3) For the vehicle path reticle,

$$C = \begin{bmatrix} 0 & 1 & (1 - C_1 Y_v V) & 1/D_1 & -C_1 & -C_1 Y_\delta \\ 1 & V/D_1 & V/D_1 & 0 & 0 & 0 \\ 0 & 1 & (1 - C_2 Y_v V) & 1/D_2 & -C_2 & -C_2 Y_\delta \\ 1 & V/D_2 & V/D_2 & 0 & 0 & 0 \end{bmatrix} \tag{A.19}$$

where C_1 and C_2 are defined by

$$C_1 = \left(1 - \frac{D_C}{D_1}\right)\frac{D_A}{V^2}, \qquad C_2 = \left(1 - \frac{D_C}{D_2}\right)\frac{D_A}{V^2}. \tag{A.20}$$

For single- and two-distance VFI, P is given by

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$P = \begin{bmatrix} (1+c)/2 & 0 & (1-c)/2 & 0 \\ 0 & 0 & 0 & 0 \\ (1-c)/2 & 0 & (1+c)/2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{A.21}$$

respectively.

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An Approach to Spatial Pattern Recognition of Solid Objects

M. BRIOT, M. RENAUD, AND Z. STOJILJKOVIĆ

***Abstract*—The tactile recognition of solid objects with the aid of gripping devices of the polyarticulated gripper type is discussed. The objects, which are assumed to have simple geometric shapes and numerous axes of summetry, are grasped by a gripper made of four symmetrical fingers and each equipped with several potentiometers to measure the angles between them. An object chosen at random and with equal probability from one of three different classes was assigned to the class to which it has the highest probability density of belonging. This density is unknown, *a priori*, and is estimated by the kernel method from the learning observations (vectors) coming from measurements made by the gripping device. In case of wrong classification an adaptive procedure (supervised learning) allows a modification of the initial distribution of the learning observations in the course of the recognition phase. Two types of kernel were used in the two following cases to estimate the probability density of belonging to a class, the unit kernel and the Cauchy kernel: 1) use of the Euclidean distance, and 2) use of a modified Euclidean distance in order to take into account the dispersion on each component of the observation vector. The corresponding strategy to each case is given and the results obtained commented upon.**

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I. Introduction

Working within the framework of research carried out in the robotics field at the L.A.A.S. of Toulouse, France, and the Mihailo Pupin Institute of Belgrade, Yugoslavia, we were particularly interested in the problem of tactile recognition of objects and their manipulation. This has both industrial and medical applications.

A certain number of transducers such as TV cameras, [1] range finders [2], etc., have already been developed to recognize objects. Our approach was different as we used tactile information. Two types of tactile information were used: 1) pressure information coming from a transducer called "artificial skin" [3], and 2) angular information.

In this correspondence we consider objects which are recognized solely from angular information given by the grasping system. To this end we developed a polyarticulated mechanical

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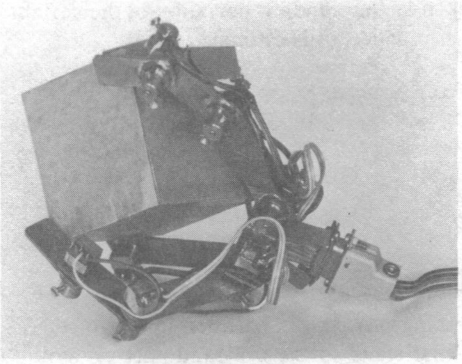

Fig. 1. Gripper grasping cube.

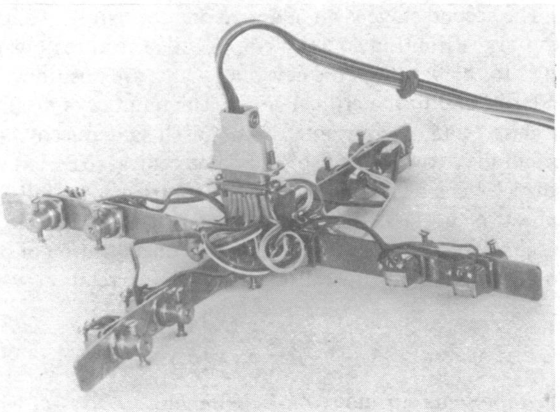

Fig. 2. Gripper with its joints at full stretch.

system giving 12 independent angular measurements. (Incidentally, other grippers have been realized in other parts of the world, e.g., the Belgrade "hand" [4], [5], and a tridigital gripper developed in Japan [6].)

