IEEE Photonics Journal
An IEEE Photonics Society Publication

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Volume 1, Number 6, December 2009

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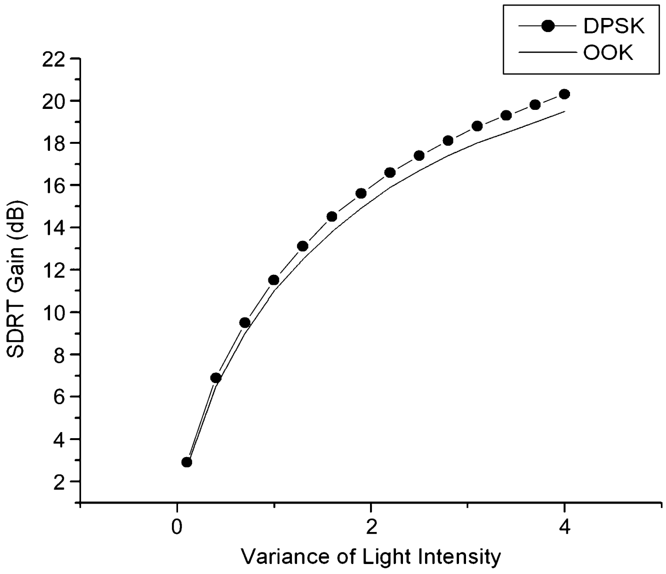

DOI: 10.1109/JPHOT.2009.2039015
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Performance Comparison of Different Modulation Formats Over Free-Space Optical (FSO) Turbulence Links With Space Diversity Reception Technique

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DOI: 10.1109/JPHOT.2009.2039015
1943-0655/$26.00 ©2009 IEEE

Manuscript received November 9, 2009; revised December 10, 2009. First published Online December 15, 2009. Current version published December 30, 2009. Corresponding author: Z. Wang (e-mail: E080008@ntu.edu.sg).

Abstract: The transmission performance of a free-space optical (FSO) link could be severely degraded due to atmospheric turbulence, which causes the temporal and spatial fluctuation of light intensity. Both the space diversity reception technique (SDRT) and advanced modulation formats can successfully mitigate the transmission impairments of the atmospheric turbulence. The purpose of this paper is to study and compare the bit-error-rate (BER) performance of several widely used modulation formats under different atmospheric turbulence scenarios with and without SDRT. The modulation formats studied in this paper include on–off keying (OOK), differential phase-shift keying (DPSK), and differential quadrature phase-shift keying (DQPSK). We derive a series-form formula for evaluating the BER performance of the DPSK format in the Gamma–Gamma distributed channel with SDRT. We use both theoretical analysis and simulation to examine the BER performance of OOK, DPSK, and DQPSK formats with and without SDRT. It is found that, in the strongly turbulent scenario, the OOK and DPSK formats can have as large as 19.5 and 20.3 dB of SDRT gains at the BER of 10^{-3}, respectively. Using SDRT, the modulation gains of the DPSK format over the OOK format are 3.2 dB in the strongly turbulent scenario and 4.5 dB in the weakly turbulent scenario, respectively. In addition, in the moderately and strongly turbulent scenarios, it is found that the DPSK and DQPSK formats have almost the same BER performance under the same symbol rate.

Index Terms: Atmospheric turbulent channel, differential phase-shift keying (DPSK), free-space optics, Gamma–Gamma distribution, selection combining, space diversity.

1. Introduction

Recently, there has been a significant resurgence of research interest in free-space optical (FSO) communications [1], [2]. The advantages of FSO communication, compared with radio frequency (RF) communications, include a much larger bandwidth/capacity, lower power consumption, more compact equipment, greater security against eavesdropping, and better protection against interference [3], [4]. However, FSO links suffer from random change of refractive index caused by the variation of air temperature and pressure [1], [4]. This impairment is shown as a temporal and spatial variation in light intensity [5], which is called scintillation, similar to the fading effect in wireless communication [4]. Presently, there are mainly three statistical models to describe the atmospheric turbulence channel, namely, the Log-normal distributed channel model [6], [20], the K-distributed channel model [7], [21] and the Gamma–Gamma distributed channel model [8], [22]. The Gamma–Gamma distributed channel model is found to be most suitable for modeling the irradiance of FSO channels in all the turbulent scenarios, from weak to strong [11], and hence, this model is used in our study.

