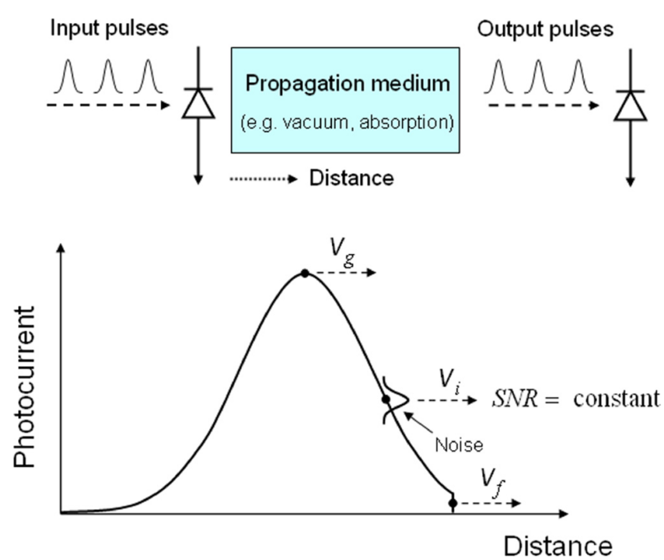
IEEE Photonics Journal
An IEEE Photonics Society Publication

Fast Light and the Speed of Information Transfer in the Presence of Detector Noise

Volume 1, Number 3, September 2009

Michael Z. Feng, Student Member, IEEE
Wayne V. Sorin, Fellow, IEEE
Rodney S. Tucker, Fellow, IEEE



DOI: 10.1109/JPHOT.2009.2031684
1943-0655/$26.00 ©2009 IEEE

Fast Light and the Speed of Information Transfer in the Presence of Detector Noise

Michael Z. Feng, *Student Member, IEEE*, Wayne V. Sorin, *Fellow, IEEE*, and Rodney S. Tucker, *Fellow, IEEE*

ARC Special Research Centre for Ultra-Broadband Information Networks,
University of Melbourne, Melbourne, Vic. 3010, Australia

DOI: 10.1109/JPHOT.2009.2031684

Manuscript received July 21, 2009; revised August 25, 2009. First published Online August 31, 2009. Current version published September 25, 2009. Corresponding author: M. Z. Feng (e-mail: mzfeng@ee.unimelb.edu.au).

Abstract: The phenomenon of fast-light pulse propagation has seen renewed interest in recent years, but its relationship to the *speed of information transfer* is still being debated. In this paper, we define the *speed of information transfer* as the propagation speed of a point of constant signal-to-noise ratio (SNR) on the leading edge of a signal pulse. We use standard telecommunication analysis to include the effects of noise so that the bit-error rate (BER) can be calculated as a function of a variable decision time. We introduce the concept of a time-dependent Q-factor so that pulse arrival times can be compared at equivalent BERs. We show that when receiver noise is included in a gain-assisted fast-light system, the measured *speed of information transfer* can exceed the speed of light for an equivalent pulse travelling through a vacuum.

Index Terms: Fast light, information transfer, pulse advancement, Q-factor.

1. Introduction

The terms "fast-light" or "superluminal" velocity can often generate confusion when one is first introduced to the concept. It implies that an optical pulse can travel faster than the speed of light in a vacuum which initially seems to violate fundamental laws. In order to try to resolve this confusion and to define concepts more precisely, many different definitions of optical velocity have been developed (see, for example, [1] and [2]). The goal of this paper is to further explore the concept of fast light by including the effects of attenuation, gain, and noise to determine if and when a fast-light medium might allow information speeds to exceed that of a vacuum medium.

Fast light occurs when the group velocity of a signal pulse exceeds the speed of light in a vacuum [3]. This effect can be understood in terms of an electrical signal passing through a linear time-invariant transfer function [4]. The apparent speeding up of the pulse occurs in regions of anomalous dispersion where the phase velocities for the different frequency components interfere constructively on the leading edge of the pulse and destructively on its trailing edge [5]. Violation of special relativity by propagating useful energy or information faster than the speed of light does not occur due to various limiting effects such as signal distortion, signal attenuation or fundamental noise effects [6].

Fig. 1(a) shows a schematic transmission spectrum where fast and slow light occurs using a general spectral transmission response. Fast light occurs at locations of positive curvature change and slow light at negative curvature locations as indicated by the "$+$" and "$-$" signs in Fig. 1(a). Fig. 1(b) shows the corresponding group index profiles for the positive and negative curvature locations. A detailed discussion on group index plots can be seen in [3]. The largest fast-light responses occur at narrow dips or notches in the transmission spectrum and the strongest slow-light

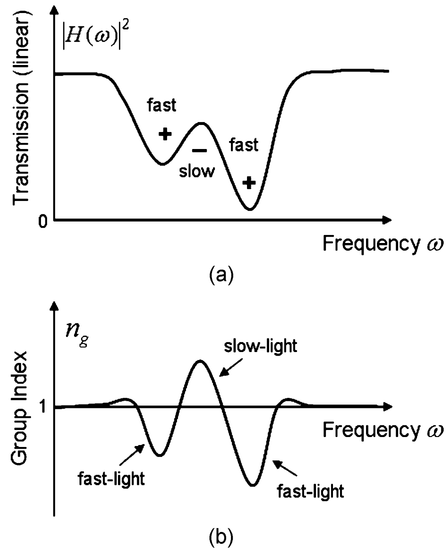


Fig. 1. (a) Locations of fast- and slow-light responses using a general spectral transmission function. Fast light occurs at locations of positive curvature and slow light at locations of negative curvature. (b) Corresponding group index profiles for the positive and negative curvature locations.

