and another with an old one (e), respectively. In Fig. 2, the results are shown for textures, where the histograms typically show the difficult cases of a broad and flat valley (c) and a unimodal peak (g). In order to appropriately illustrate the case of three-thresholding, the method has also been applied to cell images with successful results, shown in Fig. 3, where $C_0$ stands for the background, $C_1$ for the cytoplasm, and $C_2$ for the nucleus. They are indicated in (b) and (f) by ( ), (=), and (*), respectively.

A number of experimental results so far obtained for various examples indicate that the present method derived theoretically is of satisfactory practical use.

*D. Unimodality of the object function*

The object function $\sigma_B^2(k)$, or equivalently, the criterion measure $\eta(k)$, is always smooth and unimodal, as can be seen in the experimental results in Figs. 1–2. It may attest to an advantage of the suggested criterion and may also imply the stability of the method. The rigorous proof of the unimodality has not yet been obtained. However, it can be dispensed with from our standpoint concerning only the maximum.

## IV. Conclusion

A method to select a threshold automatically from a gray level histogram has been derived from the viewpoint of discriminant analysis. This directly deals with the problem of evaluating the goodness of thresholds. An optimal threshold (or set of thresholds) is selected by the discriminant criterion; namely, by maximizing the discriminant measure $\eta$ (or the measure of separability of the resultant classes in gray levels).

The proposed method is characterized by its nonparametric and unsupervised nature of threshold selection and has the following desirable advantages.

1) The procedure is very simple; only the zeroth and the first order cumulative moments of the gray-level histogram are utilized.

2) A straightforward extension to multithresholding problems is feasible by virtue of the criterion on which the method is based.

3) An optimal threshold (or set of thresholds) is selected automatically and stably, not based on the differentiation (i.e., a local property such as valley), but on the integration (i.e., a global property) of the histogram.

4) Further important aspects can also be analyzed (e.g., estimation of class mean levels, evaluation of class separability, etc.).

5) The method is quite general: it covers a wide scope of unsupervised decision procedure.

The range of its applications is not restricted only to the thresholding of the gray-level picture, such as specifically described in the foregoing, but it may also cover other cases of unsupervised classification in which a histogram of some characteristic (or feature) discriminative for classifying the objects is available.

Taking into account these points, the method suggested in this correspondence may be recommended as the most simple and standard one for automatic threshold selection that can be applied to various practical problems.

## Acknowledgment

The author wishes to thank Dr. H. Nishino, Head of the Information Science Division, for his hospitality and encouragement. Thanks are also due to Dr. S. Mori, Chief of the Picture Processing Section, for the data of characters and textures and valuable discussions, and to Dr. Y. Noguchi for cell data. The author is also very grateful to Professor S. Amari of the University of Tokyo for his cordial and helpful suggestions for revising the presentation of the manuscript.

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# Book Reviews

**Orthogonal Transforms for Digital Signal Processing**—N. Ahmed and K. R. Rao (New York: Springer-Verlag, 1975, 263 pp.). *Reviewed by Lokenath Debnath, Departments of Mathematics and Physics, East Carolina University, Greenville, NC 27834.*

With the advent of high-speed digital computers and the rapid advances in digital technology, orthogonal transforms have received considerable attention in recent years, especially in the area of digital signal processing. This book presents the theory and applications of discrete orthogonal transforms. With some elementary knowledge of Fourier series transforms, differential equations, and matrix algebra as prerequisites, this book is written as a graduate level text for electrical and computer engineering students.

The first two chapters are essentially tutorial and cover signal representation using orthogonal functions, Fourier methods of representating signals, relation between the Fourier series and the Fourier transform, and some aspects of cross correlation, autocorrelation, and convolution. These chapters provide a systematic transition from the Fourier representation of analog signals to that of digital signals.

The third chapter is concerned with the Fourier representation of discrete and digital signals through the discrete Fourier transform (DFT). Some important properties of the DFT including the convolution and correlation theorems are discussed in some detail. The concept of amplitude, power, and phase spectra is introduced. It is shown that the DFT is directly related to the Fourier transform series representation of data sequences $\{X(m)\}$. The two-dimensional DFT and its possible extension to higher dimensions are investigated, and the chapter closes with some discussion on time-varying power and phase spectra.

In order to develop a fast algorithm for efficient computation of the DFT, the fast Fourier transform (FFT) algorithm is developed in Chapter 4. This chapter includes some interesting numerical examples and applications on digital signal processing.

Chapter 5 deals with a class of nonsinusoidal orthogonal functions consisting of the Rademacher, Haar, and Walsh functions. The notion of sequency as a generalized frequency is introduced, and the frequency is used as a parameter to distinguish individual functions that belong to sets of nonsinusoidal functions.

The sixth chapter is devoted to the study of the Walsh-Hadamard transform (WHT) and algorithms to compute it. The concept of the Walsh spectra and their properties are presented with physical significance. Special attention is given to the analogy between the Walsh-Hadamard and the discrete Fourier transforms.

In Chapter 7, a study is made of the generalized Haar, Slant, and discrete cosine transforms. Fast algorithms to compute these transforms are developed.