In the first part of this correspondence we describe the grasping system used. The second part is devoted to 1) a description of the classifier based on local estimates of probability density (by the kernel method), and 2) the learning method. Finally, in the third part, we describe and comment on the experimental results obtained.

II. The Grasping System

The handling system used was composed of a four-finger gripping device, each finger having three joints; each joint was equipped with a transducer for measuring the angles between them. This gripper is shown grasping a cube in Fig. 1. The transducers on each joint are potentiometers giving information in proportion to the angles of rotation. An angle has zero value when the two corresponding phalanges are in a straight line. Fig. 2 shows the gripper with its joints at full stretch, i.e., there are no angles. Furthermore, all the angles must be positive since no joint can be bent backwards.

The information provided by the grasping system is represented by a point in a 12-dimensional space: $X = (X_1, X_2, \cdots, X_{12})^T$. The components X_1, X_2, X_3 are the angles of the joints of the first finger, being the joint nearest to the central support and the furthest away. The components X_4, X_5, X_6; X_7, X_8, X_9; X_{10}, X_{11}, X_{12} correspond to the second, third, and fourth fingers, respectively, moving in a clockwise direction.

As seen in the above paragraph, information given by the transducers are analog voltages. These voltages are read by an analog-to-digital converter which serves as an interface between the "Mitra 15" minicomputer and the grasping system. Each measurement is made to 12 bits which gives a sufficient degree of accuracy for our purposes. The data can be either recorded on magnetic tape when off-line processing is desired, or used directly to classify an object unknown to the real-time classification program.

The set of objects which we considered consisted of a sphere (6-cm diameter), a cube (7-cm edges), and a regular tetrahedron (10-cm edges). These objects were chosen at random, the only constraint being that they should not be so big as not to be totally grasped by the device. In addition, each object had to be grasped in such a way that it was in contact with each of the joints. In practice, however, this was not always possible since the joints cannot be bent backwards.

III. Classification and Learning Procedure

A. Decision Rules

We used three sets of observations (which we call learning observations): $\{X_i^1\}_{i=1}^{30}$, $\{X_i^2\}_{i=1}^{30}$, and $\{X_i^3\}_{i=1}^{30}$ from the measurements made by the gripping device grasping three objects θ_1, θ_2, and θ_3 representing the sphere, the cube, and the tetrahedron, respectively.

We made the following hypotheses.

1) The series above are series of real random variables identically distributed following the laws of probability P_{θ_1}, P_{θ_2}, and P_{θ_3}, respectively. These laws have densities $p(\cdot\,|\,\theta_1)$, $p(\cdot\,|\,\theta_2)$, and $p(\cdot\,|\,\theta_3)$.

2) The observations are considered as m dimension vectors which are written: $X_i^l = (X_{i1}^l, X_{i2}^l, \cdots, X_{im}^l)^T$.

3) The three solid objects are represented with equal probability in the gripper device.

The problem of identification of the solid object lies in finding a decision rule, or strategy, defined on the space of observations R^m and taking its values in a set of classes of solid objects δ_1, δ_2, and δ_3.

To put it another way, and based on an observation coming from θ_1, θ_2, or θ_3, one should be able to recognize if the observed solid object belongs to class δ_1, δ_2, or δ_3.

In the case where $p(\cdot\,|\,\theta_1)$, $p(\cdot\,|\,\theta_2)$, and $p(\cdot\,|\,\theta_3)$ are known, we know that the optimal decision rule (in the sense of minimizing the error probability of misrecognition) is Bayes' law.

The strategy at point $x \in R^m$ values δ_q if

$$p(x\,|\,\theta_q) = \max_{l=1,2,3} p(x\,|\,\theta_l).$$

In the application under consideration these densities are unknown. Thus a new strategy was constructed thanks to an estimate of the densities from the series given above: $\{X_i^l\}_{i=1}^n$, $l = 1, 2, 3$.

