

utility case and have been applied to a general optimal diagnostic problem.

With regard to the diagnostic example, it is apparent that, in general, an extremely rich set of diagnostic questions and tests must be available in order to insure the existence of an admissible myopic strategy. (Conditions for the existence of admissible myopic strategies for a special case of the optimal diagnostic problem are presented in [7].) Diagnostic strategies that achieve the supremum in $\sup_a F_i(x, a)$, however, often represent good suboptimal strategies, the computational impact of which are typically quite significant. We note this significance by remarking that determining the best myopic strategy (be it optimal or suboptimal) requires, in a decision tree description, only a single decision node. After an action has been selected and the diagnostic test outcome received, then the *a posteriori* over the state space is calculated and entered as the *a priori* into the same single decision node tree to determine the diagnostic decision for the next stage. Such a process can proceed for as many stages as is desired. The standard tree-folding-back procedure, together with its concomitant combinatorial problems (the "curse of dimensionality"), is thus avoided.

The above-mentioned computational implications make the use of myopic strategies as suboptimal strategies particularly attractive. Tight bounds on the difference between optimal expected utility within the class of all admissible strategies and optimal expected utility within the class of all admissible myopic strategies are not yet available, however, and are a topic of future research.

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Noise Cleaning by Iterated Local Averaging

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Abstract—A class of iterative enhancement techniques based on the neighbors of a point whose gray levels are closest to that of the point is introduced. Several such techniques are compared as to their effects on image point classification, and results of applying them to a real image are also presented.

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I. ITERATIVE TECHNIQUES FOR IMAGE ENHANCEMENT

This paper introduces a class of iterative image enhancement techniques based on the neighbors of a point whose gray levels are closest to that of the point. We will assume that images are composed of compact regions of essentially constant gray level on a similarly constituted background, i.e., that underlying the observed image is an ideal image consisting of regions of constant gray level (objects on a background), but that the observed image has been degraded by the addition of white noise.

Section II introduces a class of image enhancement procedures based on the k neighbors of a point whose gray levels are closest to that of the point. Section III gives a more adaptive technique based on the distribution of gray levels and gradients in the neighborhood of a point. Section IV discusses the effects of iterating these enhancement procedures (providing, in essence, a set of relaxation networks for image enhancement), and Section V contains a typical example of the application of the techniques to real-world images. Section VI presents our conclusions and contains suggestions for further research.

It would have been desirable to compare the results obtained in this paper with results obtained using other image smoothing methods [2]–[5]. However, this was impractical for two reasons: limitations on the size of the paper and the effort required to implement many of the other methods. In spite of the absence of comparative data, we feel that the present results are worth reporting because they have very low computational cost and yet yield substantial smoothing in a variety of cases.

II. THE k -NEAREST NEIGHBOR METHOD

The classical technique for enhancing images is to smooth the image on some fixed size neighborhood [1]. If the gray levels of the image are uncorrelated, then the smoothing is unweighted; if they are correlated and the correlations are known (or can be estimated), then an optimal set of weights can be computed to, for example, minimize the mean-square error between the smooth image and the ideal unobserved image underlying the original noisy image [2]. In the discussion that follows, we will assume that the gray levels are uncorrelated.

If our goal is to retrieve the ideal image by thresholding the observed noisy image, then we can point out that while unweighted smoothing will increase the accuracy of the thresholding procedure at interior points of the image, it will lead to incorrect results at edge points and line points (see [3] for a more exact probabilistic model of the smoothing process). Intuitively, if the objects in the image are small relative to the smoothing neighborhood or if the objects are not very compact, smoothing may actually lower the overall classification accuracy (into object and background) of the thresholding process. We will limit our consideration to smoothing neighborhoods sufficiently small so that the first problem can be ignored and concentrate our attention on dealing with the second problem.

Ideally, we would like to perform unweighted smoothing at all interior points for which the entire smoothing neighborhood contains points all drawn from the same region as the center point. At edges and lines, we would prefer to smooth only over points on, e.g., the same side of the edge as the center point or along the linear feature passing through the center point. Techniques have been proposed [4] that, on the basis of the responses of a family of edge or line detection operations, attempt to determine a reasonable set of weights for smoothing a point. These techniques require a considerable expenditure of computational effort so it is of interest to investigate simpler but similarly motivated techniques in order to gain an understanding of the trade-off in computation

and enhancement power. Consider then the following very simple enhancement strategy.

