error is the arctangent of $\pm 1/2$ pixel/25 pixels, or ± 1.15 degrees. Rotational accuracy of ± 1.15 degrees exceeded the required production accuracy of ± 3.5 degrees.

B. Reliability

Three sample sets of Darlington assemblies were provided to test the reliability of the pilot system.

The first set consisted of 270 nondefective units, all of which were properly recognized and manipulated into a reference position within the time and accuracy requirements.

The second set consisted of 33 defective assemblies in which the Darlington chip was either off the edge of the heat sink, too close to the weld cup, or rotated more than ± 30 degrees. The system was able to reject all of these defective units.

The third set consisted of 39 defective assemblies in which the Darlington chip was either missing, broken, cracked, or partial. Such units are automatically rejected by the electrical test. An effort was made, however, to visually reject these units by detecting nonuniform contrast at the sides and corners of the chip. The system was able to reject all (16) of these units for which the defect was visible to the human eye in the digitized image.

The costs required to achieve the above performance were maintained within the limits predicted by Horn [17] for such a system.

VI. State of the Art

At the time of writing (late 1975) Hitachi Central Research Laboratories in Japan had just announced [1], [9], [15], [20], [21] the development of a production (partially hardwired) machine which performs the function of automatically aligning transistor circuit chips for wire bonding. This machine is the first true production "robot" extensively employing visual image processing functions. This system appears to be very similar in approach and objectives to the system described in this correspondence; however, it appears that their application was amenable to "binary" image processing, thus simplifying the vision task considerably. In addition, the Hitachi system evidently does not perform inspection tasks.

At the time of revision of this correspondence (mid 1977) commercially available "chip alignment" systems had become available. Based on image dissector technology, however, their cost effectiveness and durability have yet to be established. IC chip manufacture indeed appears to be a natural first application for computer vision technology, since it is a highly automated, computer oriented, aggressive industry.

Other recent articles and research papers [10], [11], [13], [14], [17], [18], and [22] have also focused on electronic component manufacture as a fertile domain for computer vision technology.

Acknowledgment

The author is indebted to L. Rossol for many useful discussions during the course of this work, for suggesting the method for determining chip orientation, and for directing this project. The interest and encouragement of G. G. Dodd is deeply appreciated. The programming assistance of M. R. Ward, A. L. Martin, and W. A. Perkins is also appreciated. E. M. Kavetsky and R. Dewar of General Motors Manufacturing Development Staff produced the production version of SIGHT-I from the laboratory pilot system. G. Voorhis and others at Delco Electronics Division of General Motors were especially helpful in bringing this project to its successful completion.

