_0 or $\varphi_0 + \pi$. In DQPSK, $N = 4$, and the phases are spaced by $\pi/2$. Let us define N normalized complex amplitudes a_j such that $A_j = a_j A = \exp(i\varphi_j)A$ and assume that each phase occurs with equal probability $p = 1/N$. We will focus the analysis on differential detection, where each pulse interferes with the next pulse of the stream, with no phase shift in DPSK and phase shifted by $\varphi_d = \pm\pi/4$ in DQPSK, and the real part of the beat term is detected by a balanced receiver. The amplitude of the detected photocurrent is proportional to $I = \mathrm{Re}(\mathcal{I})$, where

$$\mathcal{I} = \exp(-i\varphi_d)\int dt\left[a_1 A\exp\left(-\frac{t^2}{2\tau^2}\right) + \sum_{j',k',l'}\Delta u_{j',k',l'}\right]^*\left[a_0 A\exp\left(-\frac{t^2}{2\tau^2}\right) + \sum_{j,k,l}\Delta u_{j,k,l}\right] \tag{4}$$

is the complex amplitude, Re stands for the real part, and $\Delta u_{j,k,l} = \Delta u_{j,k,l}(L,t)$ for short. The first sum is extended to all combinations $T_{j',k',l'} = T_{j'} - T_{k'} + T_{l'} = T_s$ and the second to all combinations $T_{j,k,l} = T_j - T_k + T_l = 0$. Using these conditions, the triple sums collapse into double ones, because the first implies that $j' - k' + l' = 1$, hence, that $k = j + l - 1$ and the second that $j - k + l = 0$, hence, that $k = j + l$. The zeroth-order term is

$$\mathcal{I} = \exp(-i\varphi_d)a_1^* a_0\int dt\ A^2\exp\left(-\frac{t^2}{\tau^2}\right) \simeq \exp(-i\varphi_d)a_1^* a_0\sqrt{\pi}A^2\tau \tag{5}$$

where, although the integral is extended to the symbol time T_s, the approximation of replacing the integration interval with the whole time axis has been used. With DPSK, $a_1^* a_0 = \pm 1$, and only a single interferometer is used. With DPSK, $a_1^* a_0 = 1, i, -1, -i$, and two interferometers are used with $\varphi_d = \pm\pi/4$ so that their outputs $I_{\mathrm{DQPSK}} = \sqrt{\pi}A^2\tau\mathrm{Re}[(1 \pm i)a_1^* a_0/\sqrt{2}]$ allow univocal selection of one of the four transmitted symbols. The capacity increase is obtained at the expenses of a $\sqrt{2}$ reduction of the detected signal when compared with DPSK. Both pulses are perturbed by the nonlinear interaction. The perturbation of the complex amplitude of the photocurrent is

$$\Delta\mathcal{I} = \exp(-i\varphi_d)\left(\Delta I_1 + \Delta I_0^*\right) \tag{6}$$

where

$$\Delta I_1 = \sum_{j,l} a_1^* a_j a_{j+l}^* a_l \mathcal{J}_{j,j+l,l}, \qquad \Delta I_0 = \sum_{j',l'} a_0^* a_{j'} a_{j'+l'-1}^* a_{l'} \mathcal{J}_{j',j'+l'-1,l'} \tag{7}$$

$$\mathcal{J}_{j,j+l,l} = i\gamma A^4\int dt\ \exp\left(-\frac{t^2}{2\tau^2}\right)\mathcal{U}_{j,j+l,l}(t + T_{j,j+l,l}). \tag{8}$$

After using (3) in (8) and integrating over time t, the parameters $\mathcal{J}_{j,k,l}$ acquire the remarkably simple expression

$$\mathcal{J}_{j,k,l} = \mathcal{J}_{j-k,l-k} = i\gamma\sqrt{2\pi^3}A^4\tau^3\int_{-z^*}^{L-z^*} f(z + z^*)G(T_j - T_k, T_l - T_k, z)dz \tag{9}$$

where the notation $\mathcal{J}_{l,j} = \mathcal{J}_{l,0,j}$ is introduced for short, defined the function of the center time of the pulses T_j and the distance z

$$G(T_1, T_2, z) = \frac{1}{2\pi\tau^2\sqrt{(z/z_d)^2 + 1}}\exp\left[-\frac{T_1^2 + T_2^2 - 2i(z/z_d)T_1 T_2}{2\tau^2\left[(z/z_d)^2 + 1\right]}\right] \tag{10}$$

and used that for equally spaced pulses $T_j - T_k = T_{j-k}$. It is interesting to notice that, formally, G is a bivariate Gaussian distribution of variance τ^2 and complex correlation $i(z/z_d)$, which somehow accounts for the dispersive evolution of the pulses. The above results can be easily extended to coherent communication employing Gaussian pulses and a continuous-wave local oscillator, if the signal is filtered before detection with a matched optical filter and sampled after detection at the center of the time slot.