Optical field has three physical attributes (intensity, phase and polarization) which can be used to transmit information. Considering that the atmospheric turbulence mainly affects on the light intensity, pulse-position modulation (PPM) [1], [4], [9], [24] is commonly used in FSO communication. Since fiber-optic technologies have been well developed and fiber-optic networks have been widely deployed from the local access networks to the long-haul intercontinental networks, some commonly used modulation formats in fiber-optic transmission system, including on–off keying (OOK) [1], and differential phase-shift keying (DPSK) [10], have also been investigated in FSO systems. Simplicity is the advantage of OOK, while DPSK format, which encodes information on its phase, can mitigate the severe effect of scintillation to some extent. Compared with the binary format such as OOK and DPSK, the differential quadrature phase-shift keying (DQPSK) format doubles spectral efficiency by taking advantage of the two signal quadratures of an optical carrier. Thus, two information bits are transmitted per symbol, being represented by four possible optical phase variations between successive symbol periods [17].

It is well known that channel coding and diversity techniques can be employed to mitigate the channel distortions and improve the performance of the transmission link. Several diversity techniques have been developed, including wavelength diversity [18], temporal diversity [19] and space diversity [11]. However, wavelength diversity techniques are found to be less effective for FSO communication systems due to the fact that the influence of atmospheric turbulence on link performance remains almost unchanged for all wavelengths [18]. Temporal diversity generally needs a longer signal processing time. Hence, space diversity becomes an attractive candidate for FSO communication.

In this paper, we investigate the bit-error-rate (BER) performance of various commonly used modulation formats together with a space diversity reception technique (SDRT) in the FSO link. We derive an analytical BER result of DPSK signal in the Gamma–Gamma distributed channel using two receivers with SDRT and carry out a comprehensive transmission performance comparison of OOK, DPSK, and DQPSK formats. It is found that, in the strong turbulent scenario, the OOK and DPSK format can have as large as 19.5 dB and 20.3 dB of SDRT gain at the BER of 10^{-3}, respectively. Using SDRT, the modulation gains of DPSK format over the OOK format are 3.2 dB in the strong turbulent scenario and 4.5 dB in the weak turbulent scenario, respectively. In addition, in the moderate and strong turbulent scenario, it is found that the DPSK and DQPSK formats have almost the same BER performance.

2. BER Performance Comparison for Various Modulation Formats

2.1. Gamma–Gamma Distributed Channel Model

In the Gamma–Gamma distributed atmospheric channel model, the normalized light intensity I can be partitioned into large-scale and small-scale atmospheric effects [7]. Both the large-scale and small-scale intensity fluctuations follow gamma distribution. Hence, the probability density function (pdf) of light intensity I is given by [7]

$$f(I) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} I^{\frac{\alpha+\beta-2}{2}} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta I}\right), I > 0 \tag{1}$$

where $K_n(\cdot)$ is the modified Bessel function of the second kind of order n, and $\Gamma(\cdot)$ is the gamma function. In (1), the positive parameters α and β represent the large-scale and small-scale optical wave intensity fluctuation, which are given by [7]

$$\alpha = \left\{\exp\left[0.49\sigma_R^2 / \left(1 + 1.11\sigma_R^{12/5}\right)^{7/6}\right] - 1\right\}^{-1} \tag{2}$$

$$\beta = \left\{\exp\left[0.51\sigma_R^2 / \left(1 + 0.69\sigma_R^{12/5}\right)^{5/6}\right] - 1\right\}^{-1} \tag{3}$$

where $\sigma_R^2 = 1.23 C_n^2 k^{7/6} L^{11/6}$ is the Rytov variance, representing the variance of log-intensity fluctuation [7], in which C_n^2 is the refractive-index structure parameter, k is the wavenumber, and L is the distance between transmitter and receiver [1].