References

[1] "Images processed to control transistor assembly machine," *Automation*, p. 8, July 1975.
[2] M. L. Baird, "A paradigm for semantic picture recognition," Ph.D. dissertation, Georgia Inst. of Technology, Atlanta, June 1973 (available from University Microfilms, Ann Arbor, MI, 48106).
[3] M. L. Baird and M. D. Kelly, "A paradigm for semantic picture recognition," *Pattern Recognition*, vol. 6, pp. 61–74, 1974.
[4] ——, "Recognizing objects by rules of inference on sequentially thresholded gray-level pictures," *Comput. Graphics Image Processing*, vol. 3, pp. 1–22, Mar. 1974.
[5] M. L. Baird, "Relational models for object location," *ACM SIGART*, vol. 55, Dec. 1975.
[6] M. L. Baird, J. T. Olsztyn, W. A. Perkins, and L. Rossol, "The GM research laboratories' machine perception project," *ACM SIGART*, vol. 55, Dec. 1975.
[7] D. I. Barnea and H. F. Silverman, "A class of algorithms for fast digital image registration," *IEEE Trans. Comput.*, vol. C-21, pp. 179–186, Feb. 1972.
[8] Y. P. Chien and K. S. Fu, "Recognition of x-ray picture patterns," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-4, pp. 145–156, Mar. 1974.
[9] "Pattern recognition capability lets robot operate unattended," *Comput. Design*, p. 54, Nov. 1975.
[10] M. G. Dreyfus, "Visual robots," *Industrial Robot*, pp. 260–264, Dec. 1974.
[11] M. Ejiri, T. Uno, M. Mese, and S. Ikeda, "A process for detecting defects in complicated patterns," *Comput. Graphics Image Processing*, vol. 2, pp. 326–339, 1973.
[12] M. A. Fischler and R. A. Elschlager, "The representation and matching of pictorial structures," *IEEE Trans. Comput.*, vol. C-22, pp. 67–92, Jan. 1973.
[13] J. A. G. Hale and P. Saraga, "Control of a PCB drilling machine by visual feedback," in *Proc. Fourth Int. Joint Conf. on Artificial Intelligence*, pp. 775–781, 1975.
[14] C. A. Harlow, S. E. Henderson, D. A. Rayfield, R. J. Johnston, and S. J. Dwyer, III, "Automatic inspection of electronic assemblies," *IEEE Trans. Comput.*, vol. C-8, pp. 36–45, Apr. 1975.
[15] "Fully automated transistor assembly system," *CRL NEWS*, Central Research Labs., Hitachi, Ltd., Japan, July 25, 1975.
[16] S. W. Holland, "A programmable computer vision system based on spatial relationships," Digital Systems Laboratory, Stanford Univ., Stanford, CA, Tech. Rep. 104, Dec. 4, 1975. (Also GM Research Publication 2078 available from the author.)
[17] B. K. P. Horn, "Orienting silicon integrated circuit chips for lead bonding," *Comput. Graphics Image Processing*, vol. 4, pp. 294–303, Sept. 1975.
[18] J. F. Jarvis, "Automatic visual inspection of Western Electric series 700 connectors," in *Proc. IEEE Comput. Soc. Conf. on Pattern Recognition and Image Processing*, pp. 153–159, June 1977.
[19] R. K. Jurgen, "Ignition systems go solid state," *IEEE Spectrum*, pp. 49–51, Sept. 1975.
[20] S. Kashioka, M. Ejiri, and Y. Sakamoto, "A transistor wire-bonding system utilizing multiple local pattern matching techniques," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-6, pp. 562–570, Aug. 1976.
[21] S. Kashioka, M. Ejiri, and Y. Sakamoto, "A transistor assembly system utilizing time shared visual image processing techniques," *Elec. Eng. in Japan*, vol. 96, pp. 118–124, 1976.
[22] E. S. McVey, G. L. Jarvis, II, and E. A. Parrish, "An example of advanced automation: printed circuit board drilling," *IEEE Trans. Ind. Electron. Contr. Instrum.*, vol. IECI-22, pp. 10–15.
[23] J. T. Olsztyn, L. Rossol, R. Dewar, and N. R. Lewis, "An application of computer vision to a simulated assembly task," in *Proc. First Int. Joint Conf. on Pattern Recognition*, pp. 505–513, 1973.
[24] "Research in computer vision systems," *SEARCH*, vol. 8, Nov.-Dec. 1973. (GM Research Publication available from the author).
[25] H. Yoda, J. Motoike, and M. Ejiri, "Direction coding method and its application to scene analysis," in *Proc. Fourth Int. Joint Conf. on Artificial Intelligence*, pp. 620–627, 1975.

On the Sequential Approach to Medial Line Transformation

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Abstract—**An iterative procedure to obtain the medial line of a binary digital figure is presented. At every iteration step, local sequential operations are employed to delete the contour elements which are neither end points nor are necessary to preserve the order of**

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connectivity both of the figure and of the background. The adopted deletability criteria, based either on the notion of crossing number or on the existence of certain neighborhood conditions, are described and the sequence according to which such criteria are applied is discussed. Some examples showing the performance of the proposed algorithm are included.