In order to choose the estimate procedure best adapted to our problem, we did a statistical analysis of the data given by the gripping device, helped by a classification algorithm of self-teaching Gaussian data [7]. This analysis showed that this is a problem concerning multimodal distributions.

Nonparametric estimate methods are all indicated in this case since they have the advantage of not requiring an hypothesis about the density form. Among the nonparametric methods, the Rosenblatt–Parzen kernel method [8], [9] is well suited to the decision problem because of the ease with which it can be put into action. We chose to use this method, taking a rectangular function (unit kernel) as the window, and a Cauchy-type continuous function.

We studied the behavior of these two kernels after previously treating the learning observation series $\{X_i^l\}$. This treatment consisted of selecting the observations so that each mode was represented in a balanced way.

Let y belong to the space R^m. We take the modified Euclidean norm

$$\|y\|_{h^l} = \left[\sum_{j=1}^{m} (y_j/h_j^l)^2\right]^{1/2}, \qquad l = 1, 2, 3$$

as the norm for y, where the components of the vector h^l are strictly positive. It was noted by v_{h^l} that the quantity (in proportion to the volume of the ellipsoid of the equation $\|y\|_{h^l} = 1$) was

$$v_{h^l} = \prod_{j=1}^{m} h_j^l, \qquad l = 1, 2, 3.$$

In the kernel method, the density estimator is written

$$p_n(x \mid \theta_l) = \frac{1}{nv_h^l} \sum_{i=1}^{n} K_{h^l}(x - X_i^l), \qquad l = 1, 2, 3;\ n \in \mathbb{N}$$

where

$$K_{h^l}(y) = K\left(\frac{y_1}{h_1^l}, \frac{y_2}{h_2^l}, \cdots, \frac{y_m}{h_m^l}\right), \qquad y \in R^m;\ l = 1, 2, 3,$$

and $K_{h^l}(\cdot)$ is a bounded measurable function of R^m called "kernel." In addition it is assumed that

$$\int_{R^m} K_{h^l}(y)\, dy = 1, \qquad l = 1, 2, 3.$$

The corresponding strategy values δ_q if

$$\frac{1}{v_h^q} \sum_{i=1}^{n} K_{h^q}(x - X_i^q) > \frac{1}{v_{h^l}} \sum_{i=1}^{n} K_{h^l}(x - X_i^l),\ l = 1, 2, 3;\ l \neq q.$$

If not, δ_q is chosen in an equiprobable way.

In R^m, Cauchy's kernel is expressed as [9]:

$$K_{h^l}(y) = \frac{\lambda_m}{1 + \|y\|_{h^l}^{m+1}}, \qquad l = 1, 2, 3$$

with

$$\lambda_m = \frac{(m+1)\Gamma(m/2) \sin [\pi/(m+1)]}{2\pi^{(m+2)/2}}.$$

The previous strategy values δ_q if

$$\frac{1}{v_{h^q}} \sum_{i=1}^{n} \frac{1}{1 + \|x - X_i^q\|_{h^q}^{m+1}} > \frac{1}{v_{h^l}} \sum_{i=1}^{n} \frac{1}{1 + \|x - X_i^l\|_{h^l}^{m+1}}, \qquad l = 1, 2, 3;\ l \neq q.$$

If not, δ_q is chosen in an equiprobable way.

To assure the convergence of the above strategies when $n \to \infty$, h^l must be a particular function of n. However, in our case, n having been given in advance, h^l must be considered as a constant.

As far as vector h^l is concerned we envisage two cases:

$$h^l = C(1, 1, \cdots, 1), \qquad l = 1, 2, 3. \quad (1)$$

$$h^l = \frac{C}{\left(\prod_{j=1}^{n} \prod_{l=1}^{3} \sigma_j^l\right)^{1/3m}} (\sigma_1^l, \sigma_2^l, \cdots, \sigma_m^l), \qquad l = 1, 2, 3. \quad (2)$$

σ_j^l is taken as the standard deviation of the variables X_{ij}^l, $i = 1, 2, \cdots, n$. In practice, σ_j^l is estimated by

$$\sigma_j^l = \left(\frac{1}{n} \sum_{i=1}^{n} (X_{ij}^l - \bar{X}_j^l)^2\right)^{1/2}$$

with

$$\bar{X}_j^l = \frac{1}{n} \sum_{i=1}^{n} X_{ij}^l.$$

B. Learning Procedure [11]

The learning procedure is divided into two stages.