E^k : Replace the gray level at a point P by the average gray level of the k neighbors of P whose gray levels are closest to that of P (in an $n \times n$ neighborhood of P). (We will refer to these neighbors from now on as the k -nearest neighbors of P .)

For all points P such that exactly k of their n^2 neighbors belong to the same population as P , strategy E^k will be optimal. Averaging with fewer neighbors will lead to an averaged population with a larger variance (hence with higher errors upon thresholding), and averaging with more neighbors will cause the mean of the averaged population to shift towards the other population (we assume throughout that thresholding will be done at the minimum error threshold for the unenhanced noisy image).

Suppose we consider 3×3 neighborhoods; then we can (incompletely) characterize interior points as points all eight of whose neighbors belong to the same population as the center point, edge points as points where between three and seven neighbors belong to the same population as the center point, and line points as points where only one or two neighbors belong to the same population as the center point. (The characterization is incomplete because it does not take into account the spatial distribution of the gray levels in the neighborhood of a point.) If one wanted to insure that lines were preserved, one would have to use E^k with $k \leq 2$. However, this turns out to have very little noise cleaning power [4]. In later sections, we will describe experiments with E^k for larger values of k . An error analysis of E^k is presented in [5, Sec. 1A].

III. A GRADIENT-BASED METHOD

The choice of any particular E^k entails trade-offs in sensitivity to the various local pictorial features. As mentioned earlier, an ideal operator would first make its best estimate of the nature of the neighborhood at each point and then compute some best set of smoothing coefficients for that neighborhood (as was done in [4]). A simpler but still powerful adaptive enhancement procedure follows.

1) At each point, decide if that point is an interior point or not (i.e., assign the point x some fuzzy membership $m_I(x)$ in the set I of interior points and then compare these membership values to some threshold).

2) If it is an interior point, simply replace it by the average of its neighborhood.

3) If it is not an interior point, replace it by the weighted average of its neighbors that have higher membership in the set of interior points than it does.

It is important to use a weighted average because large neighborhoods will often contain interior points (i.e., low gradient points) from more than one region. The weights represent the confidence that the pair of points come from the same region. It is therefore reasonable to make the weights proportional to the absolute differences in gray level between a point and its neighbors. Note that we would not expect to obtain very different enhancements from weighted versus unweighted gradient filtering over a 3×3 neighborhood. However, once the neighborhood gets larger, we would expect different results.

To understand this method, consider the neighborhood of x

$$\begin{array}{ccc} a & b & c \\ d & x & e \\ f & g & h \end{array}$$

Suppose that a vertical edge passes through x so that a , d , and f are in one region, and b , x , g , c , e , h are in a second region. If we compute membership in I based on the "Laplacian" $|x - (b + d + e + g)/4|$, then if the regions are of constant gray

level, we have $m_I(a) = m_I(d) = m_I(f) = m_I(b) = m_I(g) = m_I(x)$, but $m_I(x) < m_I(c) = m_I(e) = m_I(h)$ so that our replacement strategy will replace x by $(c + e + h)/3$. We can make the following observations about this enhancement strategy, which we will call *Laplacian smoothing* since edge points are replaced by the average of their neighbors with lower Laplacian values (i.e., higher m_I values).

1) Isolated noise points will be smoothed over, because all of the neighbors of a noise point (which has low m_I) will have higher m_I values (lower Laplacians) than the noise point.

2) Linear features will be smoothed over because points adjacent to the linear feature points will have higher membership in I than the linear feature points.

3) Ideal (vertical or horizontal) step edges will be unchanged as in the example in the previous paragraph.

Analogous remarks can be made (the details will be omitted here) if we compute m_I based on the value of a suitably chosen gradient operator rather than the Laplacian; we call this method *gradient smoothing*.