The variance of the fluctuations of the detected photocurrent $I = \mathrm{Re}(\mathcal{I})$ is $\langle \Delta I^2 \rangle = \langle (\Delta\mathcal{I} + \Delta\mathcal{I}^*)^2 \rangle/4 - \langle \Delta\mathcal{I} + \Delta\mathcal{I}^* \rangle^2/4$. When all symbols are transmitted with equal probability, a significant simplification toward its analytical estimate arises because $\langle a_j \rangle = 0$; hence, $\langle \Delta\mathcal{I} \rangle = 0$. Using this condition, the variance of ΔI becomes

$$\langle \Delta I^2 \rangle = \left\langle |\Delta I_1|^2 \right\rangle + \mathrm{Re}\Big[\cos(2\varphi_d)\langle \Delta I_1^2 + \ \exp(-2i\varphi_d)\langle \Delta I_1 \Delta I_0^* \rangle + \langle \Delta I_1 \Delta I_0 \rangle\Big] \tag{11}$$

where $\varphi_d = 0$ for DPSK, and $\varphi_d = \pm\pi/4$ for DQPSK. The terms ΔI_1 and ΔI_0 are statistically equivalent so that $\langle \Delta I_1^2 \rangle = \langle \Delta I_0^2 \rangle$, and $\langle |\Delta I_1|^2 \rangle = \langle |\Delta I_0|^2 \rangle$, and we have allowed for nonzero correlations between the terms ΔI_1 and ΔI_0 [5]. The expressions of the various terms are

$$\left\langle |\Delta I_1|^2 \right\rangle = \sum_{j,l,j',l'} \left\langle a_1^* a_j a_{j+l}^* a_l a_1 a_{j'}^* a_{j'+l'} a_{l'}^* \right\rangle \mathcal{J}_{l,j} \mathcal{J}_{l',j'}^* \tag{12}$$

$$\langle \Delta I_1^2 \rangle = \sum_{j,l,j',l'} \left\langle a_1^* a_j a_{j+l}^* a_l a_1^* a_{j'} a_{j'+l'}^* a_{l'} \right\rangle \mathcal{J}_{l,j} \mathcal{J}_{l',j'} \tag{13}$$

$$\langle \Delta I_1 \Delta I_0^* \rangle = \sum_{j,l,j',l'} \left\langle a_1^* a_j a_{j+l}^* a_l a_0 a_{j'}^* a_{j'+l'-1} a_{l'}^* \right\rangle \mathcal{J}_{l,j} \mathcal{J}_{l'-1,j'-1}^* \tag{14}$$

$$\langle \Delta I_1 \Delta I_0 \rangle = \sum_{j,l,j',l'} \left\langle a_1^* a_j a_{j+l}^* a_l a_0^* a_{j'} a_{j'+l'-1}^* a_{l'} \right\rangle \mathcal{J}_{l,j} \mathcal{J}_{l'-1,j'-1} \tag{15}$$