According to the Hufnagel–Valley (H–V) turbulence model [7], [8], the refractive-index structure parameter C_n^2 is determined by wind speed (w) and altitude (h), which is given by

$$C_n^2(h) = 0.00594(w/27)^2(10^{-5}h)^{10}\exp(-h/1000) + 2.7\times 10^{-16}\exp(-h/1500) + C_n^2(0)\exp(-h/1000) \quad (4)$$

where $C_n^2(0)$ is the value of structure constant on the ground, which is usually $1.7\times 10^{-14}\mathrm{m}^{-2/3}$.

Since this Gamma–Gamma channel model can cover all turbulence scenarios from weak to strong [7], [11], it is employed in our study on the BER performance of various widely used modulation formats. When $\sigma_R^2 < 1$, the light intensity fluctuation is weak. The moderate intensity fluctuation is defined by $\sigma_R^2 \cong 1$. The strong intensity fluctuation comes out when $\sigma_R^2 > 1$. The saturation regime is associated with $\sigma_R^2 \to \infty$ [7].

2.2. BER Performance Without SDRT

In the Gamma–Gamma distributed channel, the transmitted signal is assumed to experience an independent and identically distributed intensity fading. The received signal can be represented as $y = hx + n = \eta I x + n$, where parameter h is the Gamma–Gamma channel gain, I is the normalized light intensity, x is the transmitted signal (being logical 0 or 1), n is the additive white Gaussian noise with zero mean and variance $\mathrm{N}_0/2$, and η is the photo-current conversion ratio [1], [8]. The BER of an optical signal transmitted in the Gamma–Gamma-distributed channel can be expressed by [1], [12]

$$p_e = \int_0^\infty p(I^2\bar{\gamma}) f(I)\, dI \quad (5)$$

where $p(I^2\bar{\gamma})$ represents the BER of an optical signal transmitted in the additive white Gaussian noise (AWGN) channel, and $\bar{\gamma}$ is the average electrical signal-to-noise ratio.

The BERs of OOK, DPSK, and DQPSK formats in the AWGN channel can be calculated by (6)–(8), respectively [1], [13], [16]

$$p_{OOK} = Q\left(\sqrt{I^2\bar{\gamma}/2}\right) \quad (6)$$

$$p_{DPSK} = \frac{1}{2}\exp(-I^2\bar{\gamma}) \quad (7)$$

$$p_{DQPSK} = \exp(-2I^2\bar{\gamma})\sum_{n=0}^{\infty}(\sqrt{2}-1)^n I_n(\sqrt{2}\cdot I^2\bar{\gamma}) - \frac{1}{2} I_0(\sqrt{2} I^2\bar{\gamma})\exp(-2I^2\bar{\gamma}) \quad (8)$$

where $Q(x) = 1/2 erfc(x/\sqrt{2})$, and $erfc(x)$ is the complementary error function. By substituting (6)–(8) into (5), we can obtain the BER of OOK, DPSK, and DQPSK signals in the Gamma–Gamma distributed turbulence channel of FSO link.

2.3. BER Performance With SDRT

As mentioned earlier, an optical signal could suffer from scintillation when it propagates through the atmospheric channel. In order to mitigate the severe impact of scintillation, it is wise to use two or more receivers for further signal processing [4]. This technique is known as SDRT, which has been widely used in RF communication systems to solve the fading problem. Currently, there are three main techniques to combine the diversity signals: selection combining (SC), maximum-ratio combining (MRC), and equal-gain combining (EGC) [12]. However, both the MRC and EGC need the signal phase information, which would make the system become much more complex. Hence, the SC method is chosen in our investigation of SDRT for its simplicity.