IV. ITERATED ENHANCEMENT

A. Advantages of Iteration

As discussed in [4], better enhancements could be achieved by applying the enhancement operator to a large neighborhood of a point than would be achieved on the basis of a smaller neighborhood. However, the amount of computation required can be prohibitively greater for larger neighborhoods. As an alternative, we can consider iterating the enhancement procedure over a small neighborhood in an effort to bring more pictorial context to bear on the enhancement of individual points. The computational savings achieved by iteration can be significant. For example, suppose that we wanted to perform k -nearest neighbor enhancement over an $n \times n$ neighborhood. An efficient strategy for accomplishing this would be to store the gray levels in an $n \times n$ neighborhood of a point in a binary tree such that visiting the nodes of the tree in preorder would result in listing the gray levels of the neighborhood in ascending order. Given the binary tree, the k -nearest neighbors of a gray level x can be computed in $O(n^2)$ time—we simply start one process at the k th element in the preorder listing (b_k) and one at the first element (b_1) and find the first i such that $\max(|x - b_i|, |x - b_{i+k-1}|) < \min(|x - b_{i-1}|, |x - b_{i+k}|)$ where $b_0 = -\infty$, $b_{n+1} = +\infty$.

Given the binary tree for the $n \times n$ neighborhood of a point (i , j), the binary tree for its horizontal neighbor can be efficiently computed by deleting the n elements in column $A_{i-n/2}$ and adding the n elements in column $A_{i+n/2}$. This would require altogether $O(n \log n)$ operations because the depth of a balanced binary tree of n^2 nodes is $2 \log n$, and we are making a total of $2n$ additions and deletions. Therefore, the complexity of the k -nearest neighbor rule on an $n \times n$ neighborhood is $O(n^2)$.

Now, in order for an iterated 3×3 enhancement procedure to have an effective neighborhood size of $n \times n$, the operation must be iterated $(n-1)/2$ times. Each iteration of the nearest neighbor enhancement procedure on a 3×3 neighborhood requires approximately 36 algorithm steps to find the k -nearest neighbors (9 log 9 to sort the nine elements and up to nine sets of comparisons to find the k -nearest neighbors). If we want to account for an $n \times n$ neighborhood by iterating this basic operation, then about $36 \cdot (n-3)/2$ operations are required. Clearly as n becomes large, the iterative techniques would be computationally preferable. We should point out, however, that not all local $n \times n$ operations can be effected by iterating 3×3 operations. Furthermore, it is not always obvious what $k \times k$ operation corresponds to the iteration of a 3×3 operation (consider iterating E^3).

Knowing that it is computationally more effective to iteratively apply a local enhancement operator than to simply apply a single more global operator does not shed any light on the important questions of how many iterations of the enhancement should be computed. It is very difficult to develop models for the effects of iteratively applying, for example, a k -nearest neighbor enhancement procedure because of the following.

1) The iteration procedure introduces spatial correlations between image points that are very difficult to model.

2) Obtaining closed forms for the point distribution of the k -nearest neighbors of a point from a sample of size n is also very difficult [6].

Instead, we will describe a simple classification experiment that will, hopefully, provide some intuition about the effects of iterative enhancement. The experiment consists of iteratively applying our enhancement procedures to synthetic images such as those described in Section IVB. After each iteration, the enhanced image is thresholded at the minimum error threshold computed on the basis of the gray level distributions used to create the original synthetic image. Finally, tables of error versus iteration are compiled for several of our enhancement techniques.

B. Empirical Evaluation

The experiments described here used an image consisting of a 40×40 square embedded in a 128×128 background. The gray levels inside the square were independently normally distributed with mean $\mu = 40$ and standard deviation $\sigma = 10$. The background gray levels were independently normally distributed with $\mu = 20$ and $\sigma = 10$. Eight iterations of a) E^4 , b) E^6 , c) E^8 , and d) gradient smoothing GS (where only the points in the upper 10 percent of the gradient histogram are gradient smoothed at each iteration) were applied to this image. (The images, their thresholded versions, and the distributions of errors can all be found in [5, Sec. 3.2].) Table I lists number of errors as a function of iteration for each of the four techniques. We can make the following observations on the basis of this table.

1) All the techniques degrade after a few iterations, but E^4 and E^6 seem to degrade more slowly. This indicates that in the absence of any model for predicting how many times the enhancement operator should be iterated a k -nearest neighbor enhancement policy with $k < 8$ (but large enough to provide a noticeable enhancement) should be adopted.

