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Decentralized Stabilization of Large-Scale Dynamical Systems

M. DARWISH, H. M. SOLIMAN, AND J. FANTIN

Abstract—**The problem of stabilizing large-scale linear time invariant systems is considered. An approach is developed for stabilization using sets of decentralized controllers, and sufficient conditions are established in a form of algebraic criteria which can guarantee the stability under certain structural perturbations.**

I. Introduction

In classical control theory the systems under study are assumed to have only one control agent (called a controller) who determines the control actions based on the available information of the system. Systems of this type are called centralized systems, Li [1].

With the development of modern technology the size and complexity of the systems are increasing every day making the centralized control practically undesirable due to the cost of transmission of information to the central controller, in addition there might exist important delays and/or institutional constraints in carrying out central control actions [2], [3]. Hence large-scale systems are an essential feature of our present society.

An example of a large-scale system is a system of electric power networks belonging to several electric power generating companies connected together by tie lines where operators and dispatchers of each company have direct control of power generation and regulation of frequency and voltage in their own regions, but have no direct controls in regions belonging to different power companies. Hence dynamic behavior in these networks is influenced by several control agents acting partially independent of each other [4], [5].

Another example of a large-scale system is computer communication networks, whether land based or satellite, having a message routing between terminals where each terminal does not have access to information available to the other terminals. Here we have a team decision problem of maximizing the overall system performance [2], [6], [7].

In spite of the natural existence of large-scale systems no common precise definition for largeness has been proposed. The large-scale system we will adopt here is a system consisting of a number of interdependent subsystems which serve particular functions, share resources, and are governed by a set of interrelated goals and constraints [8], [9].

Much of the early work done in the area of large-scale systems is centered around the development of multilevel decomposition and coordination methods [9], [10], [11]. Although these techniques are conceptually simple, they require iterative solution procedures which may lead to convergent difficulties [12], [13]. Other multilevel schemes are based on characterization of interactions as perturbation signals acting in contradiction to the autonomy of the individual subsystems, then compensating signals that account for the interconnection effects are used in conjunction with locally decentralized controllers [14], [15], [16].

However, the multilevel methods can give the solution in optimal manner it requires high communication cost between the subsystems and the coordinator, besides, the stability of the system may be lost if a fault occurred in the communication links. Due to these difficulties decentralized control becomes vital in the case where one can dispense some of the optimality.

By decentralized systems we mean systems having several local control stations where at each station the controller observes only local system outputs and controls only local inputs. All the controllers are involved, however, in controlling the overall large system. In decentralized control the problem of stability of the overall system becomes very important, this problem has received the attention of many authors in the last few years [1], [3], [4], [16], [17]. Attention is given to the construction of appropriate controllability subspaces and canonical form in the subsystems descriptions and to the derivation of necessary and sufficient conditions for the existence of local controllers to stabilize a given system using either static or dynamic compensators.

In this correspondence an approach for the decentralized stabilization of a large-scale linear dynamical system is developed. Sufficient conditions for decentralized stabilization in a form of algebraic criteria are established which can guarantee the stability under certain structural perturbations. Finally the theory developed in this correspondence is illustrated by an example.

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II. Decentralized Stabilization of Large-Scale Linear Time Invariant Systems

Consider large-scale system S, which is described as an interconnection of N subsystems $S_1, S_2, \cdots, S_N$ and is represented by

$$\dot{x}_i = A_i\bar{x}_i + B_iu_i + h_i(x)$$

$$h_i(x) = \sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij}x_j, \qquad i = 1, 2, \cdots, N \tag{1}$$

where for the subsystem S_i: $x_i \in R^{n_i}$ is the state vector, $u_i \in R^{m_i}$ is the control vector, $h_i(x) \in R^{n_i}$ is the interaction vector from the other subsystems to S_i, and $A_i \in R^{n_i x n_i}$, $B_i \in R^{n_i x m_i}$, and $A_{ij} \in R^{n_i x n_j}$ are constant matrices, and all the pairs (A_i, B_i) are assumed controllable.

In this representation the composite system S can be described as

$$\dot{x} = Ax + Bu + h \tag{2}$$

where $x = [x_1^T, x_2^T, \cdots, x_N^T]^T$ is the composite state vector, $u = [u_1^T, u_2^T, \cdots, u_N^T]^T$ is the composite control vector, $h = [h_1^T, h_2^T, \cdots, h_N^T]^T$ represents the interconnection pattern of the overall system S, and $h = HX$,

$$H = \begin{bmatrix} 0 & A_{12} & A_{13} & \cdots & A_{1N} \\ A_{21} & 0 & & & A_{2N} \\ \vdots & & & & \vdots \\ A_{N1} & & & & 0 \end{bmatrix}.$$