The SC method is to select the branch with the highest SNR among all the receivers and discard the rest. Assuming that the atmospheric conditions of different branches are independent of each other, there are N branches (receivers) for space diversity, and all the receivers are identical, selecting a branch with the highest SNR is equivalent to selecting a branch with the highest light intensity I. The probability that the light intensity in one branch is less than or equal to a specified value I_s is [14]

$$P_i(I \le I_s) = \int_0^{I_s} f(I)dI. \tag{9}$$

Since the atmospheric conditions of different branches are assumed to be independent of each other, the probability that the light intensity in all N branches is simultaneously less than or equal to I_s can be calculated by

$$P_N(I \le I_s) = \prod_{i=1}^{N} P_i(I \le I_s) = \left(\int_0^{I_s} f(I)dI \right)^N. \tag{10}$$

Equation (10) is the cumulative distribution function of the light intensity I for the SC method. The probability density function (pdf) of I is given by [12]

$$p_N(I) = \frac{dP_N(I)}{dI} = Nf(I) \left(\int_0^{I} f(I_i)dI_i \right)^{N-1}. \tag{11}$$

The BER performance of different modulation formats with SDRT under the Gamma–Gamma distributed turbulent channel can be calculated by

$$P_{e,SDRT} = \int_0^{\infty} p(I^2\bar{\gamma})p_N(I)dI = \int_0^{\infty} p(I^2\bar{\gamma})Nf(I) \left(\int_0^{I} f(I_i)dI_i \right)^{N-1} dI. \tag{12}$$

Replacing $p(I^2\bar{\gamma})$ in (12) with p_{OOK}, p_{DPSK}, and p_{DQPSK} in (6)–(8), respectively, we can numerically calculate the BER of OOK, DPSK, and DQPSK signals with SDRT.

In the rest of this section, we show the derivation of an analytical BER formula (series form) of DPSK format with two receivers $(N = 2)$ for the purpose of SDRT (in practice, this is the simplest)

$$P_{e,DPSK,SDRT} = \int_0^{\infty} \exp(-I^2\bar{\gamma})f(I) \left(\int_0^{I} f(I_i)dI_i \right) dI. \tag{13}$$

The cumulative distribution function of the gamma–gamma distribution is given by [7]

$$\int_0^{I} f(I_i)dI_i = \frac{\pi}{\sin[\pi(\alpha-\beta)]\Gamma(\alpha)\Gamma(\beta)} \times \left\{ \frac{(\alpha\beta I)^{\beta}}{\beta\Gamma(\beta-\alpha+1)} {}_1F_2(\beta;\beta+1,\beta-\alpha+1;\alpha\beta I) - \frac{(\alpha\beta I)^{\alpha}}{\alpha\Gamma(\alpha-\beta+1)} {}_1F_2(\alpha;\alpha+1,\alpha-\beta+1;\alpha\beta I) \right\}. \tag{14}$$

Hence, substituting (14) into (13), we have

$$P_{e,DPSK,SDRT} = \left\{ \frac{\pi}{\sin[\pi(\alpha-\beta)]\Gamma(\alpha)\Gamma(\beta)} \right\}^2 \times 2(\alpha\beta)^{\frac{\alpha+\beta}{2}}(p_1 - p_2 - p_3 + p_4). \tag{15}$$

The definition of ${}_pF_q(\cdot)$ in (14) and full derivation for p_1 to p_4 in (15) are given in the Appendix.