2) E^6 , E^8 , and GS all eliminate interior point misclassifications; however, E^8 introduces more edge point misclassifications than either E^6 or GS and eventually propagates these errors into the object and background interiors (by creating a large ramp edge between object and background).

In order to add confirming evidence to these conclusions, a second example was run using a 40×40 square ($\mu = 40$, $\sigma = 10$) in a 128×128 image (background $\mu = 30$, $\sigma = 10$). Table II lists errors as a function of iteration. The results are very similar to those obtained in the first example. The $k = 4$ procedure does not do nearly as well in reducing the overall classification error as any of the other techniques. This is not at all surprising since in the underlying noise-free images there are only four points (the corners of the square) having four neighborhoods split between the object and background. We note that *all* of the enhancement techniques yield considerable increases in overall classification accuracy when compared to thresholding the original image at the minimum error threshold and that there are apparent advantages to iterating the enhancement procedures (e.g., E^6 , E^8 , and GS all halve their error rates between one and two iterations).

It is worth pointing out that E^8 , E^7 , and E^6 , if iterated sufficiently often, will produce a constant image since only con-

TABLE I
ERRORS AS A FUNCTION OF NUMBER OF ITERATIONS
IN THE FIRST EXPERIMENT

Iteration	Method	E^4	E^6	E^8	GS
1		723	242	141	121
2		280	87	87	81
3		166	80	105	92
4		130	84	122	114
5		134	90	148	120
6		132	94	175	146
7		134	94	213	171
8		142	100	244	190

TABLE II
ERRORS AS A FUNCTION OF NUMBER OF ITERATIONS
IN THE SECOND EXPERIMENT

Iteration	Method	E^4	E^6	E^8	GS
1		3206	1764	616	708
2		2123	721	237	309
3		1546	514	251	320
4		1177	505	292	360
5		959	551	365	422
6		841	600	423	491
7		768	622	496	565
8		746	751	589	659

stant images are invariant under these operations (see the Appendix for a proof of this). However, this does not rule out using, for example, E^6 for a limited number of iterations to enhance an image.

V. EXAMPLES

In this section, we present the results of applying the enhancement techniques described in Sections II and III to a portion of a LANDSAT image. (Several other examples are presented in [5, Sec. 4].) Fig. 1 shows eight iterations of E^2 , E^4 , E^6 , E^8 and gradient smoothing (using thresholds on the gradient corresponding to the lower 20, 40, 60, and 80 percent of the gradient values) as well as median filtering for this image. We will make the following (subjective) evaluations of the different enhancement techniques.

1) E^2 : Using only the two nearest neighbors, while preserving the integrity of fine image detail, does so, of course, at the expense of having very little noise cleaning power. It seems to be an inadequate procedure.

2) E^4 : Four nearest neighbor enhancement tends to form clumps from the noise. It does preserve edges and other detail relatively well.

3) E^6 : Six nearest neighbor enhancement produces substantially less mottle than E^4 but does not preserve fine detail through as many iterations as E^4 . However, as we have seen in the previous section, there is probably little reason to iterate any of these procedures more than a small number of times so that that overall E^6 is preferable to E^4 .