I. Introduction

Operations which transform a digital figure into a smaller one by deleting a suitable subset of it, have been widely considered in the past [1]. For sake of simplicity, in the sequel we will be concerned with binary digital pictures and will mean by figures any connected set of digital elements having state 1; such elements will also be mentioned as points. Depending on the nature of the problem at hand, the deleting operations are required to lead, in correspondence of every figure, either to a single point, e.g., [2], or to a subset H of S which is still meaningful for description and classification purposes. The aim of this correspondence is to discuss this second type of transformation, with a special emphasis on the sequential approach. Without losing in generality, we will assume that only one figure is contained in the digital picture and will indicate with $\bar{S}$ the background, i.e., the set of elements having state 0. The wanted H should result as a connected set made by the union of simple curves and arcs [3], generated in correspondence with the elongated regions of S and lying along their medial lines. H should be obtained by the repeated application of operations which delete points of S provided that the order of connectivity is preserved for both S and $\bar{S}$ and that the extremes of the found arcs, i.e., the so-called end points, are never erased. Unfortunately, end points may be produced, during the deletion process, which are not in correspondence with the meaningful prominences of the figure. As a consequence, it may happen that, depending on the employed algorithms, a number of arcs different from the expected one may be found, so originating rather unlike H's. This kind of ambiguity in the figure's description can be avoided if one considers all the convex regions of S as significant prominences. The resulting H is now in a 1-to-1 correspondence with S and the initial figure can be recovered whenever an inverse transformation is applied. In this framework, several algorithms have been proposed, e.g., [4]–[6], all of which inspired by a model of the visual system perception [7] employing the medial axes as the most significant shape descriptors. However, due to the difficulty of operating on the discrete plane rather than on the Euclidean plane, H is not always connected so that, if classification is the purpose of the task at hand, its structural description is harder to be achieved. Furthermore, also the computational effort necessary to obtain H is sometimes heavy.

As a matter of fact, up to now the transformation $S \rightarrow H$ has mostly been considered for figure generalization rather than discrimination. In many application fields, for instance OCR, the interest is one of regarding discriminable figures belonging to the same class as being equivalent, so that a unique prototype can be associated with them all. In this case the transformation is requested to be independent on the small inflexions of the contour and to provide an H whose simple arcs are generated only in correspondence with the more relevant prominences of S. Before illustrating the procedure we propose to reach this goal, it is interesting to outline the features both of the parallel and of the sequential approach. First of all, let us recall that to work correctly on digital pictures, two different metrics must be chosen for S and $\bar{S}$ [8]. Since one of the aims of the transformation is to associate to S a subset H made by a minimum number of points, it is natural to choose 8-connectedness for S and 4-connectedness for $\bar{S}$. As a consequence, to preserve the order of connectivity of S, it will be possible to erase only those points of S which are 4-adjacent to $\bar{S}$.

Parallel Approach: Let $P = S \cup \bar{S}$ be an $n \times m$ binary picture and let p_{ij} indicate an element of P. Any local operation applied in parallel to P generates a transformed picture $P^* = S^* \cup \bar{S}^*$, every element p^*_{ij} of which has a state that depends on the state of p_{ij} and of those elements of P lying in a given neighborhood of p_{ij}. Therefore, the state of p^*_{ij} is determined independently from the state of any other element p^*_{hk} of P^* and, moreover, whenever a parallel processor is employed, the transformation occurs simultaneously for all the elements of P. Both to simplify the transformation function and to minimize the hardware complexity of the parallel machinery, the size of the neighborhood, the state of p_{ij} depends on, must be as small as possible, and it is usually chosen to be a 3×3 window centered in p_{ij}. Unfortunately, this size is not sufficient [8] to allow a symmetric erosion of S, while this would be mostly desirable. In turn, this can be achieved by considering a sequence of four asymmetric parallel operations, each one compressing S from top, right, down, left directions, respectively. Rather simple neighborhood conditions [9] must be satisfied by the deletable points, but the complexity of such conditions increases as soon as faster processing is needed. For this reason, the use of parallel algorithms seems to be profitable only if parallel processors are available.