The last three chapters are primarily concerned with applications of orthogonal transforms to the generalized Wiener filtering, data compression, and feature selection in pattern recognition. It is shown that orthogonal transforms can be used to generalize Wiener filtering to digital signal processing with an emphasis on reduction of computational requirements. Applications of data compression in the areas of image processing and electrocardiographic data processing are discussed. Two pattern recognition experiments are included in the final chapter.

All chapters are provided with problems for solution and a set of references to the original papers and books. The authors have produced an excellent textbook on a subject that is receiving increasing attention in all areas of signal processing and can be strongly recommended to electrical and computer engineering students.

**Walsh Functions and Their Applications**—K. G. Beauchamp (New York: Academic, 1975, 236 pp.). *Reviewed by Lokenath Debnath, Departments of Mathematics and Physics, East Carolina University, Greenville, NC 27834.*

The Walsh functions were invented by the American mathematician J. L. Walsh in 1923. These functions are defined on the interval $0 \leq x \leq 1$ and assume only the values $+1$ and $-1$. They have many properties similar to those of the Haar functions and trigonometric series, and form a complete orthogonal system. These functions are now most widely used in communication engineering and other applied sciences.

With some reader background in the Fast Fourier transforms, this book gives a self-contained systematic treatment of the theory and applications of the Walsh, Haar, and related transforms.

The book begins with a chapter on orthogonal functions and their simple properties. The next three chapters deal with the Walsh function series, Walsh transformation, and the Haar functions. These functions are compared with the sine-cosine functions used in Fourier analysis. These chapters also cover the properties of the discrete transforms and the derivation of past transform algorithms with programs for implementation on the digital computers.

Chapters 5 and 6 are concerned with the details of the Walsh spectral analysis, sequency filtering, correlation, and convolution. With respect to the operation of correlation and convolution, there is a striking difference between the Walsh transform theory and the Fourier analysis. This effects the way in which filtering of discrete sampled data is carried out and discussed with reference to the classical process of filtering originally due to Wiener. In fact, these chapters are devoted to the general principles of sequency analysis and filtering that form the basis of many applications of the Walsh theory of functions.

The last two chapters describe applications of Walsh transforms to various problems in communications, image processing and pattern recognition, electromagnetic radiation, radar systems, speech processing, and medical signal processing.

The first appendix on Fortran 4 subroutines for the fast Walsh, fast Haar, and other transforms and the second on Tables for Modulo $-2$ addition $R \oplus S$ are added at the end of the book. A set of important references to the original papers and books is included at the end of each chapter. However, [1]–[3] escaped the notice of the author.

In summary, this book is well written and successful and certainly represents a welcome addition to the literature. Despite the lack of problems and exercises, it can be used as an excellent text on the subject. However, the book would become more useful for the students if problems and exercises had been included. It should also occupy a place on the shelves of applied mathematicians, computer scientists and engineers as a useful reference book.

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**Digital Filters**—Richard W. Hamming (Englewood Cliffs, NJ: Prentice-Hall, 1977, 226 pp.). *Reviewed by Samuel D. Stearns, Sandia Laboratories, Albuquerque, NM 87115.*

In writing this textbook on digital filters, Richard Hamming has been able to draw on his own rich and varied background in numerical analysis, computing, statistics, and applied mathematics. Probably no living author can claim to so much experience and so many fundamental contributions in the disciplines that are basic to modern digital signal processing. Hamming's ability to draw from this experience and to illustrate clearly the fundamental ideas of digital filtering makes this an unusually good text for one new to the subject. Using this text, the reader can build a solid foundation in the theory as well as an appreciation for some of the applications.

Most texts on digital signal processing are meant especially for electrical engineers, but this one is not. The author has purposely tried to steer clear of engineering jargon and to introduce the necessary engineering fundamentals such as the frequency domain, the transfer function, the Fourier series and integral, etc. One of Hamming's reasons for doing this is to make the subject matter available to those in nonengineering disciplines such as statistics, numerical analysis, physics, economics, etc.

The first five chapters of *Digital Filters* are devoted to introductory material. The frequency approach is introduced, and the sine and cosine functions are presented as the eigenfunctions to be used with linear digital systems. The Fourier series is introduced and discussed in detail, with emphasis on the different forms of the series, convergence, etc. There is also some introductory discussion on convolution, Gibbs phenomenon, and data windows.

Chapters 6 and 7 are on nonrecursive filter design. Simple and practical design methods, including the design of filters with ripple-free gain characteristics, are presented. At the end of Chapter 6, there is some interesting new material on sharpening the response of a filter by making multiple passes through the same filter. Chapter 8 returns to a discussion of the properties of signals and covers mainly the sampling theorem and the Fourier integral, and Chapter 9 returns again to a discussion of windows for nonrecursive filters, with emphasis on the Kaiser window. Chapter 10 is on the discrete Fourier transform, with a section on the FFT. The remaining three chapters are on recursive digital filter design (Butterworth, Chebyshev, and Elliptic) plus some practical details, including finite word-length effects.

The book is not primarily for advanced readers; it sticks mainly to fundamentals and omits some of the material on optimum filter designs, spectral analysis methods, etc., found in other modern texts. There is a relatively small number of practical exercises. But the beginning reader and the student in digital signal processing will appreciate Hamming's ability to select and order the fundamentals and his ability to present the basic ideas of digital filtering in a style that is easy and pleasant to follow.