H is the interconnection matrix, $A = \text{diag }(A_i)$, $B = \text{diag }(B_i)$, $A \in R^{n \times n}$ and $B \in R^{n \times m}$ matrices where $n = \sum_{i=1}^{N} n_i$ and $m = \sum_{i=1}^{N} m_i$ equation (2) can be rewritten as

$$\dot{x} = \bar{A}x + Bu \tag{3}$$

where

$$\bar{A} = A + H = \begin{bmatrix} A_1 & A_{12} & \cdots & A_{1N} \\ A_{21} & A_2 & \cdots & A_{2N} \\ \vdots & & & \vdots \\ A_{N1} & A_{N2} & \cdots & A_N \end{bmatrix}.$$

Associated with each subsystem S_i a performance index J_i of the form

$$J_i = \tfrac{1}{2}\int_0^{\infty} (x_i^TQ_ix_i + u_i^TR_iu_i)\, dt \tag{4}$$

where $Q_i \in R^{n_i \times n_i}$ is a positive semidefinite matrix, and $R_i \in R^{m_i \times m_i}$ is a positive definite matrix such that

$$J = \sum_{i=1}^{N} J_i \tag{5}$$

that is, each subsystem is optimized with respect to its own performance index (4), regardless of the behavior of the other subsystems.

The problem is how to find decentralized controller u_i of the form

$$u_i = -W_ix_i, \qquad i = 1, 2, \cdots, N \tag{6}$$

which minimizes J_i and insures the stability of the overall system S given by (2).

We adopt here the notions of exponential stability [3], [18], that is, the solution of (1) should satisfy $xe^{\alpha t} \to 0$ as $t \to \infty$, and where $\alpha > 0$.

Consider the isolated decoupled subsystems $S_1^0, S_2^0, \cdots, S_N^0$ in which the interaction vectors $h_i(x)$ are assumed to be zero

$$\dot{x}_i = A_ix_i + B_iu_i, \qquad i = 1, 2, \cdots, N \tag{7}$$

each of subsystems $S_1^0, S_2^0, \cdots, S_N^0$ described by (7) can be exponentially stabilized with degree α and minimizes

$$J = \tfrac{1}{2}\int_0^{\infty} e^{2\alpha t}(x_i^TQ_ix_i + u_i^TR_iu_i)\, dt \tag{8}$$

when the control vectors are given by

$$u_i = -R_i^{-1}B_i^TK_ix_i, \qquad i = 1, 2, \cdots, N \tag{9}$$

where K_i is symmetric positive definite solution of matrix Riccati equation

$$A_i^TK_i + K_iA_i - K_i(B_iR_i^{-1}B_i^T)K_i + 2\alpha K_i + Q_i = 0. \tag{10}$$

Then the closed loop decoupled subsystems are given by

$$x_i = (A_i - B_iR_i^{-1}B_i^TK_i)x_i, \qquad i = 1, 2, \cdots, N \tag{11}$$

and has the property that $x_ie^{\alpha t} \to 0$ as $t \to \infty$ using the control u_i of (9), the composite system (2) is described by

$$\dot{x} = (A - BR^{-1}B^TK)x + h \tag{12}$$

$$h = Hx$$

where $R^{-1} = \text{diag }(R_i^{-1})$, $K = \text{diag }(K_i)$ and is the solution of

$$A^TK + KA - K(BR^{-1}B^T)K + Q + 2\alpha K = 0 \tag{13}$$

where $Q = \text{diag }(Q_i)$. The presence of interconnection h will change the stability, and it is necessary to obtain sufficient conditions to guarantee the stability of the overall system S. This is given by the following theorem.

Theorem: The original system $\dot{x} = \bar{A}x + Bu$ can be stabilized in a decentralized form by the control $u = -R^{-1}B^TKx$, if the matrix $G = [2\alpha K + P - (KH + H^TK)]$ is positive definite where $P = Q + KBR^{-1}B^TK$.

Proof: The proof is based on Lyapunov theory. Consider the positive definite Lyapunov function V for the overall system S as

$$V = x^TKx \tag{14}$$

taking the time derivative of (14) along (12) gives

$$\dot{V} = x^T(A^TK - KBR^{-1}B^TK)x + h^TKx + x^T(KA - KBR^{-1}B^T)x + x^TKh. \tag{15}$$

Using (12), then (15) will be

$$\dot{V} = x^T(A^TK + KA - 2KBR^{-1}B^TK)x + x^TH^TKx + x^TKHx. \tag{16}$$

Then using (13) and (16) $\dot{V}$ can be written as

$$\dot{V} = -x^T[2\alpha K + P - (KH + H^TK)]x. \tag{17}$$

For (12) to be stable, $\dot{V}$ should be negative definite, then the matrix

$$G = [2\alpha K + P - (KH + H^TK)] \tag{18}$$

should be positive definite. Q.E.D.