Sequential Approach: When local operations are applied sequentially to P, the picture elements are examined and transformed according to some *a priori* established sequence. In this way the state of each element p^*_{ij} of the transformed picture P^* will depend, besides on the state of p_{ij}, on the state of the neighboring elements, some of which have already been transformed. Since only one element at a time is processed, there arise no problems with regards the connectedness preservation both of S and of $\bar{S}$ and any sequential operation defined on a 3×3 window is sufficient, if suitably chosen, to obtain a connected thin H. On the other hand, such an H is seldom meaningful if a row-by-row (or a column-by-column) scanning sequence is considered. In fact, in this case the resulting H is mainly biased toward one peripheral part of S and, moreover, it does not exhibit the majority of those topological and geometrical features one should expect in order to obtain a satisfactory description of S. To achieve better results, only the contour elements of S should be sequentially examined and a suitable procedure could be to adopt a contour following algorithm which, while turning round the contour, assigns to $\bar{S}$ all the encountered deletable elements of S. In this way, a fairly good H is obtained by means of an algorithm which is not too complicated if S has no holes. Vice versa, if S has order of multiplicity $\nu > 1$, the application of the algorithm is not straightforward. In fact, the contour of S is now expressed by the union of ν curves, each of which should be separately followed to cause a symmetric erosion of S. Then the overall process could be schematized as an application, repeated as far as no deletable elements exist in S, of a border following algorithm on ν contour curves. As an example, in [10] a method inspired by such an approach can be found.

Since sequential digital computers are commonly employed in picture processing and their use is more advantageous [4] when sequential rather than parallel algorithms are considered, this second approach to the transformation we are interested in should be preferable. Nevertheless, to avoid the complexity of a fully sequential algorithm, an iterative procedure has been tried in which only the contour elements are taken into account at every step; but such elements are not removed, if deletable, until the whole current picture has been examined. In this way, some sort of parallelism is introduced which biases H toward the medial line

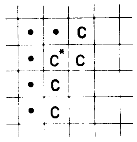

Fig. 1. Dots and c's represent inner elements and contour elements of S^k, respectively. The deletion of c^* is allowed only after that at least one of the 4-adjacent c's has been removed.

of S and valuable results can be obtained when specific tasks are concerned, e.g., chromosome analysis [11].

Nevertheless, the sequential processing employed within every step may locally alter the contour configurations, so that the correspondence between the detected end points and the significant prominences of S is not always verified. Therefore, such regions should be detected independently from the sequential processing and every step should be featured as made up by two distinct parts: the first tailored to detect all the nonspurious end points, the second devoted to the deletion of those elements of S which do not change the order of connectivity both of S and of $\bar{S}$. An algorithm following this scheme is described in the next section.

II. A Hybrid Procedure

Let us indicate with S^0 the input digital figure and with S^k, $k \geq 1$, the set of 1-elements obtained after k applications of a sequence of local operations which erase from S^{k-1} all the deletable elements. The process terminates at step h, if $S^h \equiv S^{h-1}$. Within every kth step, the process can be envisaged as consisting of several stages, every one of which is devoted to the identification of meaningful substructures in S^{k-1} and is accomplished by means of the application of a specific local operation. The local operations are applied only to the 1-elements and the new state of every element is defined in terms of the states of all the elements lying in the 3×3 window including it. As a consequence, in order that the local operations we consider are wherever defined, we will assume that no elements of S^0 lie on the first and on the last row and column of the binary picture. Let us consider now the processing to be performed on the set S^k. First of all, the contour C^k is to be detected. Since 8-connectedness holds in S^k, C^k will be constituted by those elements of S^k which have at least one element of $\bar{S}^k$ in their 8-neighborhood, i.e., $C^k = \{p_{ij}; p_{ij} \in S^k, d_8(p_{ij}, \bar{S}^k) = 1\}$. For clarity's sake, let us recall that the 4-distance between two elements p_{ij} and p_{hk} is given by $d_4(p_{ij}, p_{hk}) = |i-h| + |j-k|$, while their 8-distance is given by $d_8(p_{ij}, p_{hk}) = \max(|i-h|, |j-k|)$. Although all contour elements are deletable, provided that neither the order of connectivity both of S^k and of $\bar{S}^k$ is changed nor the end points erased, it is convenient to assign different labels to the elements with $d_4 = 1$ from $\bar{S}^k$ and to the ones with $d_4 = 2$ from $\bar{S}^k$. In fact, to avoid the creation of holes inside S^k, some elements of the second type (see Fig. 1) can be turned to 0 only after that at least one of the neighboring elements of the first type has been deleted. Furthermore, as it will be seen later (refer to Fig. 7), different labeling of the contour elements may help in disregarding some spurious end points originated from nonelongated regions. In the following, we will contradistinguish with 2 and 3, respectively, every element of the former and of the latter type.