where $\mathcal{J}_{j,k,l} = \mathcal{J}_{j-k,0,l-k} = \mathcal{J}_{j-k,l-k}$. First of all, note that all expressions have the exchange symmetry $j \leftrightarrow l$ and $j' \leftrightarrow l'$. Condition $\langle a_j \rangle = 0$ implies that nonzero average is obtained when the terms in the averages are equal in couples. Let us consider first (12) and (13). The average is nonzero if a) $j = j'$ and $l = l'$, or if $j = l'$ and $l = j'$. This second condition is fully equivalent to the first by exchange symmetry. It is convenient to group these two cases into a single, two-fold degenerate one. The only exception is the case $j = j'$ where the two conditions coincide; hence, there is no degeneracy. The average is also nonzero if b) $j = 0$ or $l = 0$, and $j' = 0$ or $l' = 0$ and the other two nonzero indices arbitrary. This case corresponds to the average of four-wave mixing (FWM) terms where the pulses acting on pulse 0 collapse into a single one, hence, to the average of cross-phase modulation (XPM) terms. Because any combination of a zero primed index with a zero unprimed index is allowed, this case is a four-fold degenerate one. Also, in this case, there are exceptions to the four-fold degeneracy. If two primed indices are simultaneously zero or two of the unprimed indices are simultaneously zero, there is only two-fold degeneracy, and there is no degeneracy when all indices are simultaneously zero. If conditions a) or b) are not met, the average is zero. Let us now consider (15) and (14). The average is nonzero if c) $j' = 1$ or $l' = 1$ and $j = 0$ or $l = 0$, and the other two indices are arbitrary, d) if $l = 1$, $l' = 0$, and $j' = j + 1$, with again all four combinations, and, finally, if e) $j = j'$, $l = l'$ and $j = 1 - l$. Cases c) and d) are four-fold degenerate, and case e) is two-fold degenerate. Again, there are exceptions. In case d), there is two-fold degeneracy if $j = 1, -1$. In case c), there is two-fold degeneracy if the two primed indices are simultaneously one or if the two unprimed indices are simultaneously zero, and there is no degeneracy for the single case $j' = 1$, $l' = 1$, $j = 0$, and $l = 0$. Physically, c) is caused by nondegenerate FWM terms where one of the pulse is the interfering pulse at the detector and the contribution of the other two, complex conjugate, collapse into the intensity of a single one. This case accounts for the correlation of the XPM interaction caused on the two interfering pulses by the same pulse. Cases d) and e) are instead caused by correlated FWM terms. Gathering together all these cases, we get

$$\left\langle |\Delta I_1|^2 \right\rangle = \sum_{j,l} f_{j,l} \left\langle |a_1|^2 |a_j|^2 |a_{j+l}|^2 |a_l|^2 \right\rangle |\mathcal{J}_{l,j}|^2 + \sum_{j \neq j'} g_{j,j'} \left\langle |a_1|^2 |a_0|^2 |a_j|^2 |a_{j'}|^2 \right\rangle \mathcal{J}_{0,j} \mathcal{J}^*_{0,j'} \tag{16}$$

$$\left\langle \Delta I_1^2 \right\rangle = \sum_{j,l} f_{j,l} \left\langle a_1^{*2} a_j^2 a_{j+l}^{*2} a_l^2 \right\rangle \mathcal{J}^2_{l,j} + \sum_{j \neq j'} g_{j,j'} \left\langle a_1^{*2} a_0^2 |a_j|^2 |a_{j'}|^2 \right\rangle \mathcal{J}_{0,j} \mathcal{J}_{0,j'} \tag{17}$$

$$\left\langle \Delta I_1 \Delta I_0^* \right\rangle = \sum_{j,j'} h_{j,j'} \left\langle a_1^{*2} |a_j|^2 a_0^2 |a_{j'}|^2 \right\rangle \mathcal{J}_{0,j} \mathcal{J}^*_{0,j'-1} + \sum_{j \neq 0} q_j \left\langle |a_1|^2 a_{j+1}^{*2} |a_0|^2 a_j^2 \right\rangle \mathcal{J}_{1,j} \mathcal{J}^*_{-1,j}$$
$$+ \sum_{j \neq 0,1} 2 \left\langle a_1^{*2} |a_j|^2 a_0^2 |a_{1-j}|^2 \right\rangle |\mathcal{J}_{j,1-j}|^2 \tag{18}$$

$$\left\langle \Delta I_1 \Delta I_0 \right\rangle = \sum_{j,j'} h_{j,j'} \left\langle |a_1|^2 |a_j|^2 |a_0|^2 |a_{j'}|^2 \right\rangle \mathcal{J}_{0,j} \mathcal{J}_{0,j'-1} + \sum_{j \neq 0} q_j \left\langle |a_1|^2 |a_{j+1}|^2 |a_0|^2 |a_j|^2 \right\rangle \mathcal{J}_{1,j} \mathcal{J}_{-1,j}$$
$$+ \sum_{j \neq 0,1} 2 \left\langle a_1^{*2} a_j^2 a_0^{*2} a_{1-j}^2 \right\rangle \mathcal{J}^2_{j,1-j} \tag{19}$$