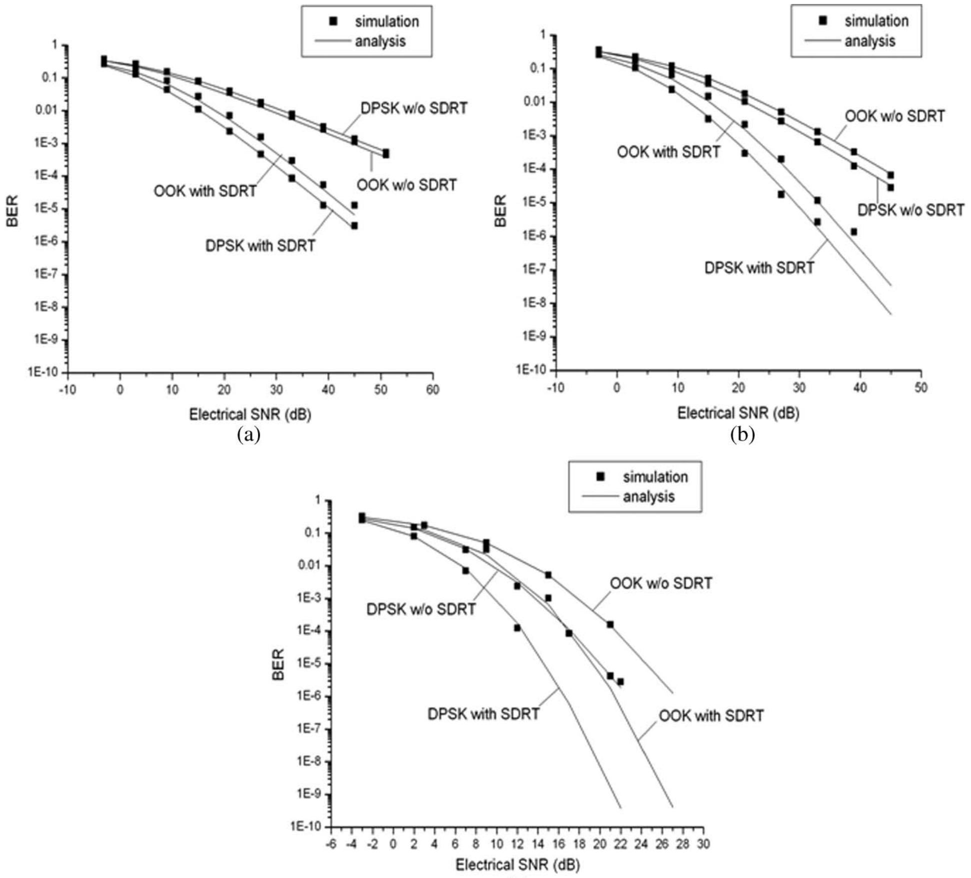
3. Analytical and Simulation Results

We have conducted both simulation and numerical calculation to compare the performance of various modulation formats with/without SDRT. In the SDRT case, only two receivers were used (i.e., $N = 2$). In both the simulation and analysis, we selected 0.16, 1.2, and 4 as the variance of light intensity, which can represent the weak, moderate and strong turbulent scenarios, respectively. Accordingly, the two parameters of the Gamma–Gamma channel became $(\alpha, \beta) = (14.11, 12.54)$, (4.20, 2.27), and (4.34, 1.31), respectively. In the simulation, the Monte Carlo Method was adopted in order to verify our numerical results by over 1 million times of random attempts. We use the BER level of 10^{-3} as the benchmark BER without using the forward error correction (FEC) coding technique, since it has been demonstrated that the error-free transmission can be achieved when the FEC coding technique is employed [23]. In order to evaluate the BER performance of the three modulation formats, we define modulation gain (in decibels) as the SNR improvement of one modulation format over another to achieve a BER of 10^{-3}. The SDRT gain (in decibels) is defined to be the amount of reduction in the electrical SNR required for each modulation format to achieve a BER of 10^{-3} due to the use of SDRT.

Fig. 1(a) and (b) show the BER performance of DPSK and OOK formats in the strong and moderate turbulent channel, respectively. As shown in the figures, the numerical analytical results are in good agreement with the Monte Carlo simulation results. Obviously, DPSK format performs better than that of OOK format. In the strong turbulent channel, as shown in Fig. 1(a), OOK format can have 19.5-dB SDRT gain, and the DPSK format can have 20.3-dB SDRT gain, which is 0.8 dB more than that of OOK format; the modulation gains of DPSK format over OOK format is 2.4 dB without SDRT and 3.2 dB with SDRT, respectively. In the moderate turbulent channel, as shown in Fig. 1(b), OOK format can have 12.1-dB SDRT gain, and DPSK format can have 12.7-dB SDRT gain; the modulation gains of DPSK format over OOK format are 3.1 dB without SDRT and 3.7 dB with SDRT, respectively. It is interesting to find that in the weak turbulent scenario, as shown in Fig. 1(c), DPSK without SDRT performs almost the same as OOK format with SDRT at the BER of 10^{-3}. Thus, it is suggested to use DPSK format in the weak turbulent channel. Fig. 1 also shows that the modulation gain of DPSK format over OOK format gets smaller and smaller as the turbulent scenario changes from weak to strong, especially when SDRT is not adopted. Adopting SDRT, the modulation gains of DPSK format over OOK format are 3.2 dB in the strong turbulent channel and 4.5 dB in the weak turbulent channel. Without SDRT, those modulation gains become 2.4 dB and 4.1 dB, respectively.