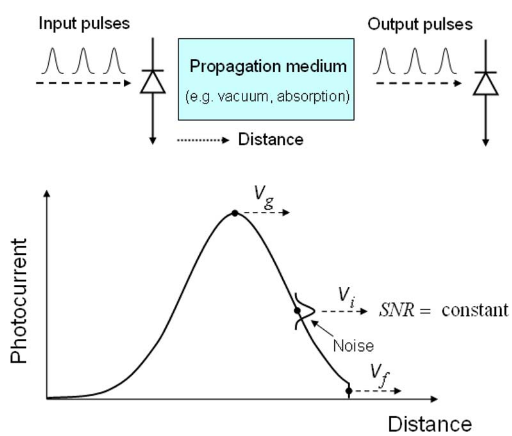


Fig. 2. Photocurrent versus distance for illustrating group velocity (V_g), front velocity (V_f), and speed of information transfer (V_i).

responses occur at narrow peaks or bandpass locations. Optically, these sharp variations in the spectral transmission can be achieved through the use of narrow-band optical absorption or gain resonances [3].

To avoid signal distortion during fast-light propagation, the input pulse spectrum should be much narrower than the width of the absorption dip. Although time advancement increases as the absorption width is narrowed, the input pulse bandwidth constraint fundamentally limits the maximum amount of fractional pulse width advancement to about one or two pulse widths [3]. Attempting to increase the fractional pulse advancement by using shorter pulses results in a large differential gain for the wider spectral tails, hence causing pulse distortion. In contrast, since slow-light pulses occur at transmission peaks their spectral tails do not experience this differential gain and can therefore be delayed by a very large number of pulse widths [3].

Fig. 2 illustrates several velocity definitions that have been developed to describe the speed of light in a dispersive medium. For example, for a narrow-band signal, "group velocity" (V_g) can be defined as the propagation speed for the peak of a signal pulse [2]. Another common definition is the "front velocity" (V_f), which is equivalent to the propagation speed of an abrupt sharp edge in the

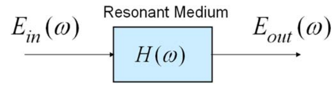


Fig. 3. Electric field transfer function describing transmission through a narrow-band resonant medium.

signal shape [2]. However, neither of these two velocities applies directly to information transfer because information is intimately coupled to signal-to-noise ratio (SNR), which is not employed in either of the above two velocity definitions. In order to address this fact, we define a term called the "*speed of information transfer*" (V_i) as the propagation speed of a measurable point of constant SNR on the leading edge of a signal pulse [7]. This definition takes into account the fact that any measurement of information requires the use of an optical detector (i.e., receiver). To measure this propagation speed, identical detectors are required at both the input and output sides of the propagation medium (see Fig. 2). Otherwise, the SNR for vacuum propagation would not remain constant resulting in a measurement speed unequal to "c" (the speed of light in a vacuum).

The *speed of information transfer* is a practical definition that incorporates many of the propagation details such as the signal strength at the detector, the effects of noise, and the efficiency of the detection process. It is generally accepted that the speed at which information is transferred cannot exceed the speed of light in a vacuum [1], [2], [6]. Recent experimental demonstrations have also implied this result is fundamental, even in the presence of receiver noise [8], [9].

In this paper, we show that when receiver noise is included in a gain-assisted fast-light enhanced communication link, it becomes possible to measure a *speed of information transfer* that exceeds the speed of light for a pulse propagating through an equivalent distance in vacuum. We use standard optical communication concepts to define a time-dependent Q-factor to compare the bit-error rates (BERs) at the leading pulse edges of an incoming data stream. We show that when the receiver noise is eliminated, the *speed of information transfer* cannot exceed the speed of light in a vacuum which is in agreement with the generally accepted understanding.

This paper is an extension to our previous work [7]. In the present paper, we have included a more detailed analysis in our modeling of the fast-light pulse propagation and noise sources. We have also expanded the number of transmission configurations that we consider to provide further insight into our investigation.

This paper is structured as follows. In Section 2, we review some basic concepts of fast-light pulse propagation using standard electrical engineering terminology. Section 3 introduces typical noise sources and defines the concept of a time-dependent Q-factor for determining BER. In Section 4, we present a variety of transmission configurations that feature different arrangements of absorption and gain mediums to illustrate some basic concepts related to fast light and the *speed of information transfer*. Finally, we discuss a few special conditions to generalize some of the concepts.

2. Basics of Fast Light

The concepts of fast or slow light are a straightforward effect of an optical signal passing through a medium that can be characterized by a linear time-invariant transfer function. Fig. 3 shows a signal passing through a narrow-band resonant medium, where $E_{in}(\omega)$ is the input electric field of the signal, $H(\omega)$ the transfer function of the resonant medium and $E_{out}(\omega)$ the output electric field of the signal. The group velocity (hence, group delay) of an optical signal is related to the phase slope of the transfer function. This phase slope can become large in a medium with a narrow-band optical resonance. The analysis below describes some of these basic concepts in more detail.