where the symmetry properties of $\mathcal{J}_{l,j}$ have been used, and we have defined the degeneracy functions: $f_{j,l} = 2$ always, except $f_{j,j} = 1$; $g_{j,j'} = 4$, except $g_{j,0} = g_{0,j'} = 2$; $h_{j,j'} = 4$, except $h_{0,j'} = h_{j,1} = 2$, and $h_{0,1} = 1$; and finally $q_j = 4$ except $q_1 = q_{-1} = 2$. Some indices are excluded to include all nonzero terms of the sums in (12)–(14) only once. For instance, the case $j = j'$ has been omitted in the last sum of (16) and (17), because this case coincides with its degeneracy factor 4, with the two double degenerate cases $l = 0$ and $j = 0$ of the first term of the same equations.

Let us now consider separately the cases of DPSK and DQPSK. For DPSK, $|a_j|^2 = 1$, and $a_j^2 = 1$ for every j. After using these properties, we get

$$\left\langle |\Delta I_1|^2 \right\rangle = \mathcal{A}_{\rm fwm} + \mathcal{A}_{\rm xpm}, \qquad \left\langle \Delta I_1^2 \right\rangle = \mathcal{B}_{\rm fwm} + \mathcal{B}_{\rm xpm}, \qquad \left\langle \Delta I_1 \Delta I_0^* \right\rangle = \mathcal{A}_{\rm corr}, \qquad \left\langle \Delta I_1 \Delta I_0 \right\rangle = \mathcal{B}_{\rm corr} \tag{20}$$

where $\mathcal{A}_{\rm corr} = \mathcal{A}_{\rm c,xpm} + \mathcal{A}_{\rm c,fwm,1} + \mathcal{A}_{\rm c,fwm,2}$

$$\mathcal{A}_{\rm fwm} = \sum_{j,l} f_{j,l} \mathcal{J}_{l,j} \mathcal{J}^*_{l,j}, \qquad \mathcal{A}_{\rm xpm} = \sum_{j \neq j'} g_{j,j'} \mathcal{J}_{0,j} \mathcal{J}^*_{0,j'}, \qquad \mathcal{A}_{\rm c,xpm} = \sum_{j,j'} h_{j,j'} \mathcal{J}_{0,j} \mathcal{J}^*_{0,j'-1} \tag{21}$$

$$\mathcal{A}_{\rm c,fwm,1} = \sum_{j \neq 0} q_j \mathcal{J}_{1,j} \mathcal{J}^*_{-1,j}, \qquad \mathcal{A}_{\rm c,fwm,2} = \sum_{j \neq 0,1} 2 \mathcal{J}_{j,1-j} \mathcal{J}^*_{j,1-j} \tag{22}$$

and the $\mathcal{B}$ are obtained from the corresponding $\mathcal{A}$ by removing the complex conjugation on the right-hand side. Inserting (20)–(22) into (11), one may obtain

$$\left\langle \Delta I^2_{\rm DPSK} \right\rangle = \mathcal{A}_{\rm fwm} + {\rm Re}(\mathcal{B}_{\rm fwm} + \mathcal{A}_{\rm c,fwm,1} + \mathcal{B}_{\rm c,fwm,1} + \mathcal{A}_{\rm c,fwm,2} + \mathcal{B}_{\rm c,fwm,2}). \tag{23}$$

$\mathcal{B}_{\rm xpm}$ is real such that $\mathcal{B}_{\rm xpm} = -\mathcal{A}_{\rm xpm}$ and such that $\mathcal{B}_{\rm c,xpm} = -\mathcal{A}_{\rm c,xpm}$. The terms related to XPM interactions and their correlations disappear. This fact should not be surprising. Phase fluctuations do not contribute to first order to the noise of DPSK because the receiver is sensitive only to the in-phase component of the fluctuations; hence, their correlations do not affect the performance of a DPSK system to first order either.