1) The first stage consists of randomly obtaining the initial n samples for each class. In this case the classification gives accurate results if X falls within the central region of a class, but much less so if X falls near the boundaries between classes, especially if these are close or overlap.

2) The second stage is an adaptive one consisting of destroying the initial distribution so as to empty the central regions of their samples to the benefit of the new ones which are positioned on the boundaries and in the critical zones. The number of n samples in each class remains constant. From this it is deduced that if an observation X to be classified "falls" in a central region, it will still be correctly classified but that, on the contrary, if it "falls" on the periphery of a class, the precision of the classification will be better than before because of the greater concentration of samples.

To accomplish this operation the following relation was used:

$$E_{iq}(X) = \frac{p_n(X \mid \delta_i)}{p_n(X \mid \delta_q)}$$

which represents an index of X belonging to class δ_i when the decision gives class δ_q.

All the $E_{iq}(X)$ are calculated, and we look for the maximum value for $i \neq q$:

$$\max_{i \neq q} E_{iq}(X) = E_{pq}(X).$$

Class δ_p is the one to which X is nearest. A test is then carried out to determine whether $E_{pq}(X)$ is above a certain threshold E_0 beyond which X can be taken to modify the initial learning. To see which sample has to be replaced, all the observations of δ_q are examined to find the one which is least like those of class δ_p, i.e.,

$$E_{pq}(X_j^q) = \min_i E_{pq}(X_i^q), \qquad i = 1, 2, \cdots, n.$$

The values of indices of vectors X and X_j^q belonging to class δ_p are compared with each other. If $E_{pq}(X_j^q) < E_{pq}(X)$, then X_j^q is replaced by X in the initial observations. If $E_{pq}(X_j^q) \geq E_{pq}(X)$, then X_j^q is retained in the initial observations.

This procedure has the advantage of improving the classification without increasing the amount of information to be stored in the computer memory. The flowchart in Fig. 3 shows how the algorithm makes it possible to pass readily from classification to adaptive supervised learning.

IV. Experimental Results

The results shown in Figs. 4 and 5 are given in the form of curves representing variations of the probability of error of misclassification as a function of the C parameter which characterizes the window width. This probability of error is estimated by counting wrongly classed samples coming from test observations as distinct from observations which serve to calculate the strategies (learning observations). For this operation, 30 test observations per class were used. We studied the influence of the