The frequency-domain transfer function for the optical electric field can be written as

$$H(\omega) = \frac{E_{out}(\omega)}{E_{in}(\omega)} = |H(\omega)| \cdot e^{j\theta(\omega)} \tag{1}$$

where the optical phase imparted onto the input spectrum is given by $\theta(\omega)$, and the transmission for the optical power is given by $T(\omega) = |H(\omega)|^2$.

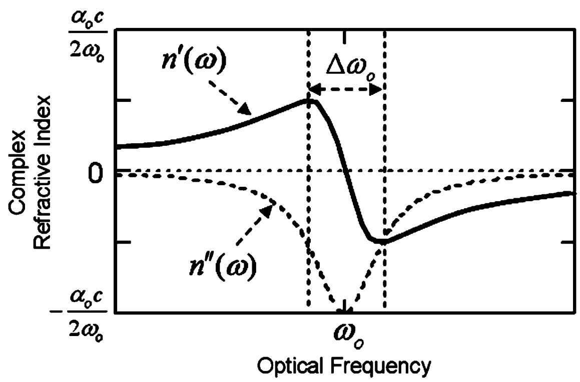


Fig. 4. Complex refractive index due to a narrow-band absorption resonance. The real part of the index is given by the solid line and the imaginary part by the dashed line. Fast light occurs in the anomalous dispersion region within the absorption width.

Speeding up or slowing down of a pulse can be understood by using standard Fourier analysis concepts which state that a translation of a pulse in the time domain is equivalent to adding a linear phase shift to its Fourier components. This can be expressed as [10]

$$\mathrm{E}(t-\tau_d) \Leftrightarrow e^{-j\omega\tau_d}E(\omega) \tag{2}$$

where $\mathrm{E}(t)$ and $E(\omega)$ are Fourier transform pairs. Equation (2) implies that if a system has a frequency-domain transfer function that is primarily dominated by a linear phase shift $\theta(\omega) = -\omega\tau_d$ then the signal pulse will retain its original shape and experience only a relative time translation. The group velocity can be defined as $v_g = L/\tau_d$, where L is the physical propagation length and $\tau_d = -d\theta/d\omega$ is the group delay measured at the pulse peak [4]. The validity of the group velocity is based on the assumptions that the signal spectrum experiences an approximately linear phase change and that the relative amplitudes of the spectral Fourier components are not significantly altered by transmission through the transfer medium.

A fast- or slow-light response can be demonstrated using a narrow-band optical resonance which can consist of either an atomic absorption or gain or a combination of both. Analytically this type of resonance can be completely characterized by the use of a frequency-dependent complex refractive index, where the real part represents the normal index term and the imaginary part describes changes in signal amplitude. Using the model of a standard classical electron oscillator [11], the complex refractive index for a narrow-band absorption (or gain) can be approximated quite accurately by

$$n(\omega) \approx n_o - j\frac{\alpha_o}{2}\frac{c}{\omega}\left(\frac{1}{1+j\Delta\hat{\omega}}\right) \tag{3}$$

where the first term n_o represents a constant background host index and the second term represents the effects of the narrow-band atomic resonance. The parameter α_o characterizes the strength of the peak absorption (or gain) and can be equated to the optical power transmission at resonance by $T_o = e^{-\alpha_o L}$. The bracketed term in (3) is the complex Lorentzian function normalized to unity at resonance [11]. The normalized frequency is given by $\Delta\hat{\omega} = 2(\omega-\omega_o)/\Delta\omega_o$, where ω_o is the center of the atomic resonant frequency, and $\Delta\omega_o$ is the width of the atomic resonance. The real $(n'(\omega))$ and imaginary $(n''(\omega))$ index terms can be obtained from (3) using the relation $n(\omega) = n'(\omega) + jn''(\omega)$, where $n'(\omega)$ and $n''(\omega)$ are related through the Hilbert transform [11]. This means that if the transmission response through a resonant absorption is known, then its phase response can be determined and therefore the amount of pulse advancement can be evaluated.

Fig. 4 graphically illustrates the complex refractive index due to the narrow-band absorption term given in (3). The total index is a combination of this result and the index of the background host material.

A fast-light response will occur for an input pulse spectrum centered at the resonance frequency ω_o where anomalous dispersion occurs. To avoid pulse distortion, the input spectrum should be narrow (i.e., $\ll \Delta\omega_o$) so that it experiences only the linear phase region and to also avoid excessive changes in its spectral shape. This puts a limit on the maximum fractional pulse advancement in a fast-light system to about one or two pulse widths [3]. If the value for the absorption constant α_o is made negative, the expression in (3) describes a resonant gain and exhibits a slow-light response due to the slope change in the real part of the index.

Using the complex index given in (3), the transfer function through a resonant medium of length L can be written as

$$H(\omega) = e^{-j\beta(\omega)L} = e^{\frac{\omega}{c}n''(\omega)L}e^{-j\frac{\omega}{c}n'(\omega)L} \tag{4}$$

where $\beta(\omega) = \omega n(\omega)/c$ is the propagation constant, and c is the speed of light in a vacuum. By comparing the right-hand side of (4) with (1), it can be seen that the phase shift of the transfer function is determined by the real part of the refractive index (i.e., $\theta(\omega) = -\omega n'(\omega)L/c$) and the magnitude of the transfer function is determined by the imaginary part. It should be pointed out that confusion may occur due to different conventions for the sign of the optical phase. In this paper, we will use the electrical engineering convention of defining the optical phase of the electric field as $\phi(t, z) = \omega t - \beta z$. The convention in physics is to reverse the signs on the frequency and distance variables.