Fig. 2 shows the relationship between SDRT gain and the variance of light intensity σ_R^2 (i.e., the strength of the scintillation). As can be seen, the SDRT gain of both DPSK and OOK formats increases as the variance of light intensity grows. This means that the SDRT is very effective when the turbulence becomes stronger. The SDRT gain of DPSK format is slightly higher than that of OOK format.

Simulation results also reveal that DPSK and DQPSK formats perform almost the same in the strong and moderate turbulent scenarios. As shown in Fig. 3(a) and (b), the BER curves for DQPSK and DPSK almost overlap, with negligible differences. Since two information bits are transmitted per symbol for DQPSK format, the effective data rate of DQPSK format is twice as high as that of DPSK format; hence DQPSK should be a promising candidate for high-data-rate FSO communication systems. In the weak turbulent scenario, the modulation gain of DPSK over DQPSK becomes 0.79 dB without SDRT and 0.75 dB with SDRT, respectively. It should be noted that in the AWGN channel, the modulation gain of DPSK format over DQPSK format is 1.25 dB to achieve a BER of 10^{-3}. Thus, we can conclude that when the light intensity is less affected by the propagation channel (weak or no turbulence channel), the difference of SNR between DPSK format and DQPSK format to achieve 10^{-3} is increased.



(a) (b)

(c)

Fig. 1. BER performance of DPSK and OOK formats in (a) the strong turbulent channel, (b) the moderate turbulent channel, and (c) the weak turbulent channel.

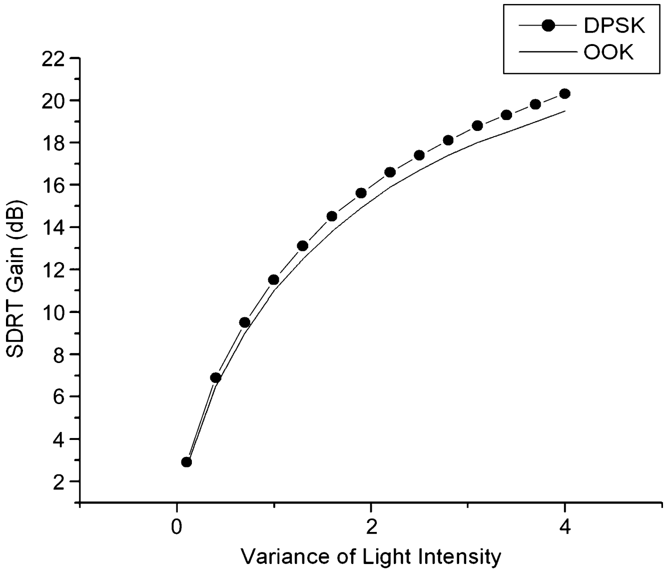
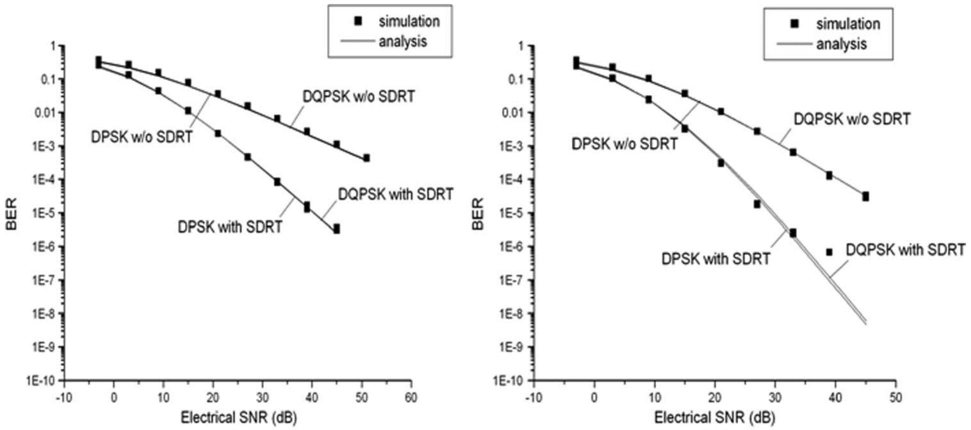