Lemma: A sufficient condition for G to be positive definite can be given as

$$2\alpha \min_i \lambda_{\min}(K_i) + \min_i \lambda_{\min}(P_i) > 2C \max_i \lambda_{\max}(K_i)$$

where $C = \|H\|$ norm of H, $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote, respectively, the minimum and the maximum eigenvalues of the argument matrix.

$$\min_i \lambda_{\min}(K_i) \quad \text{and} \quad \max_i \lambda_{\max}(K_i)$$

are performed for all the N subsystems.

Proof: For G to be positive definite it is sufficient to show that

$$x^T[2\alpha K + P - KH + H^T K)]x \tag{19}$$

is positive definite or $2\alpha x^T Kx + x^T Px > 2x^T KHx$. Then

$$2\alpha\lambda_{\min}(K) + \lambda_{\min}(P) > 2\|H\|\lambda_{\max}(K)$$

but

$$\lambda_{\min}(K) = \min_i \lambda_{\min}(K_i)$$

$$\lambda_{\min}(P) = \min_i \lambda_{\min}(P_i)$$

and

$$\lambda_{\max}(K) = \max_i \lambda_{\max}(K_i).$$

Hence a sufficient condition for G to be positive definite is

$$2\alpha \min_i \lambda_{\min}(K_i) + \min_i \lambda_{\min}(P_i) > 2C \max_i \lambda_{\max}(K_i). \tag{20}$$

Conditions (18) or (20) guarantee the stability of the overall system S, but it does not yield any measure on the degree of stability.

III. Case Study

To illustrate the above approach let us consider the second order system

$$\begin{aligned} \dot{x}_1 &= 5x_1 - 3x_2 + u_1 \\ \dot{x}_2 &= -4x_1 + 6x_2 + u_2. \end{aligned} \tag{21}$$

In this case we have two subsystems, each is a single order and it is required to design decentralized controllers u_1 and u_2 such that the overall system is stable.

The interconnection matrix H is given by

$$H = \begin{bmatrix} 0 & -3 \\ -4 & 0 \end{bmatrix}.$$

Consider the decoupled subsystems

$$\begin{aligned} \dot{x}_1 &= 5x_1 + u_1 \\ \dot{x}_2 &= 6x_2 + u_2 \end{aligned} \tag{22}$$

$$\begin{aligned} u_1 &= -K_1 x_1 \\ u_2 &= -K_2 x_2 \end{aligned} \tag{23}$$

u_1 and u_2 should stabilize the overall system and minimize the performance index J_1 and J_2, respectively

$$J_1 = \tfrac{1}{2}\int_0^\infty (x_1^2 + u_1^2)\,dt$$

$$J_2 = \tfrac{1}{2}\int_0^\infty (x_2^2 + u_2^2)\,dt$$

K_1 and K_2 are the solution of

$$10K_1 - K_1^2 + 1 + 2\alpha K_1 = 0$$

$$12K_2 - K_2^2 + 1 + 2\alpha K_2 = 0.$$

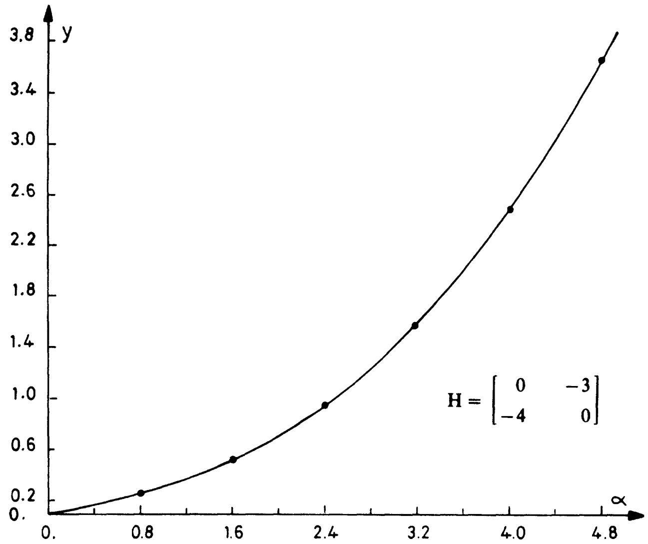

Fig. 1.

Hence the matrix G is given by

$$G = \begin{bmatrix} (4\alpha + 10)K_1 + 2 & +(3K_1 + 4K_2) \\ +(3K_1 + 4K_2) & (4\alpha + 12)K_2 + 2 \end{bmatrix}.$$

Using the theorem described above, to insure the stability of the overall system (21), the matrix G should be positive definite, that is

$$y = \{[(4\alpha + 10)K_1 + 2][(4\alpha + 12)K_2 + 2] - (3K_1 + 4K_2)^2\} > 0.$$

Fig. 1 shows the variation of y with α, from which we see that the overall system can be stabilized through the decentralized controllers u_1 and u_2 for any value of α.