Before describing the remaining stages, let us introduce a few notations and make some remarks on the conditions which must hold inside a 3×3 window centered on $p_{ij} \in S^k$ in order that the topology both of S^k and of $\bar{S}^k$ is not changed when p_{ij} is deleted. The topology will be globally preserved if it will not be changed inside every local neighborhood. The symbols of the cardinal points N, NE, E, $\cdots$, NW will be used for the 8-neighbors of p_{ij} and Σ will indicate the sum of the states of all such elements. When $\Sigma = 0$, p_{ij} is an isolated point, while when $\Sigma = 1$, p_{ij} is an end point. The number CN of transitions from an element of $S^k(\bar{S}^k)$ to an element of $\bar{S}^k(S^k)$, occurring when a 4-path [8] around p_{ij} is followed starting from a given neighbor and coming back to it, is the crossing number associated to p_{ij}. Half its value indicates how many 4-connected components of 0-elements (1-elements) lie in the 8-neighborhood of p_{ij}. The notion of crossing number can greatly help in establishing general deletability conditions without examining one by one all the possible 3×3 configurations.

$CN = 0$. The element p_{ij} is either an isolated point or an inner point of S^k; in both cases its deletion is forbidden since the order of connectivity of $\bar{S}^k$ and of S^k would be respectively changed.

$CN = 2$. The deletion of p_{ij} does not locally alter the connectedness of S^k, but if $\Sigma = 7$ and the product $F = \text{N} \cdot \text{E} \cdot \text{S} \cdot \text{W}$ is not zero, one hole is created in S^k. Then, $F = 0$ is a necessary and sufficient condition for the deletability of p_{ij}, provided that this one is not an end point.

$CN = 4$. There are two distinct components of 0-elements which separate two 4-connected components, say D_1 and D_2, of 1-elements. A necessary and sufficient condition to remove p_{ij} is that both $F = 0$, i.e., no holes are created in S^k, and that one of the components of 0-elements is constituted by only one element placed in one of the four positions NW, NE, SW, and SE, i.e., D_1 and D_2 are 8-connected to each other.

$CN = 6$. Let D_1, D_2, D_3 be the three 4-connected components of 1-elements. A necessary and sufficient condition for the deletability of p_{ij} is that both $F = 0$ and $D_1 \cup D_2 \cup D_3$ is an 8-connected set. Since no more than five 1-elements can exist in the neighborhood of p_{ij}, at least one of the components D_1, D_2, D_3 is constituted by only one element. In turn this component, say D_1, must be 4-adjacent to p_{ij}, in order to be not 8-isolated with respect to the remaining components. Now, in order that $F = 0$, neither D_2 nor D_3 can be constituted by three elements, therefore either D_2 and D_3 have two elements each and are both 8-connected to D_1, or one of them, say D_2, has only one element and D_3 is 8-connected either to D_1 or to D_2. Summarizing, p_{ij} is deletable if and only if, among the four positions N, E, S, W, one is occupied by one 0-element, while the remaining three are occupied by 1-elements belonging to the three distinct components D_1, D_2, D_3.

$CN = 8$. The neighborhood of p_{ij} is constituted by four 1-elements, placed either in the position SW, NW, NE, SE or in the positions S, W, N, E. In both cases, p_{ij} is not deletable since either disconnection or presence of one hole is originated in S^k.

It is also possible to give deletability conditions which do not involve the crossing number. For instance, once assumed that a given pattern of 1-elements exists in the neighborhood of p_{ij}, it is possible to find deletability conditions which hold for all the neighborhood configurations containing such a pattern. From this point of view it is advantageous, if an 8-connected H is of interest, to consider the pattern constituted by two 1-elements 4-adjacent to p_{ij} and 8-connected to each other. In this case, the general condition can be expressed as follows. Any element p_{ij}, for which $F = 0$, is deletable whenever one of the relations

$$\alpha)\ \text{W} \cdot \text{N} \cdot (\text{E} + \text{S} - \text{SE} + 1) \geq 1$$

$$\beta)\ \text{N} \cdot \text{E} \cdot (\text{S} + \text{W} - \text{SW} + 1) \geq 1$$

$$\gamma)\ \text{E} \cdot \text{S} \cdot (\text{W} + \text{N} - \text{NW} + 1) \geq 1$$

$$\delta)\ \text{S} \cdot \text{W} \cdot (\text{N} + \text{E} - \text{NE} + 1) \geq 1$$

is verified.