Fig. 2. SDRT gain curve of DPSK and OOK formats in different turbulent scenarios.



(a) (b)

Fig. 3. BER performance of DPSK and DQPSK formats in (a) the strong turbulent channel and (b) the moderate turbulent channel.

4. Conclusion

In this paper, the BER performances of DPSK, OOK, and DQPSK signals in the Gamma–Gamma distributed atmospheric turbulence channel from the weak- to the strong-turbulence scenarios have been investigated and compared. We have derived a series-form BER formula of DPSK signal in the Gamma–Gamma distributed channel with SDRT. It is found that, in the strong turbulent scenario, the OOK and DPSK format can have as large as 19.5 dB and 20.3 dB of SDRT gains at the BER of 10^{-3}, respectively. Using SDRT, the modulation gains of DPSK format over the OOK format are 3.2 dB in the strong turbulent scenario and 4.5 dB in the weak turbulent scenario, respectively. In the moderate and strong turbulent scenarios, it is also found that the DPSK and DQPSK formats have almost the same BER performance under the same symbol rate.

Appendix

The complete forms of p_1 to p_4 in (15) are given as[1]

$$p_1=\frac{1}{2}\int\limits_0^\infty \exp(-I^2\bar{\gamma})I^{\frac{\alpha+\beta}{2}-1}I_{-(\alpha-\beta)}(2\sqrt{\alpha\beta I})\times\frac{(\alpha\beta I)^\beta}{\beta\Gamma(\beta-\alpha+1)}{}_1F_2(\beta;\beta+1,\beta-\alpha+1;\alpha\beta I)dI$$

$$p_2=\frac{1}{2}\int\limits_0^\infty \exp(-I^2\bar{\gamma})I^{\frac{\alpha+\beta}{2}-1}I_{-(\alpha-\beta)}(2\sqrt{\alpha\beta I})\times\frac{(\alpha\beta I)^\alpha}{\alpha\Gamma(\alpha-\beta+1)}{}_1F_2(\alpha;\alpha+1,\alpha-\beta+1;\alpha\beta I)dI$$

$$p_3=\frac{1}{2}\int\limits_0^\infty \exp(-I^2\bar{\gamma})I^{\frac{\alpha+\beta}{2}-1}I_{\alpha-\beta}(2\sqrt{\alpha\beta I})\times\frac{(\alpha\beta I)^\beta}{\beta\Gamma(\beta-\alpha+1)}{}_1F_2(\beta;\beta+1,\beta-\alpha+1;\alpha\beta I)dI$$

$$p_4=\frac{1}{2}\int\limits_0^\infty \exp(-I^2\bar{\gamma})I^{\frac{\alpha+\beta}{2}-1}I_{\alpha-\beta}(2\sqrt{\alpha\beta I})\times\frac{(\alpha\beta I)^\alpha}{\alpha\Gamma(\alpha-\beta+1)}{}_1F_2(\alpha;\alpha+1,\alpha-\beta+1;\alpha\beta I)dI.$$

In deriving p_1 to p_4, we use the equation $K_\nu(z)=\pi(I_{-\nu}(z)-I_\nu(z))/(2\sin(\pi\nu))$. The generalized hypergeometric function is given by ${}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z)=\sum_{n=0}^{\infty}((a_1)_n\ldots(a_p)_n z^n/$

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[1] Note that $I_\nu(z)$ represents the modified Bessel function of the first kind rather than light intensity, I which appears in the same equation.