For DQPSK, but also for more dense formats like 8-ary differential phase-shift keying (D8PSK), we have $|a_j|^2 = 1$, and $\langle a_j^2 \rangle = 0$. This means that, in all averages, terms like a_j^2 average to zero unless they have a partner like a_j^{*2}, or like a_j^2 being $a_j^4 = 1$, with which to saturate. Again, using (11), one may obtain

$$\left\langle \Delta I^2_{\rm DQPSK} \right\rangle = \mathcal{A}_{\rm fwm} + \mathcal{A}_{\rm xpm} + \mathcal{B}_{\rm c,xpm} + \mathcal{B}_{\rm c,fwm,1}. \tag{24}$$

In DQPSK, XPM interactions (through the term $\mathcal{A}_{\rm c,xpm}$) and their correlations (through the term $\mathcal{B}_{\rm c,xpm}$) do affect the photocurrent fluctuations. Again, this is no surprise. In DQPSK, the signal is contained in both quadratures of the field; hence, to extract the signal, a projection onto two axis at 45° to the symbol constellation is required. In this case, phase fluctuations are not orthogonal to the axis where the signal is projected; hence, they do contribute to the fluctuations of the detected photocurrent. The term $\mathcal{B}_{\text{c,xpm}}$ accounts for the correlations of the phase fluctuations induced, by any given pulse, through XPM on the two pulses overlapping at the receiver. Correlations are beneficial in DQPSK, because a differential receiver cancels perfectly correlated fluctuations.

When $f(z)$ is a symmetric function about $z = L/2$, which is a condition that can be approximated by Raman amplification with a counter-propagating pump, and $z^* = L/2$, the photocurrent fluctuations for DPSK are zero. This result, which is exact within first-order perturbation theory, may be simply shown by observing that when this symmetric condition is met and their phases are multiple of 180 °, the time-integrated fluctuations $\mathcal{J}_{j,l}$ are in quadrature with the pulse, as it may be easily verified by the change of variable $z' = z - L/2$ in the integral in (10). The amplitude fluctuations of the pulses, hence the fluctuations of the detected eye, therefore vanish to first order. With loss and lumped amplifiers, in-phase fluctuations cannot be nulled. It will be shown below by a numerical example that, similarly to OOK-IMDD [2], in this case, that they are minimized by a precompensation $z^*_{\text{opt}} < L/2$.

With DQPSK, instead, the reduction of the in-phase fluctuations does not in general improve system performance because phase fluctuations do affect the photocurrent fluctuations at the receiver, as discussed previously.

4. Numerical Examples

Let us now evaluate the contribution of the nonlinear fluctuations to the Q factor at the receiver, which is defined in our case as $Q_{\text{Mod}} = 2\langle I_{\text{Mod}}\rangle / 2\sqrt{\langle \Delta I^2_{\text{Mod}}\rangle}$, where Mod stands for DPSK or DQPSK. The inverse of the Q factor is the standard deviation of the detected fluctuations normalized to the detected signal. With the average transmitted power $P_{\text{av}} = \sqrt{\pi}A^2\tau/T_s$, the average signal square at detection in DPSK is $\langle I_{\text{DPSK}}\rangle^2 = \pi A^4\tau^2 = P^2_{\text{av}}T^2_s$, whereas in DQPSK, because of the aforementioned $\sqrt{2}$ reduction of the signal power, it is $\langle I_{\text{DQPSK}}\rangle^2 = \pi A^4\tau^2/2 = P^2_{\text{av}}T^2_s/2$. We have, therefore, $Q_{\text{DPSK}} = P^2_{\text{av}}T^2_s/\sqrt{\langle \Delta I^2_{\text{DPSK}}\rangle}$ and $Q_{\text{DQPSK}} = P^2_{\text{av}}T^2_s/2\sqrt{\langle \Delta I^2_{\text{DQPSK}}\rangle}$, where the variance of the nonlinear fluctuations is given by (23) and (24). Being, for a given pulse-width, $\langle I_{\text{Mod}}\rangle \propto P_{\text{av}}$ and $\sqrt{\langle \Delta I^2_{\text{Mod}}\rangle} \propto |\mathcal{J}_{j,l}| \propto A^4 \propto P^2_s$, the nonlinear contribution to the Q is inversely proportional to the transmitted average power P_{av}.

To illustrate these results, the Q factors for a system with the parameters listed in Table 1 [6] have been plotted. Notice that the bit rate of the DQPSK system (two bit/symbol) is 80 Gbit/s, whereas that of the DPSK (one bit/symbol) is 40 Gbit/s. A Matlab routine is used, and in particular, the Matlab command "quadv" that performs integrals that depend on matrices, in our case, that containing T_j and T_l, simultaneously and efficiently, and the property $\mathcal{J}_{j,l} = -\mathcal{J}^*_{j,-l} = \mathcal{J}_{-j,-l} = -\mathcal{J}^*_{-j,l}$.