As an illustrative example of a fast-light response we can calculate the time delay at the resonance peak of the narrow-band absorption. Assuming an input signal spectrum much narrower than $\Delta\omega_o$, the propagation time (or group delay) at resonance becomes

$$\tau_d = -\left.\frac{d\theta}{d\omega}\right|_{\omega=\omega_o} = \frac{L}{c}\left[n_o - \frac{\alpha_o c}{\Delta\omega_o}\right] = \frac{L}{v_g}. \tag{5}$$

The resonant absorption results in a decrease in the group delay of $\Delta\tau_d = \alpha_o L/\Delta\omega_o$, which depends only on the absorption strength and its spectral width. The result inside the brackets in (5) can be identified as the group index n_g at resonance. A fast-light response occurs if n_g becomes less than unity. Group indexes less than unity and even having negative values have been experimentally demonstrated by many researchers [8], [12], [13]. A negative group index (hence, negative group velocity) means that the pulse peak shows up at the output before it enters the input. However, the concepts of negative or infinite group velocities can be somewhat confusing and they do not provide much insight into more intuitive concepts such as the physical shape changes of a pulse as it propagates through the dispersive medium.

3. Noise Sources

A complete understanding of the *speed of information transfer* requires knowledge of both the signal and noise strengths plus the details of the detection process [2]. For our analysis, we consider a signal consisting of a random return-to-zero (RZ) pulsed data stream and define the *speed of information transfer* as $v_i = \Delta L/\Delta t$ where ΔL is the propagation distance and Δt is the earliest detected propagation time that gives a predetermined BER. The propagation time is measured relative to a global system clock synchronized to the pulsed data rate.

Although the SNR is a commonly used term to quantify the effects of noise [6], in this paper, we will use a similar but slightly different quantity that has been developed for analyzing optical transmission systems. This quantity is called the Q-factor, which allows for relatively easy estimates of the corresponding BER in an optical communication link [14]. Typically, the Q-factor is calculated at the maximum signal strength of the optical pulse which usually provides the lowest BER. In our analysis, we expand the concept of the Q-factor to make it a time-dependent quantity so that the BER can be determined at any point relative to the incoming data stream.

Fig. 5 graphically illustrates the parameters used to define the time-dependent Q-factor. In practice the decision time for determining the Q-factor can be set by adjusting the phase from a clock and data recovery (CDR) circuit. The parameter I_q in Fig. 5 refers to the decision-level setting

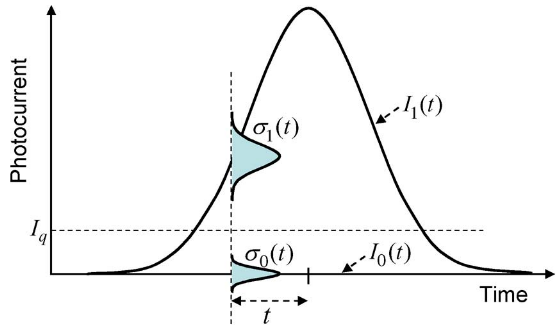


Fig. 5. Photocurrent pulse and associated noise variances for describing a time-dependent Q-factor. The BER is a function of where the data pulse is sampled.

for the CDR circuit. In our analysis, this value will be set to the optimum value which gives equal probability of error from either a one or zero bit. The BER will be a function of the two independent variables relating to decision time and decision threshold level.

The following equation describes the time-dependent Q-factor:

$$Q(t) = \frac{I_1(t) - I_0(t)}{\sigma_1(t) + \sigma_0(t)} \tag{6}$$

where $I_1(t)$ and $I_0(t)$ refer to either the "1 bit" or "0 bit" of the receiver photocurrent, respectively. The time variable t is measured from the peak power of the reference vacuum pulse. The standard deviation of the photocurrent noise on the "1 bit" and "0 bit" is given by $\sigma_1(t)$ and $\sigma_0(t)$, respectively. To avoid unnecessary complexity the "0-bit" signal will be set to zero (i.e., $I_0(t) = 0$), but the effects of noise on the zero level will still be included.

The photocurrent noise is a function of many factors. In this paper, we will include four major noise sources: thermal noise from the receiver σ_{th}, shot noise on the signal σ_{sn}, beat noise due to the mixing between the signal and any amplified spontaneous emission (ASE) noise $\sigma_{sig-ase}$ (i.e., signal–spontaneous beat noise) and beat noise due to the ASE mixing with itself $\sigma_{ase-ase}$ (i.e., spontaneous–spontaneous beat noise). Since these noise effects are uncorrelated, the total standard deviation can be written as

$$\sigma_{1,0}(t) = \sqrt{\sigma_{th}^2 + \sigma_{sn}^2 + \sigma_{sig-ase}^2 + \sigma_{ase-ase}^2} \tag{7}$$

where the subscripts on $\sigma_{1,0}(t)$ refer to the noise on either a "1 bit" or a "0 bit." The time argument highlights the fact that the noise changes in time since some of the noise terms are dependent on the strength of the signal pulse.

To provide a first-order understanding of the contributions and scaling of each of the individual noise sources, simple expressions are provided below. These simple expressions allow the basic effects and trends to be more easily understood. The results provided in this paper have also been compared by using a more exact and complex numerical noise analysis, and the differences are minor.