4) E^8 : Iterated 3×3 smoothing destroys most of the visible

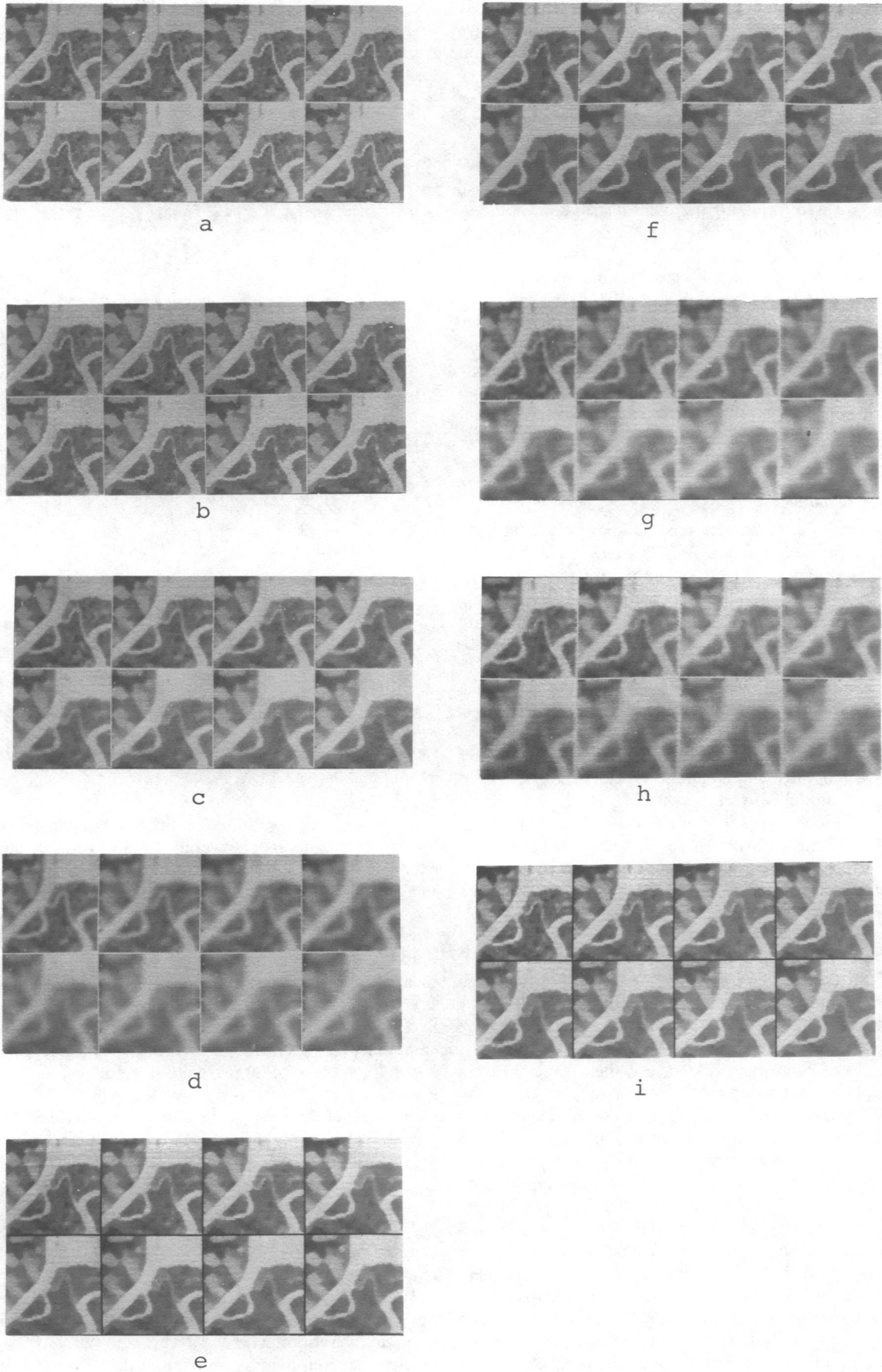


Fig. 1. Smoothings of LANDSAT image. (a) E^2 . (b) E^4 . (c) E^6 . (d) E^8 . (e) GS (20 percent). (f) GS (40 percent). (g) GS (60 percent). (h) GS (80 percent). (i) Median filtering.

structure in the image; therefore it is not a desirable enhancement technique.

5) Gradient smoothing: This approach (using any of the p -tiles tested) does not preserve the linear features and edges as well as E^6 but introduces less mottle.

6) Median filtering: This method produces more mottle than E^6 on noisy images while smoothing over linear features and many edges. It is, therefore, less desirable than either E^6 or gradient smoothing.

VI. CONCLUSION

E^6 and gradient smoothing provide the best subjective enhancements over a variety of images. In images containing large regions, gradient smoothing is more desirable because it will introduce less mottle into the regions. In images with smaller regions and linear features, E^6 will generally provide better enhancements.

It would be desirable to extend these enhancement techniques to deal with multispectral data. They could then be applied to LANDSAT images as a preprocessing step to improve classification results. This could be done either on the multispectral data or on the multi-image of likelihoods of spectral class membership at each point.

APPENDIX
PATTERNS STABLE UNDER E^k

For $k \leq 5$, there are many different patterns that are stable under E^k ; e.g., any pattern consisting of constant gray level vertical (or horizontal) strips of width ≥ 2 , such as

aabbcc
aabbcc.
aabbcc
⋮
⋮

On the other hand for $k \geq 6$, we can prove that the only stable patterns must have constant gray level.