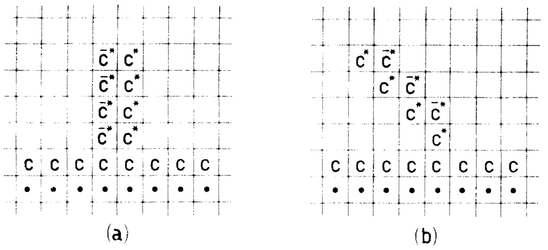

Fig. 2. Elongated regions, the elements of which are starred, vanish if the c's with $F = 0$ and $CN = 2$ are sequentially removed.

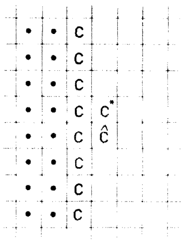

Fig. 5. A spur $\hat{c}$ is generated when the processing ϕ, which causes the deletion of c^*, is first performed.

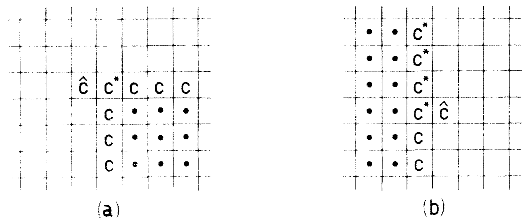

Fig. 3. The elements $\hat{c}$ are detected as end points if the c^*'s are first removed.

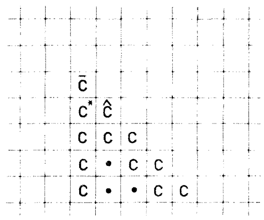

Fig. 6. The $\pi/4$ contour configurations disappear if the sequence of operations ψ, ϕ, ψ, each of which removing the elements $\hat{c}$, c^*, $\hat{c}$, respectively, is considered.

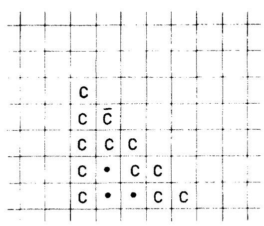

Fig. 4. A contour configuration which identifies an elongated region of S^k.

Without losing any generality, we will restrict ourselves to show only the validity of relation α). Readily, since $F = 0$, the deletion of p_{ij} does not originate any hole in S^k. As for the connectedness of S^k once W and N have both state 1, it is not preserved only when SE has state 1, while both E and S have state 0. Since this case contradicts α), the deletability of p_{ij} is ensured.

To avoid the detection of a wrong number of end points, the order of application of the above criteria cannot be free, but an *a priori* fixed sequence must be taken into account. For instance, most of the regions of S^k, constituted by digital contours arcs which double back on themselves might disappear, instead of originating an end point, when the picture is sequentially examined. In fact, with reference to Fig. 2, the starred elements of S^k which are in turn examined in a forward raster sequence have both $F = 0$ and $CN = 2$, so that they all could be deleted. On the contrary, an end point could be originated by a spur either if the contour elements with $CN > 2$ are examined first (see Fig. 3(a)) or if the end point test and a general deletability test are subsequently performed during the same stage (see Fig. 3(b)). Without quoting any other examples, it is clear that some strategy must be adopted in order to distinguish the really elongated regions. In this correspondence we consider as really elongated all those regions of S^0 which, after k processing steps, originate in S^k either end point or configurations made up by contour arcs which either double back on themselves (Fig. 2) or (if the barycenters of their successive elements 4-adjacent to $\bar{S}^k$ are joined by straight line segments) are represented by $\pi/4$ convex polygonal curves (see Fig. 4). Such configurations can be characterized as containing at least one element bearing an 8-distance greater than 2 from the elements of S^k not in the contour. Since, due to the local nature of the employed processing operations, the elongated regions of the second type can be detected only if an end point can be associated with them, let us illustrate how to reach this goal and, meanwhile, how to disregard the effect of the spurs. For the time being, only contour elements labeled 2, i.e., having $d_4 = 1$ from $\bar{S}^k$, will be candidates to deletion. Readily, let us recall that, to avoid the generation of spurious end points (see Fig. 3(a)), the contour elements with $CN > 2$ cannot be analyzed first. Then, let us be concerned with the configurations of the type of Fig. 2(a) and Fig. 4, where the elements whose deletion should originate an end point are dashed, and with those of the type of Fig. 3, where the contour elements labeled $\hat{c}$ should be erased. The elements belonging to both types of configurations can be distinguished from the remaining contour elements with $CN = 2$ by observing that all their neighbors are contour elements. This can be expressed by introducing the step function $u(t - \theta)$, with $u(t - \theta) = 1$ if $t > \theta$ and $u(t - \theta) = 0$ if $t \leq \theta$, and by computing the crossing number CNN associated to p_{ij} in terms of the values assumed by $u(t - \theta)$ on the 8-neighbors of p_{ij}, with $\theta = 1$. Only the contour elements with $CNN = 2$ belong to the thick contour components we are actually concerned with, while the remaining contour elements are characterized by $CNN > 2$. In turn, the neighborhood's configuration allows to discriminate the elements belonging to the first type from those belonging to the second type. In fact, the former possess at least one pair of 4-adjacent contour elements, i.e., there results