The first term in (7) models the thermal noise in the receiver and is given by

$$\sigma_{th}^2 = \frac{4kTB_e}{R_{eff}} \tag{8}$$

where k is Boltzman's constant, T the temperature in Kelvin, B_e the electrical noise bandwidth, and R_{eff} the effective equivalent noise resistance of the receiver. The second term in (7) is shot noise and given by

$$\sigma_{sn}^2(t) = 2q\Re P_{sig}(t)B_e \tag{9}$$

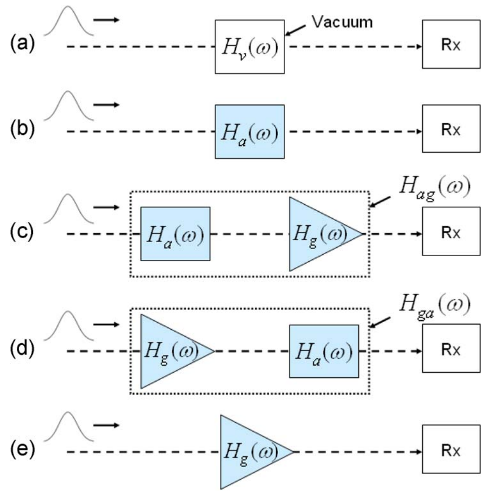


Fig. 6. Block diagrams showing five different transmission configurations and their associated electric-field transfer functions. (a) Vacuum, (b) absorption-only, (c) absorption–gain, (d) gain–absorption, and (e) gain-only.

where $\Re$ is the detector responsivity, q the electron charge, and $P_{sig}(t)$ the time-varying optical pulse power incident on the detector.

The third and fourth terms in (7) are a consequence of the ASE noise that accompanies any optical amplification process. Since exact expressions describing these two noise sources can be cumbersome, we present expressions for the simplified case when the shape of the generated optical noise spectrum is rectangular with an optical bandwidth B_o (where $B_o \gg B_e$). Although these simplifying assumptions are not fully valid in practice, the main concepts and trends are covered.

The noise variance due to the signal–spontaneous beat noise is given by

$$\sigma^2_{sig-ase}(t) \sim 4\Re^2 P_{sig}(t) P_{ase} \frac{B_e}{B_o} \tag{10}$$

where P_{ase} is the ASE noise power in a single-polarization incident onto the detector [15], [16]. The single-polarization noise power can be calculated from the amplifier gain G by $P_{ase} = n_{sp}(G-1)h\nu B_o$ where n_{sp} is the spontaneous emission factor and $h\nu$ is the energy per photon [15]–[16]. It should be noted that for unpolarized ASE, the total ASE noise power will double, but this does not affect the variance calculated using (10).

The noise term associated with the spontaneous-spontaneous beat noise has a similar form to (10) and is given by

$$\sigma^2_{ase-ase} \sim 4\Re^2 P_{ase} P_{ase} \frac{B_e}{B_o} \tag{11}$$

where this result describes the total noise generated by both polarizations, even though P_{ase} is the noise power in a single polarization.

4. Comparison of Five Basic Configurations

In this section, we analyze five basic transmission configurations to illustrate that under specific conditions the *speed of information transfer* can exceed the speed of light in a vacuum. The *speed of information transfer* will be deduced from the Q-factor measured at the receiver. The five configurations are illustrated in Fig. 6. They consist of (a) vacuum $H_v(\omega)$ and four different configurations of absorption and gain, including (b) absorption-only $H_a(\omega)$, (c) absorption followed by gain $H_{ag}(\omega)$, (d) gain followed by absorption $H_{ga}(\omega)$, and (e) gain-only $H_g(\omega)$. The overall transfer function for configurations (c) and (d) is obtained by multiplying $H_a(\omega)$ and $H_g(\omega)$ in the frequency domain. The transfer functions are calculated using the equations in (3) and (4).

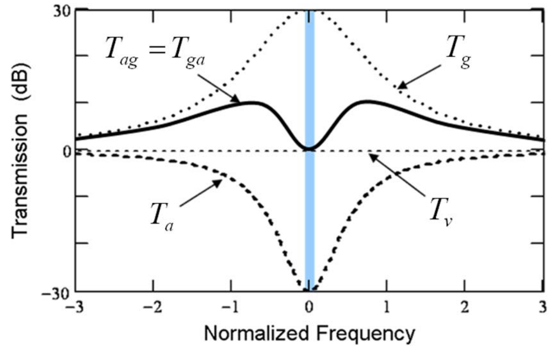


Fig. 7. Transmission characteristics for the five configurations shown in Fig. 6. The subscripts on the T parameter refer to $v =$ vacuum, $a =$ absorption-only, $ag =$ absorption followed by gain, $ga =$ gain followed by absorption, and $g =$ gain-only. The gain spectrum has twice the spectral width as the absorption spectrum. The shaded region represents the spectral width of the input signal pulse, which is 20% of the absorption width. *Tga* has the same response as *Tag*.

4.1. Normalized Pulse Transmission

The vacuum configuration in Fig. 6(a) serves as a reference for comparison with the other configurations. The other four configurations can be though of as a modification to the vacuum configuration by adding dilute resonant atoms to provide absorption, gain or a combination of both [5]. This conceptually highlights that all five configurations will have equal physical lengths and a background host index of $n_0 \approx 1$. The total transfer function for the two combinations of gain and absorption in Fig. 6(c) and (d) are identical since their order of multiplication is not important [i.e., $H_a(\omega)H_g(\omega) = H_g(\omega)H_a(\omega)$]. Although they have the same transmission characteristics, they demonstrate largely different noise characteristics. The impact of the combination of gain and absorption resonance and its resulting noise effect has been investigated in [17].