To see this, let z be the highest gray level in the picture. If the picture is stable under E^k , then at least k of the z neighbors must be z since z must be the average of its k nearest gray level neighbors and none of these neighbors can be $> z$. Thus for $k = 8$, we see that every neighbor of a point having gray level z must also have gray level z , which readily implies that the picture has constant gray level z .

More generally, let w be a neighbor of some z such that w has lowest possible gray level. If $w = z$, then every neighbor of every z must also be a z , and the picture is again constant. Suppose that $w < z$; thus the neighborhood looks like

$$\begin{matrix} \dots \\ .zw \\ \dots \end{matrix} \tag{1}$$

or

$$\begin{matrix} \dots \\ .zw \\ .z. \\ \dots \end{matrix} \tag{2}$$

(or a rotation of one of these). If $k = 7$, all seven of the dots must be z since z is the average of these seven neighbors. Moreover, the z that are adjacent to the w must each have seven z 's as neighbors so that the neighborhood looks like

$$\begin{matrix} zzz \\ zzzzz \\ zzwzz \\ zzzzz \\ zzz \end{matrix} \tag{1}$$

or

$$\begin{matrix} zzz \\ zzzzz \\ zzwzz. \\ zzzzz \\ zzz \end{matrix} \tag{2}$$

But w must be the average of its seven nearest gray level neighbors, and this is impossible since its neighbors are all $z > w$. Thus for $k = 7$, we cannot have $w < z$, and so the picture is constant.

Finally if $k = 6$, then six of the seven dots must be z , while the seventh dot must be $\geq w$ by definition of w . Now w must have at least one neighbor with gray level $v < w$ since it must be the average of its six nearest gray level neighbors, some of which are $z (> w)$. This v cannot be a neighbor of any z by definition of w . Hence in case (1), the dots above and below the w cannot both be z , since any neighbor of w would then be a neighbor of some z . Thus the neighborhood in case (1) must look like, e.g.,

$$\begin{matrix} zzz \\ zzw \\ zzxv \end{matrix}$$

where $z > x \geq w > v$. But the z to the left of x must have at least six z 's as neighbors. So that in particular, the point below x must be z , and this is adjacent to $v < w$, contradiction.

In case (2), the neighborhood could look like

$$\begin{matrix} .zw^v \\ .zz \\ \dots \end{matrix} \tag{2a}$$

or

$$\begin{matrix} zzw^v \\ zzx \\ zzz \end{matrix} \tag{2b}$$

or

$$\begin{matrix} zzwv \\ zzx \\ zzz \end{matrix} \tag{2c}$$

where $z > x \geq w > v$. In cases (2b, c), the z to the left of the w has two neighbors $< z$ so that all its other neighbors must be z , and in particular, the point above the w must be z . This is adjacent to $v < w$, contradiction. In case (2a), none of the points adjacent to the v can be z so that the neighborhood looks like

$$\begin{matrix} rv \\ .zws \\ .zz \\ \dots \end{matrix}$$

where $z > r, s \geq w > v$. Thus the z 's adjacent to the w each have two non- z 's as neighbors implying that their other neighbors are all z , i.e.,

$$\begin{matrix} zzrv \\ zzws \\ zzzz. \\ .zzz \end{matrix}$$

(1) Similarly, the z 's adjacent to the r and s have two non- z 's as neighbors so that their other neighbors are all z ; and this implies that the points above the r and to the right of the s —which are adjacent to v —are z 's, contradiction.

In the above discussion, we have ignored the possibility that the points in question might be adjacent to the picture border, in which case some of the neighbors used in the proofs might not exist. Note, however, that if any z is adjacent to the border, then (for $k \geq 5$) all its neighbors must be z 's so that the cases involving a neighbor $w < z$ must indeed be interior to the picture. Of course, if we take border effects into account, the striped pattern shown at the beginning of the Appendix is no longer stable since, e.g., the right "a" on the top row now has fewer than five a's as neighbors. Note also that in the Appendix we are regarding the gray levels as real-valued; if they are converted into integers after averaging, there are other possibilities for stable patterns. (The authors are indebted to one of the referees for these remarks.)

