Correspondence

Erratum and Corrigendum for "Structured Programming With and Without GO TO Statements"

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ERRATUM

The author wishes to thank J. C. Shepherdson for calling to his attention that, in the above paper,[1] Theorem 7.1 is false with Fig. 19 witness to its falsity. Since the proof of Theorem 7.2 depends on Theorem 7.1, it is rendered invalid. Fortunately, Theorem 7.2, the point of the section, remains valid as the proof given below shows. An independent proof, based upon the same idea but different in detail and logical organization, was found by Shepherdson.

CORRIGENDUM

The proof we give of Theorem 7.2 actually proves much more. Before stating the stronger result embodied in the Lemma and Theorem below, we make some preliminary observations.

If $F \in \mathcal{F}(\Pi, \Omega)$, where Π is finite, is a flowchart scheme, v is a vertex of F and a is an atom (i.e., an evaluation sequence) in $\mathcal{B}(\Pi)$, let $\mathbf{p}(F, v, a)$ be the path (cf., Section 6: "The evaluation sequence determines a path in $F \cdots$"[1]) in F starting with v which is compatible with a (i.e., "determined by a"). Let $\mathbf{t}(F, v, a)$ be the terminal vertex (either an operation vertex or an exit of F) $\mathbf{p}(F, v, a)$ if it is finite, otherwise $\mathbf{t}(F, v, a) = \infty$.

Observation 1: If v' is a vertex which occurs, nonterminally, in the path $\mathbf{p}(F, v, a)$, then $\mathbf{p}(F, v', a)$ is a suffix of $\mathbf{p}(F, v, a)$ and $\mathbf{t}(F, v, a) = \mathbf{t}(F, v', a)$.

Observation 2: If $\mathfrak{M}$ is a set of scalar schemes in $\mathcal{F}(\Pi, \Omega)$ *closed under the CASCI operations* (composition, alternation, separated conditional iteration) and $\mathrm{BI}(\mathfrak{M})$ is the set of all biscalar schemes in $\mathfrak{M}$, then $\mathrm{BI}(\mathfrak{M})$ is closed under the CASCI operations. In particular, the set of biscalar CASCI schemes is the smallest set of biscalar schemes containing 1_1, Ω and closed under the CASCI operations.

Now let $\Pi = \{\pi_1, \pi_2\}$, $\pi_1 \neq \pi_2$, $\omega_1 \in \Omega$, $\omega_2 \in \Omega$, $\omega_1 \neq \omega_2$. Let a_1 be the atom $\phi\tau$ and let a_2 be the atom $\tau\phi$. An $a_1 \omega_1 a_2 \omega_2$-circle in $F \in \mathcal{F}(\Pi, \Omega)$ is a circle C in F with s operation vertices labeled ω_1, s operation vertices labeled ω_2, where $s > 0$, such that for all $i, j \in [2]$, $i \neq j$ and for all ω_j-vertices v in C:

1) $\mathbf{t}(F, v, a_i)$ is an ω_i-vertex in C; and

2) $\mathbf{t}(F, v, a_j)$ is the exit of F.

Lemma: If $F \in \mathcal{F}(\Pi, \Omega)$ is weakly equivalent to the scheme of Fig. 23, then there is an $a_1 \omega_1 a_2 \omega_2$-circle in F.

Proof: We note that for each $m \geqslant 0$, $(a_1 \omega_1 a_2 \omega_2)^m a_2$ and $(a_1 \omega_1 a_2 \omega_2)^m a_1 \omega_1$ are in the weak behavior of Fig. 23 and hence in the weak behavior $|F|$ of F. Since $a_1 \omega_1 a_1$, $a_1 \omega_1 a_2 \omega_2 a_2$, $a_1 \omega_1 a_2 \omega_2 a_1 \omega_1 a_1, \cdots$ are all in $|F|$, if we define $v_1 = \mathbf{t}(F, b_F, a_1)$, where b_F is the begin of F, $v_2 = \mathbf{t}(F, v_1, a_2)$, $v_3 = \mathbf{t}(F, v_2, a_1)$ etc., we ultimately obtain (since F is finite) for some $r > 0, s > 0$ an operation vertex $v_r = v_{r+2s}$. Moreover, v_1 is an ω_1-vertex, v_2 is an ω_2-vertex, v_3 is an ω_1-vertex and $\mathbf{t}(F, v_1, a_1) = \mathbf{t}(F, v_2, a_2) = \mathbf{t}(F, v_3, a_1) = \text{exit } F$, etc. This information is indicated by $*$ below.