In our calculations, we used −30 dB and +30 dB for the peak values of the absorption and gain resonances in order to simulate approximate practical system parameters [18]. To avoid cancellation of the transfer functions through the absorption and gain pairs the spectral width of the gain resonance was made to be twice as wide as that of the absorption resonance. This produces a gain doublet in the transmission characteristics.

Fig. 7 shows the spectral power transmission characteristics $(T(\omega) = |H(\omega)|^2)$ in optical decibels for the five configurations illustrated in Fig. 6. The two configurations that use the gain and absorption pairs have identical transmission characteristics and provide unity transmission at resonance. Away from resonance they display a gain doublet profile. Using the illustrative diagram in Fig. 1, a fast-light response should occur at the positive curvatures that exist in the absorption-only configuration and the two absorption–gain configurations.

Fig. 8 shows the optical phases resulting from the transmission through each of the five configurations illustrated in Fig. 6. The phase change for the reference vacuum configuration was set equal to zero to remove the effects of the common physical distance for each of the five configurations. The absorption-only and gain-only configurations display opposite phase transition profiles, and the linear phase region of the gain-only configuration is twice as wide and has one half the slope as that of the absorption-only configuration. The gain–absorption and absorption–gain pairs have the same phase response which is obtained by adding the phase curves of the absorption-only and gain-only configurations.

Fig. 9 shows the calculated normalized output powers as a function of time (referenced to the propagation time for the vacuum configuration) when a Gaussian pulse is transmitted through each of the five configurations presented in Fig. 6. To avoid excessive pulse distortion, the spectral width of the Gaussian pulse was limited to 20% of the absorption width. The full-width-half-maximum (FWHM) for the Gaussian pulse and the absorption width were 100 MHz and 500 MHz, respectively.

When comparing the normalized output pulses in Fig. 9, the absorption-only configuration (P_a) demonstrates the largest pulse advancement in time. The amount of pulse advancement for the

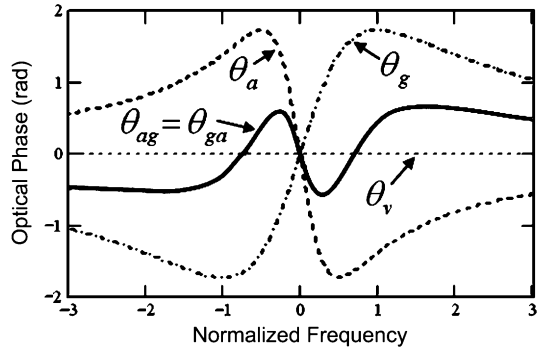


Fig. 8. Phase responses for the five transfer functions shown in Fig. 6. The phases are referenced to the vacuum case whose phase is arbitrarily set to zero. θ_{ga} has the same response as θ_{ag}.

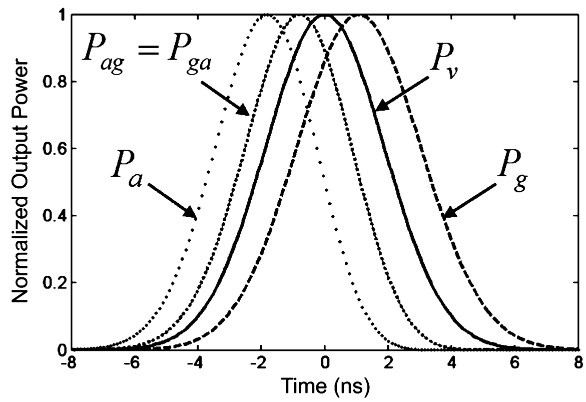


Fig. 9. Normalized output power for pulses after transmission through the five configurations shown in Fig. 6. The relative peak power strengths are $P_v = 0$ dB, $P_a = -28.3$ dB, $P_{ag} = P_{ga} = +1.2$ dB, and $P_g = +29.6$ dB.

absorption–gain (P_{ag}) and gain–absorption (P_{ga}) pairs is the same and is equal to one half of the advancement from the absorption-only configuration. This occurs because the gain resonance slows the pulse by 50% relative to the advancement from the absorption resonance.

The output pulse resulting from the gain-only configuration (P_g) exhibits a slow-light response and is delayed relative to the vacuum pulse as discussed in Section 2. However, since these curves are plotted by using normalized powers rather than absolute powers, they do not offer any representation of the signal strength at the receiver. Based on our results from the numerical analysis, the differential gain (or loss) for the peak power (referenced to the vacuum configuration) in each pulse is as follows: 0 dB for vacuum, -28.3 dB for absorption-only, $+1.2$ dB for both absorption–gain and gain–absorption, and $+29.6$ dB for gain-only. Since the normalized output power plots do not reflect these parameters, they provide no information regarding the SNR or the level of difficulty to detect the output signal at the receiver. In order to address this deficiency, we will next look at Q-factor plots in our analysis of fast light and its relation to the *speed of information transfer*.