Taking $\alpha = 0$, then

$$\begin{aligned} u_1 &= -10.099x_1 \\ u_2 &= -12.083x_2. \end{aligned} \tag{24}$$

The overall system (21) can be rewritten as

$$\dot{x} = \begin{bmatrix} 5 & -3 \\ -4 & 6 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u \tag{25}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

using the local controllers (24), system (25) will be

$$\dot{x} = \begin{bmatrix} -5.099 & -3 \\ -4 & -6.083 \end{bmatrix} x = Fx.$$

The eigenvalues of F are given by $(-2.1$ and $-9.1)$, that is, the real parts of the eigenvalues are negative, hence the system (25) is stable under the local controllers (24), and this satisfy the proposed technique.

Now suppose that the interconnection term from x_2 to x_1 is opened, that is,

$$H = \begin{bmatrix} 0 & 0 \\ -4 & 0 \end{bmatrix}.$$

Then in this case the matrix F will be

$$F = \begin{bmatrix} -5.099 & 0 \\ -4 & -6.083 \end{bmatrix}.$$

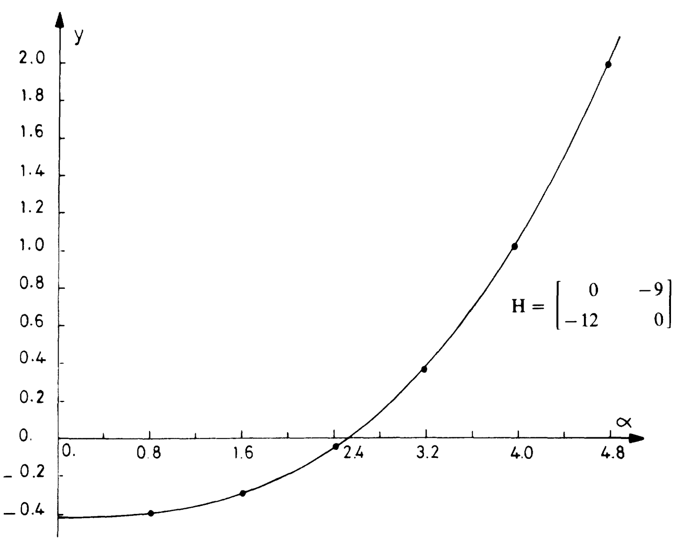

Fig. 2.

The eigenvalues of F are given by $(-5.1$ and $-6.1)$ which indicate that the system is still stable under this perturbation in the interconnection. This very important aspect of the proposed approach guarantees the stability of the system even under structural perturbation in the interconnection. We note that the reliability of the local controllers cannot insure the stability of the system under all perturbation in the interconnection, to show this aspect, suppose that the interconnection matrix is changed to

$$H = \begin{bmatrix} 0 & -9 \\ -12 & 0 \end{bmatrix}.$$

Then to insure the stability of the overall system, it is required that

$$y = \{[(4\alpha + 10)K_1 + 2][(4\alpha + 12)K_2 + 2] - (9K_1 + 12K_2)^2\} > 0.$$

Fig. 2 shows the variations of y with α, from which we see that the overall system cannot be stabilized using (23) except for values of $\alpha > 2.5$. From the above results we see that the local controllers guarantee the stability of the overall system under the perturbations which reduce the size of the interconnection and this can be seen from (20) in which the inequality is always satisfied for smaller values of the norm of the interconnection matrix C. This aspect is very important for practical systems, like power systems for which some interconnection lines between the generators may be opened during the operation of the system. In this case using local controllers based on the proposed technique insure the stability of the overall system [19], [20].

Conclusion

The problem of decentralized stabilization of large-scale dynamical systems is examined in this correspondence. Sufficient conditions for stabilization using decentralized controllers are established in a form of algebraic criteria. The proposed approach can insure in some sense the stability under structural perturbation in the interconnection. It also has the advantage of eliminating the cost of communication links between the subsystems and the coordinator as compared with multilevel methods.

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Measures of Complexity of Fault Diagnosis Tasks

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***Abstract*—The literature of complexity is reviewed and the distinction between perceptual complexity and problem solving complexity is discussed. Within the context of two particular fault diagnosis tasks, four measures of complexity are considered. These measures are evaluated using data from two previously reported experiments which employed eighty-eight subjects. It is shown that two particular measures of complexity, one based on information theory and the other based on the number of relevant relationships within the problem, are reasonably good predictors of human performance in fault diagnosis tasks. The success of these measures is explained by the fact that they incorporate the human's understanding of the problem and specific solution strategy as well as properties of the problem itself.**

Introduction

This correspondence is concerned with how humans cope with complexity. Perhaps complexity confronts us most when something does not work. Airline reservation systems and computerized banking are quite convenient until something goes

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