In the first example, assume dispersion compensation complete at each span. Being the analysis based on linearization, and being the unperturbed evolution identical after each span, the amplitude of the perturbation is N times the perturbation of a single span. Consequently, the variance of the nonlinear noise is N^2 times the variance of the noise of the single span and the Q factor $1/N$ times the Q factor of the single span. Fig. 1(a) shows the Q factor versus the zero dispersion length of each span z^*, in kilometers. The blue solid line refers to DPSK and the red dashed line to DQPSK. The peaks in the Q factor correspond in DPSK to the value of z^* where the in-phase component of the perturbation is minimum and in DQPSK to the value of z^* where the correlation of the phase fluctuations induced by XPM is maximum. The two values of z^* coincide, although the maxima are generated by different mechanisms.

On the opposite side, there is the case in which dispersion compensation is applied only at the input and output of the line. The results are reported in Fig. 1(b). Here, the solid blue line shows the Q factor for DPSK versus the zero dispersion length z^*, in kilometers, whereas the red dashed line shows the Q factor for DQPSK. For DPSK, the peaks in the Q factor are again caused

TABLE 1

Numerical Parameters (FWHM = full-width at half maximum)

Quantity	Symbol	Value	Units
Fiber loss	α	0.25	dB/km
Fiber dispersion	β''	-20.4	ps^2/km
Nonlinear coefficient	γ	1.3	$W^{-1}km^{-1}$
Pulse-width (FWHM)	$\tau_{\rm FWHM}$	5	ps
Symbol time (inverse of baud rate)	T_s	25	ps
Input power	$P_{\rm dBm}$	10	dBm
Number of spans	N	7	
Span length	z_s	100	km

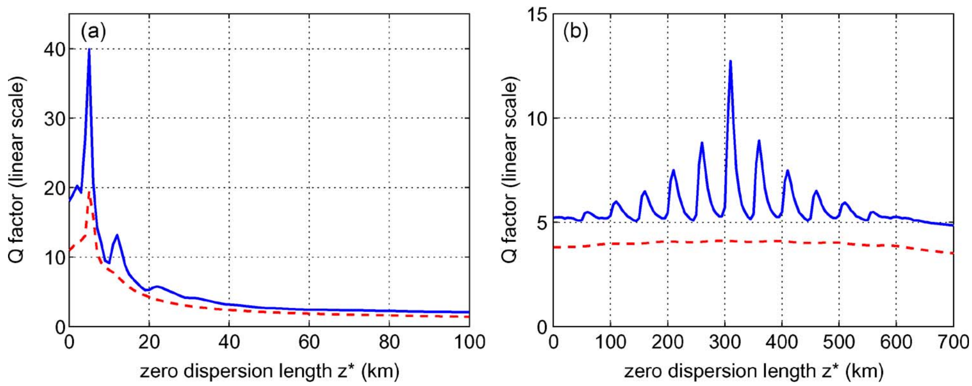

Fig. 1. (a) Q factor versus the zero dispersion length z^*, when dispersion compensation is complete at each span. Solid blue line: $Q_{\rm DPSK}$. Dashed red line: $Q_{\rm DQPSK}$. (b) Q factor versus the zero dispersion length z^* when compensation is all at the fiber ends. Solid blue line: $Q_{\rm DPSK}$. Dashed red line: $Q_{\rm DQPSK}$.

by local minima of the in-phase component of the perturbation. These are identical to the peaks of the inverse of the normalized amplitude noise of OOK-IMDD under similar conditions [2]. The Q factor of DQPSK shows, instead, a weaker dependence on precompensation $Q = 3.79$ for $z = 0$ and $Q = 4.14$ for $z^*_{\rm opt} = 295$ km. This is because, when no in-line compensation is used, a large number of pulses overlap along the fiber. Therefore, XPM is dominated by the contribution of distant pulses producing, no matter the precompensation, almost equal phase fluctuations on the two, consecutive, pulses interfering at the receiver. Being that the receiver is a differential one, it removes the correlated component of the phase fluctuations, and the uncorrelated component left only weakly depends on precompensation.

5. Conclusion

Based on first-order perturbation theory, it is shown that the phase distribution of the field and the detection scheme strongly affect the strength of the nonlinear impairments. In DPSK, the impairments are minimized by a dispersion profile that minimizes the in-phase component of the nonlinear displacement, whereas in DQPSK, the impairments are minimized by a dispersion profile that maximizes the correlation of the phase fluctuations on two consecutive pulses.

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