4.2. Receiver With Noise

Fig. 10 shows the plots of the time-dependent Q-factors for the five system configurations in Fig. 6 when receiver noise is included. The plots in Fig. 10 used a peak input power of -30 dBm (i.e., 1 μW) injected into each configuration. This value is similar to power levels used in practical communication systems and limits the receiver power to 0 dBm for the gain-only configuration (peak gain $\sim +30$ dB). To simplify our analysis, we have assumed that there is no gain saturation. The

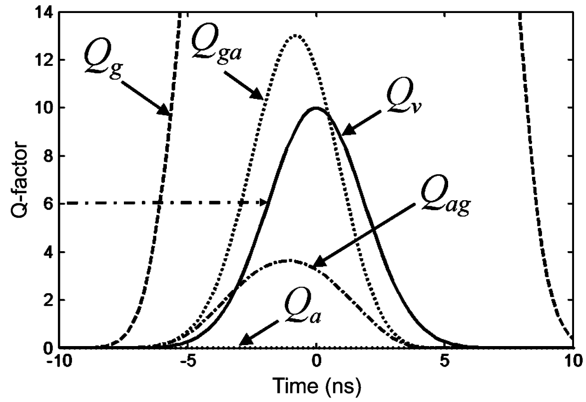


Fig. 10. Time-dependent Q-factors representing the BER for the example configurations shown in Fig. 6 when the receiver noise is nonzero.

electrical bandwidth B_e for the receiver was set to be 100 MHz (equivalent to the signal bandwidth), and the optical ASE bandwidth B_o was set equal to 10 times larger than the electrical bandwidth (i.e., $B_o = 1$ GHz). This factor of 10 in bandwidth results from our earlier assumptions of a two-fold increase between the absorption and gain widths coupled with a five-fold increase between the signal and absorption widths.

We used a detector responsivity $\Re$ of 1 A/W and an effective receiver noise resistance R_{eff} of 670 Ω to generate a peak Q-factor of 10 for the reference vacuum configuration. We chose a spontaneous emission factor n_{sp} of 1 for the calculation of ASE noise power P_{ase}.

The Q-factor plots describe the BER as a function of decision time for a pulse train consisting of an arbitrary distribution of ones and zeros. The leading edge (i.e., negative times) of the curves show how early the pulses can be detected for a given Q-factor (or BER). The earliest detection times to reach a Q-factor of 6 (approximately equivalent to achieving a BER of 10^{-9}) are shown in Fig. 10. Although these Q-factor curves use the simplified noise expressions given in Section 3 (which assumed a rectangular ASE noise spectrum), more rigorous methods that take into account the actual spectral shapes for both the ASE and the signal provide similar results.

From the Q-factor plots shown in Fig. 10, the absorption-only configuration (which displayed the largest pulse advancement in the normalized output pulse power plot) has such a poor signal quality that no information can be detected with a reasonable Q value (or BER), whereas the gain-only configuration (exhibiting a slow-light response) allows much earlier signal detection due to its superior SNR. If we draw a line at a detection threshold of $Q = 6$ (i.e., a BER of 10^{-9}), it will intersect the different curves in the order of gain-only, gain–absorption, and then vacuum. The curves of absorption–gain and absorption-only would not have sufficient signal quality to be detected at this BER.

Fig. 10 shows that when practical communication parameters are used for pulse powers and detector noise the measured *speed of information transfer* can be increased compared to that of a pulse traveling through a vacuum. This analysis implies that under specific conditions measured the *speed of information transfer* for a fast-light system (e.g., a gain–absorption pair) can exceed the speed of light in a vacuum.

4.3. Receiver Without Noise

In the above subsection, we have shown that when using a practical thermal-noise-limited receiver, both the gain-only and the gain-assisted fast-light configurations can result in a measured *speed of information transfer* that exceeds c, i.e., the speed of light in a vacuum. We will now show that when an ideal noiseless receiver is used, the *speed of information transfer* will always be limited to less than or equal to c. If we consider a noiseless optical receiver, then the dominant noise

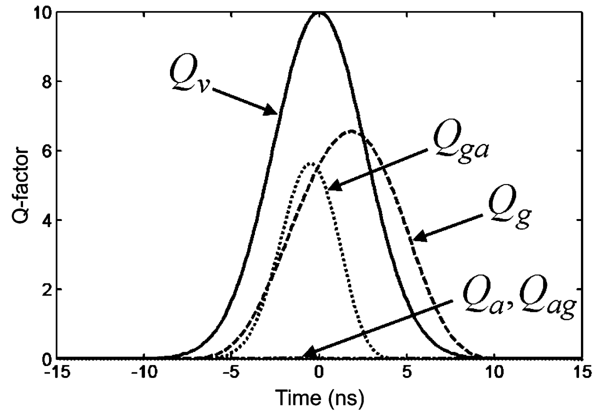


Fig. 11. Time-dependent Q-factors for an ideal noiseless receiver. When using a noiseless receiver, the vacuum case always exhibits the fastest *speed of information transfer*.

source for the vacuum configuration becomes the signal-dependent shot noise. To compare BERs similar to that in Fig. 10, the input pulse power was reduced by approximately 25 dB.

Fig. 11 shows Q-factor plots for the five configurations illustrated in Fig. 6, when the receiver noise is eliminated and the input power is reduced to give the same BER for the reference vacuum configuration. Now the earliest arrival time occurs for vacuum propagation and all other configurations result in a *speed of information transfer* less than c. This result remained true, regardless of any parameter changes we made to the system configurations. Thus, for a noiseless receiver, a pulse traveling through a vacuum will always result in the fastest *speed of information transfer*.

When compared with Fig. 10, we can see that removing the receiver noise results in reducing the Q-factor for the gain-only and the gain–absorption configurations. The reason for this is that although gain can increase the signal strength, the added amplified spontaneous noise will always reduce the signal quality when compared with the shot noise limit [19].