$$P = u(S - \theta) \cdot u(W - \theta) + u(W - \theta) \cdot u(N - \theta) + u(N - \theta) \cdot u(E - \theta) + u(E - \theta) \cdot u(S - \theta) \geq 1;$$

but this is not the case for the latter. For simplicity's sake, let us indicate with ϕ and ψ the processing involving the deletion of the elements with $P > 0$ and with $P = 0$, respectively. Then, ψ should follow ϕ since this one may originate additional spurs (see Fig. 5). The possibility that ψ, besides following, also precedes ϕ is prevented by the fact that the $\pi/4$ convex configurations are destroyed by the sequence of the employed operations (see Fig. 6). Only the configurations of the type of Fig. 7 seem to require that ψ precedes ϕ in order to avoid that spurious end points (the element

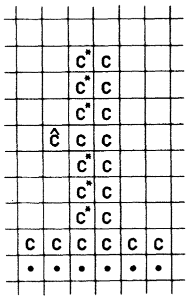

Fig. 7. Since the starred elements are removed by the processing ϕ, $\hat{c}$ is detected as end point if ϕ is not preceded by ψ.

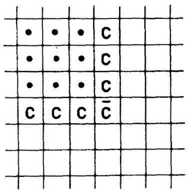

Fig. 8. The corner element $\bar{c}$ is removed by the same processing which deletes the $\bar{c}$'s of Fig. 2(b).

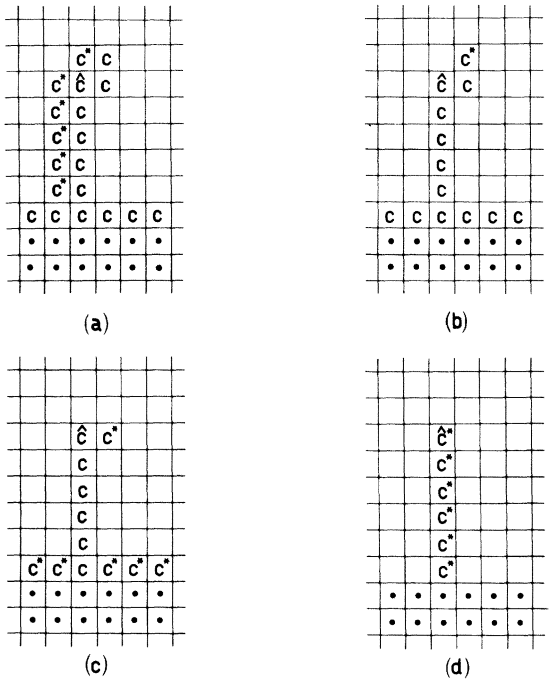

Fig. 9. The starred elements in a), b), c) are, respectively removed at stages 2, 4, and 7. The contour element $\hat{c}$, not 4-adjacent to $\bar{S}^k$, is then removed by the second processing of the contour elements with $CN = 2$, as well as the c^*'s shown in d). This fact is avoided if $\hat{c}$ is marked as end point.