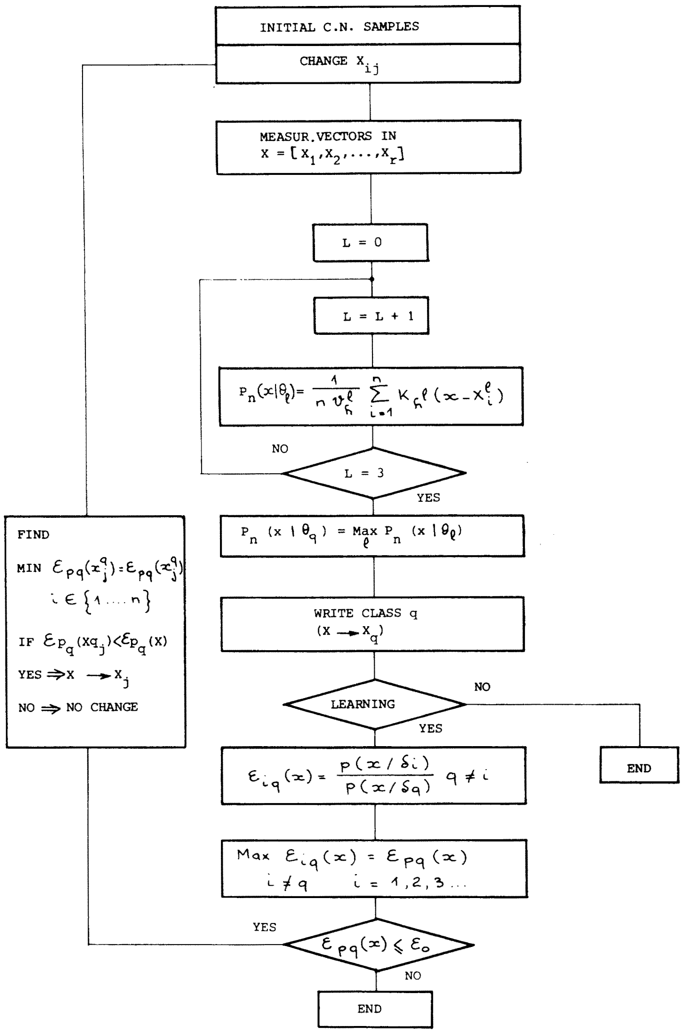

Fig. 3.

C parameter on the calculation of the probability density in the case where, on the one hand, the Euclidean distance is used, and on the other, the modified Euclidean distance.

In addition we studied the behavior of these strategies after treating a series of learning observations. We compared the following cases: a) where no treatment was carried out, and b) where observations were selected, making sure of having the same number of observations for each mode (subclass). We sum up in Table I the minimum value of the probability of wrongly classifying an object for the various cases considered.

V. Remarks

The observation given in the previous paragraph calls for a certain number of remarks.

1) The selection of learning observations used for the calculation of strategies brings about a sharp decrease in the probability of wrongly classed objects. This selection, which takes into account the position of the object in the gripper, allows a taking into account in its turn of the parameters linked to the object's structure. In this case the parameters are masked by the probabilistic classification method used.

2) The use of the modified Euclidean distance, which allows the dispersion of each component of the vector observation to be considered, i.e., it makes the distributions of each class more homogeneous, also serves to decrease the probability of wrongly classed objects.

TABLE I

Distance	unselected	selected
Euclidean	14 %	5 %
modified Euclidean	3 %	2 %

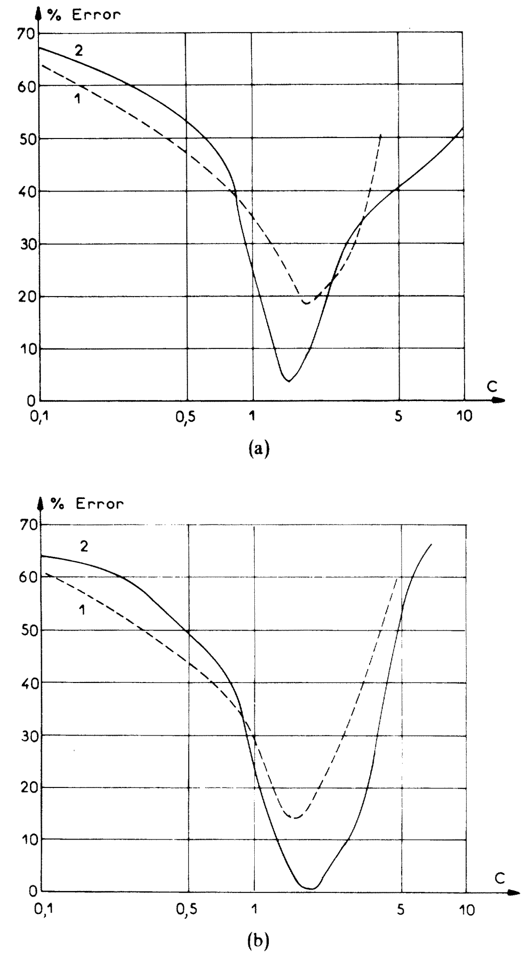

Fig. 4. Error probability of unit kernel method. (a) Unselected (1) Euclidean distance. (2) modified Euclidean distance. (b) Selected (1) Euclidean distance, (2) modified Euclidean distance.

3) It is interesting to note the insensitivity of the C parameter to low values of the strategy using the Cauchy kernel. This gives it a definite advantage over the method using the unit kernel, for which the choice of C is very critical. This result can be explained in the following way. When the density functions are cut into a finite number of points, e.g., Cauchy's kernel, one can always decide whether or not an observation belongs to a given class, which is not the case when a function of the unit kernel type (cf. Fig. 6) is used. In fact, it is very interesting to use a Cauchy kernel (or any function of the same type) in the case of a small number of learning observations.