$(b_q)_n \cdots (b_q)_n n!)$ [15], where $(a)_n$ is the Pochhammer symbol, which is defined as $(a)_n = \Gamma(a+n)/\Gamma(a)$. The derivation of the series form for p1 (15) is shown as follows: Let $u = I^2$, we have

$$p_1 = \frac{(\alpha\beta)^{\beta}}{4\beta\Gamma(\beta-\alpha+1)} \sum_{m=0}^{\infty} \frac{1}{m!\Gamma(m-\nu+1)} (\sqrt{\alpha\beta})^{2m-\nu} \times \left[\int_0^{\infty} \exp(-B\bar{\gamma}u) u^{\frac{2\beta+m}{2}-1} \times {}_1F_2\left(\beta; \beta+1, \beta-\alpha+1; \alpha\beta u^{\frac{1}{2}}\right) du \right]. \tag{16}$$

By making use of

$$\begin{aligned} (a)_{n+k} &= (a)_k (a+k)_n \\ (a)_{2k} &= 2^{2k} (a/2)_k [(1+a)/2]_k \\ (2k+1)! &= 2^{2k} (3/2)_k k! [15] \end{aligned}$$

we can obtain

$$\begin{aligned} {}_1F_2\left(\beta; \beta+1, \beta-\alpha+1; \alpha\beta u^{\frac{1}{2}}\right) = {}_1F_4\left(\frac{\beta}{2}; \frac{\beta+2}{2}, \frac{\beta-\alpha+1}{2}, \frac{\beta-\alpha+2}{2}, \frac{1}{2}; \frac{(\alpha\beta)^2}{16} u\right) \\ + \left(\alpha\beta u^{\frac{1}{2}}\right) \beta / [(\beta+1)(\beta-\alpha+1)] \times {}_1F_4\left(\frac{\beta+1}{2}; \frac{\beta+3}{2}, \frac{\beta-\alpha+2}{2}, \frac{\beta-\alpha+3}{2}, \frac{3}{2}; \frac{(\alpha\beta)^2}{16} u\right). \end{aligned}$$

Considering

$$\int_0^{\infty} t^{\delta-1} \exp(-t)\, {}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; \tau u) du = \Gamma(\delta)\, {}_{p+1}F_q(\delta, a_1, \ldots, a_p; b_1, \ldots, b_q; \tau) \tag{17}$$

the series form of p_1 is

$$\begin{aligned} p_1 = \frac{(\alpha\beta)^{\beta}}{4\beta\Gamma(\beta-\alpha+1)} \sum_{m=0}^{\infty} \frac{1}{m!\Gamma(m-(\alpha-\beta)+1)} (\sqrt{\alpha\beta})^{2m-(\alpha-\beta)} \\ \times \left[(\bar{\gamma})^{-\frac{2\beta+m}{2}} \Gamma\left(\frac{2\beta+m}{2}\right) \times {}_2F_4\left(\frac{2\beta+m}{2}, \frac{\beta}{2}; \frac{\beta+2}{2}, \frac{\beta-\alpha+1}{2}, \frac{\beta-\alpha+2}{2}, \frac{1}{2}; \frac{(\alpha\beta)^2}{16\bar{\gamma}}\right) \right. \\ + \frac{(\alpha\beta)\beta}{(\beta+1)(\beta-\alpha+1)} (\bar{\gamma})^{-\frac{2\beta+m+1}{2}} \times \Gamma\left(\frac{2\beta+m+1}{2}\right) \\ \left. \times {}_2F_4\left(\frac{2\beta+m+1}{2}, \frac{\beta+1}{2}; \frac{\beta+3}{2}, \frac{\beta-\alpha+2}{2}, \frac{\beta-\alpha+3}{2}, \frac{3}{2}; \frac{(\alpha\beta)^2}{16\bar{\gamma}}\right) \right]. \end{aligned} \tag{18}$$

Here, we only show the derivation for the series form of p_1. Following the same way, we can also derive the series forms of p_2 to p_4. Substituting the series forms of p_1 to p_4 into (15), we obtain the series form of (15). It is noted that, as shown in (17), the series form can be only derived when the BER in the AWGN channel is of an exponential form.

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