		ω_1		ω_2		ω_1		ω_j		ω_i		ω_j
$*$:	$\cdots$	v_1	$\cdots$	v_2	$\cdots$	v_3	$\cdots$	v_r	$\cdots$	v_{r+1}	$\cdots$	$v_{r+2s} = v_r$.
		a_1		a_2		a_1		a_j		a_i		a_j
		to exit		to exit		to exit		to exit		to exit		to exit

Thus, $v_r \cdots v_{r+1} \cdots v_{r+2s} = v_r$ is a linearization of an $a_1 \omega_1 a_2 \omega_2$-circle in F. □

Theorem: The set $\mathcal{S}$ of biscalar schemes F in $\mathcal{F}(\Pi, \Omega)$ satisfying

$**$: there is no $a_1 \omega_1 a_2 \omega_2$-circle in F

is closed under the CASCI operations.

Proof: Suppose C is an $a_1 \omega_1 a_2 \omega_2$-circle in F.

Case 1: $F = G \cdot H$, where $G, H \in \mathcal{S}$. Then C is an $a_1 \omega_1 a_2 \omega_2$-circle in G or in H. Contradiction. Hence, $F \in \mathcal{S}$.

Case 2: $F = T \cdot (G_1, G_2, \cdots, G_n)$, where $G_i \in \mathcal{S}$ for $i \in [n]$. The circle C is not in T since C contains operation vertices. Hence, C is an $a_1 \omega_1 a_2 \omega_2$-circle in G_i for some $i \in [n]$. Contradiction. Hence, $F \in \mathcal{S}$.

Case 3: $F = [T \cdot [1_1 \oplus G]]^\dagger$, where $G \in \mathcal{S}$ and $T: 1 \to 2$ is a test. The circle C is not in T since C contains operation vertices. Hence, either C lies wholly in G or $b_F = b_T$ is in C. In the former eventuality, we have for each ω_j-vertex v in C, $\mathbf{t}(F, v, a_j) = \text{exit } F$ and $\mathbf{p}(F, v, a_j)$ contains b_F since v is in G. Thus, $\mathbf{t}(G, v, a_j) = \text{exit } G$ and it follows C is an $a_1 \omega_1 a_2 \omega_2$-circle in G—which contradicts $G \in \mathcal{S}$.

The latter eventuality leads to the crux of our considerations. Suppose then that b_F is a vertex of C. Then for some operation vertex v in C, labeled $\omega_j, j \in [2]$, we have assuming $i \neq j$, $\mathbf{t}(F, v, a_i)$ is an ω_i-vertex in C and $\mathbf{p}(F, v, a_i)$ passes through b_F. Thus, by Observation 1, $\mathbf{t}(F, b_F, a_i) = \mathbf{t}(F, v, a_i)$ is an ω_i-vertex. On the other hand, where w is an ω_i-vertex in C, we have $\mathbf{t}(F, w, a_i) = \text{exit } F$ so that $\mathbf{p}(F, w, a_i)$ passes through b_F and, again by Observation 1, $\mathbf{t}(F, b_F, a_i) = \text{exit } F$. But we concluded above that $\mathbf{t}(F, b_F, a_i)$ is an ω_i-vertex. This contradiction implies $F \in \mathcal{S}$ and concludes the argument. □

Corollary: No CASCI scheme is weakly equivalent to the scheme of Fig. 23.

Proof: Suppose F is a biscalar scheme weakly equivalent to Fig. 23. By the lemma there is an $a_1 \omega_1 a_2 \omega_2$-circle in F. By Observation 2 and the fact that the atomic schemes $\omega \in \Omega$ and trivial scheme 1_1 are in $\mathcal{S}$, it follows from the Theorem that every biscalar CASCI scheme is in $\mathcal{S}$. Thus, F is not a CASCI scheme. □

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[1] C. C. Elgot, *IEEE Trans. Software Eng.*, vol. SE-2, pp. 41–54, Mar. 1976.