VI. Conclusion

The approach described in this correspondence shows that angle measurements can be usefully employed for the problem of the tactile recognition of simple objects. It is likely that the recognition performance could be improved by a combination of pressure information (using an "artificial skin") and angle measurements. The two types of transducers were compared in

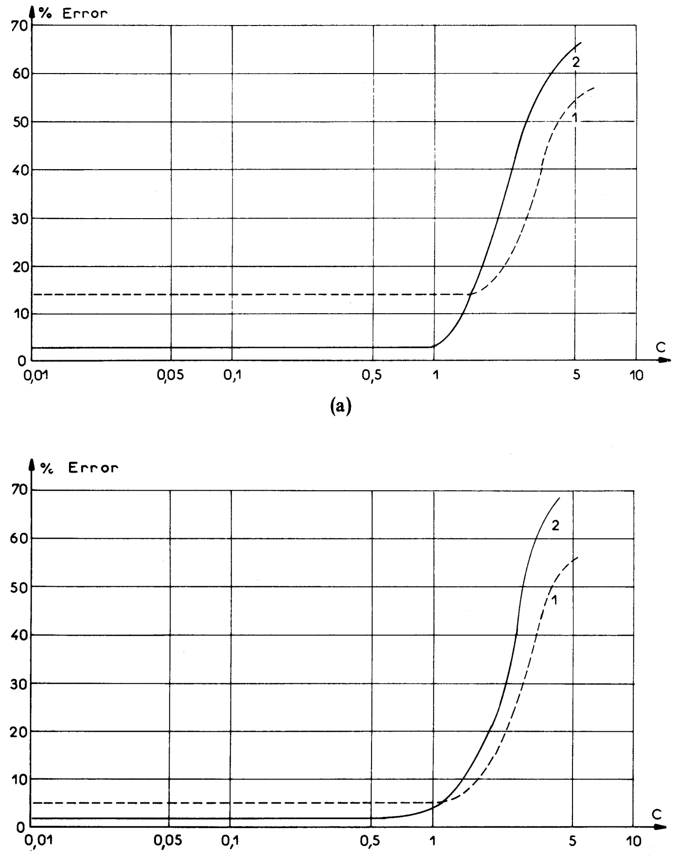

Fig. 5. Error probability of Cauchy's kernel method. (a) Unselected (1) Euclidean distance, (2) modified Euclidean distance. (b) Selected (1) Euclidean distance, (2) modified Euclidean distance.

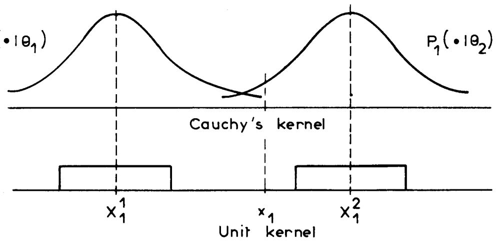

Fig. 6.

[12]. A detailed study concerning the recognition of solid objects with the aid of the grasping devices such as artificial hands, polyarticulated grippers, and grippers with parallel surfaces was also developed in [13]. Another way of improving recognition is to extend the research on learning and adaptive learning. In particular, one could change either the information vector by increasing the number of fingers, joints, or both, or the number of memorized elements that determine the classes. It may also be useful to carry out a statistical multivariate analysis of the working space. Finally, the use of an Euclidean distance in the probability density estimator gives a simple solution to the recognition algorithm.

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Random Mosaic Models for Textures

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Abstract—**Several models for generating isotropic, "cellular" textures are discussed. These models tessellate a region into cells, and assign gray-level probability densities to the cells. The models can, in principle, be used to predict statistical texture properties such as those commonly used for texture classification.**

I. Introduction

Understanding texture is an important aspect of image analysis. It plays a prominent role in applications as diverse as cytology, radiology, and remote sensing (see [1] for a review of texture analysis studies through the end of the 1960's). Most current texture models are descriptive models based on second-order statistics of the gray levels in a texture. The most widely known models of this type are based on matrices of co-occurring gray levels (Haralick *et al.* [2]) and histograms of gray-level differences

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