5. Discussions

The earlier arrival time for the gain-only configuration shown in Fig. 10 is not that surprising since the detected pulse power is about 1000 times larger than for the vacuum case. The larger pulse amplitude results in a larger photocurrent which dominates over the receiver noise and results in an earlier detection time.

The more interesting situation is the gain–absorption pair which could be thought of as modeling a possible practical gain-assisted fast-light system. For this arrangement the received pulse power is approximately equal to that for the vacuum configuration, and therefore, one cannot use the same argument that the larger photocurrent dominates over the receiver noise. This situation requires the advancement of the pulse shape caused by the fast-light mechanism (i.e., the absorption resonance). This illustrates that in a communication link that uses a receiver with practical noise level and has gain to compensate for the absorption that typically accompanies a fast-light response, the measured *speed of information transfer* can exceed the speed of light through an equivalent length of vacuum. In typical fast-light systems (without any gain), pulse attenuation is so strong that it dwarfs any pulse advancement resulting from the anomalous dispersion.

6. Conclusion

We have investigated the *speed of information transfer* in fast- and slow-light systems when practical levels of detector noise are included. To analyze these concepts we use a time-dependent version of the Q-factor commonly used in optical communications. This approach allows the arrival time of information to be determined based on achieving a predetermined BER. Five basic transmission configurations have been used to illustrate the effect of fast light and its relation to the *speed of information transfer*. We have shown that when receiver noise is included in the detection process, it is possible for the measured *speed of information transfer* in a gain-assisted fast-light system to be greater than that for the same pulse traveling through an equivalent length of vacuum. Although this result appears to violate a fundamental limitation on the *speed of information transfer*, it applies only when a nonfundamental noise source (e.g., thermal noise) is added to the system. Also note that this result does not violate the fundamental law of causality because the input pulse has a leading edge that precedes the sampling point. If an ideal noiseless receiver is used, the *speed of information transfer* can never exceed that for the vacuum configuration, which is in agreement with the commonly accepted understanding.

References

[1] L. Brillouin, *Wave Propagation and Group Velocity*. New York: Academic, 1960.
[2] P. W. Milonni, *Fast Light, Slow Light and Left-Handed Light,* 1st ed. New York: Taylor & Francis, 2005.
[3] R. W. Boyd and P. Narum, "Slow- and fast-light: Fundamental limitations," *J. Mod. Opt.*, vol. 54, no. 16, pp. 2403–2411, Nov. 2007.
[4] M. Kitano, T. Nakanishi, and K. Sugiyama, "Negative group delay and superluminal propagation: An electronic circuit approach," *IEEE J. Sel. Topics Quantum Electron.*, vol. 9, no. 1, pp. 43–51, Jan. 2003.
[5] D. J. Gauthier and R. W. Boyd, "Fast light, slow light and optical precursors: What does it all mean," *Photon. Spectra*, vol. 41, no. 1, pp. 82–90, Jan. 2007.
[6] A. Kuzmich, A. Dogariu, L. J. Wang, P. W. Milonni, and R. Y. Chiao, "Signal velocity, causality, and quantum noise in superluminal light pulse propagation," *Phys. Rev. Lett.*, vol. 86, no. 18, pp. 3925–3928, Apr. 2001.
[7] M. Z. Feng, W. V. Sorin, and R. S. Tucker, "Fast-light and the speed of information transfer," in *Proc. Slow Fast Light Conf.*, Jul. 2009.
[8] M. D. Stenner, D. J. Gauthier, and M. A. Neifeld, "The speed of information in a 'fast-light' optical medium," *Nature*, vol. 425, no. 6959, pp. 695–698, Oct. 2003.
[9] M. Centini, M. Bloemer, K. Myneni, M. Scalora, C. Sibilia, M. Bertolotti, and G. D'Aguanno, "Signal velocity and group velocity for an optical pulse propagating through a GaAs cavity," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 68, no. 1, p. 016602, Jul. 2003.
[10] R. Bracewell, *The Fourier Transform and Its Applications,* 3rd ed. New York: McGraw-Hill, 2000.
[11] A. E. Siegman, *Lasers*. Mill Valley, CA: Univ. Sci., 1986.
[12] L. J. Wang, A. Kuzmich, and A. Dogariu, "Gain-assisted superluminal light propagation," *Nature*, vol. 406, no. 6793, pp. 277–279, Jul. 2000.
[13] L. Thevenaz, "Slow and fast light in optical fibres," *Nat. Photon.*, vol. 2, no. 8, pp. 474–480, Aug. 2008.
[14] G. P. Agrawal, *Fiber-Optic Communication Systems,* 2nd ed. New York: Wiley, 1997.
[15] D. Derickson, *Fiber Optic Test and Measurement*. Englewood Cliffs, NJ: Prentice-Hall, 1998.
[16] G. Keiser, *Optical Fibre Communications,* 3rd ed. New York: McGraw-Hill, 2000.
[17] J. B. Khurgin, "Power dissipation in slow light devices: A comparative analysis," *Opt. Lett.*, vol. 32, no. 2, pp. 163–165, Jan. 2007.
[18] R. Ramaswami and K. Sivarajan, *Optical Networks: A Practical Perspective*. New York: Morgan Kaufmann, 1998.
[19] A. Yariv and P. Yeh, *Photonics: Optical Electronics in Modern Communications,* 6th ed. London, U.K.: Oxford Univ. Press, 2006.