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A Sliding Scale for Hospital Reimbursement

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Abstract—Reimbursement by governmental agencies and by third party payors of general and voluntary hospitals currently does not take cognizance of the special financial burdens of inner city teaching hospitals. It is proposed that a formulation, previously described, be used to provide a sliding scale for reimbursement purposes which would ameliorate the situation in an auditable, justifiable, and fair manner.

In a recent paper in this TRANSACTIONS, we derived a function $\beta_{P=0}$ which described how to set the ratio of charges to cost such that profit would be zero. This function was then expressed totally in terms of such dimensionless values (ranging from 0 to 1), as the ratio of inpatients to outpatients, fraction of patients (both in and out) paid on a cost basis versus a charge basis, fraction of bad debts in each category, and fractional loss due to reimbursement for Medicaid patients where only a portion of charges are paid. It was further proven that the function $\beta_{P=0}$ is independent of either total charges or costs and was therefore applicable to any hospital, independent of size, as long as the accounting and statistical parameters were known.

We now wish to propose that the function $\beta_{P=0}$ has a much wider applicability than first envisioned and could serve as an index to be used by government (federal, state, and municipal) and by noncommercial third party payors (Blue Cross, etc.) to

assess, on a sliding scale, the true needs of various hospitals with respect to reimbursement formulas.

At present, each payor (other than the individual payor or commercial insurers) determines how it will evaluate a given hospital's efficiency and then reimburses the hospital for that portion of its patients that represents the payor's obligation. Affluent or reasonably stable community hospitals which serve a paying or totally insured clientele do fairly well under this form of reimbursement. Teaching and/or inner city hospitals which serve a poorer patient population, many of whom are on welfare or who are above welfare standards but still cannot pay their hospital bills, find themselves being increasingly squeezed by the reimbursement formulas into a deficit situation. Government and other noncommercial third party payors refuse to obligate themselves for patients' payments if those patients are not their subscribers. Although it can be argued that charity is not an obligation of noncommercial third party payors, the role of government is more obscure in this regard.

One of the main reasons that government (the federal government at least) finds it difficult to support the hospital needs of persons who are not covered directly by one of its agencies or by state or city programs or by commercial or noncommercial insurance is that it would be extremely difficult to draft legislation which would reimburse some hospitals at a higher or more favorable rate than other hospitals, and this is exactly what needs to be done. The bureaucratic attitude with respect to reimbursement formulas (and who can say they are wrong) is to treat each and every hospital by the same set of standards. Unfortunately, this Procrustean bed standard gives to some affluent hospitals funds they do not need, does not affect the vast majority of community and general hospitals adversely since it results in reasonable payments for reasonable charges, but penalizes the teaching and inner city hospitals for every "charity" patient whose care they assume.

If we now look at the function $\beta_{P=0}$, we note that although it was derived for use in pricing the laboratory, all the variables which go into the formula, although derived from laboratory statistics and accounting data, are applicable to gauging the financial needs of the hospital independent of the hospital's size, scope of operation, or efficiency. Thus the smaller $\beta_{P=0}$ is (with a lower limiting value of unity) the less that hospital needs federal intervention. Conversely, the higher $\beta_{P=0}$ is (with no upper limit as would be the case if there were zero reimbursement) the more need there is for governmental intervention. This intervention need not be in the form of a direct cash subsidy which would be difficult to calculate but could take the form of a percentage increase in the reimbursement formula which would be in proportion to $\beta_{P=0}$ for that hospital. Note that such a method of calculation is independent of the size and cost of the hospital, since $\beta_{P=0}$ is itself independent of size and cost and achieves the desideration of reimbursing for nonpaying patients without, at the same time, compensating for poor fiscal management or inefficiency. The possible objection that inefficiency in hospital operation would result in an increased value of $\beta_{P=0}$, thus resulting in undeserved reimbursement, can be countered by stating that the numbers which go into the formulation of $\beta_{P=0}$ are themselves clearly verifiable and auditable. Government and third-party auditors should not, and presumably would not, sanction payment for obvious mismanagement or lack of proper billing procedures, but can allow an increased level of payments according to the formulation where applicable.

We therefore propose that the $\beta_{P=0}$ function, which we have previously described in this TRANSACTIONS, might serve as a

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