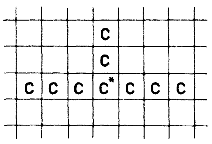

Fig. 10. The starred element, if not removed, may contribute to a better shaped H.

$\hat{c}$, in our case) are generated. On the other hand, since these end points may be featured as elements labeled 2 which are 4-adjacent to an element labeled 3, it is still possible to discriminate them with respect to the true end points. Summarizing, the two processes to be subsequently performed are 1) deletion of the 2's for which $CNN = 2$ and $P > 0$, and 2) deletion of the 2's with $CNN = 2$, $P = 0$ and which are not end points (unless the adjacent element is a 4-adjacent 3). Finally, let us note that the second processing cannot be implemented sequentially without completely erasing the starred elements shown in Fig. 2(b). To avoid such a drawback, a parallel process must be envisaged which considers two complete scanning of the picture: spur labeling and deletion are then performed separately and independently.

To exhaust the examination of the elongated regions, the diagonally oriented contour arcs which double back on themselves are considered at the next stage. Their elements, both with $P > 0$ and with $CN = 4$, that should assume the state 0 in order to obtain the end point configuration, are dashed in Fig. 2(b). Since other deletable elements with $P > 0$, but $CN \neq 4$, exist (for instance, refer to Fig. 8), it is advisable to candidate them all to the deletion, at this same stage. Therefore, whichever the value of CN, all the elements with $P > 0$ are removed, provided that connectedness is preserved, i.e., that at least one of the relations α), β), γ), δ) given before is satisfied. In this way, besides obtaining the wanted end points, an additional smoothing of S^k is achieved by the deletion of the corner elements ($\bar{c}$ in Fig. 8), so preventing the rise of spurs during the sequential processing of the remaining contour elements.

After the detection of the elongated regions of S^k, the second part of the process is started: this one can be described rather briefly. All the remaining contour elements could be sequentially examined within two stages; first, the elements with only one 8-adjacent neighbor are marked as end points, then the remaining ones with $CN = 2$ are erased. During these stages, besides the 2's also the 3's are deleted, provided that, when encountered, they are 4-adjacent to $\bar{S}^k$. Although, due to the sequential nature of the employed process, some 3's are not removed (refer to c^* in Fig. 1), these elements cannot be neglected at step k. In fact, their maintenance affects the isotropy of the transformation by shifting the position of the final subset H away from the medial line of S^0. A further processing of the contour elements with $CN = 2$ is therefore to be performed. Moreover, before it, also the stage concerning the detection of the end points must be repeated. In fact, the removal of some 2's may force a 3 to become an end point (see Fig. 9) so that, if such an element with $CN = 2$ is not marked, the corresponding elongated region completely vanishes. Summarizing, the stages at step k will be ordered and described according to the following.

1) The contour elements are labeled 2 if $F = 0$, 3 otherwise.
2) The 2's with $CNN = 2$ and $P > 0$ are deleted.
3) The 2's with $CNN = 2$ and with $\Sigma \neq 1$ or, alternatively, $\Sigma = 1$, the only 8-neighbor being a 3, are marked.
4) The elements marked at the preceding stage are deleted.
5) The 2's satisfying at least one of the relations α), β), γ), δ) and with $P > 0$ are deleted.
6) The contour elements with $\Sigma = 1$ are marked as end points.
7) The 2's and the 3's with $F = 0$ and $CN = 2$ are deleted.
8) The same as stage 6).
9) The same as stage 7).

III. Concluding Remarks

Let us note that the obtained subset H can be partially modified by allowing the presence of T-junctions (Fig. 10) among the digital arcs constituting it. This fact, which sometimes originates a better shaped H, is achieved by preserving from deletion at stage 5

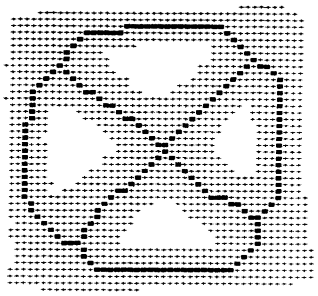

Fig. 11. The shape of the thinned figure H, shown as superimposed on the original figure S^0, does not depend on the local noise present on the contour of S^0.

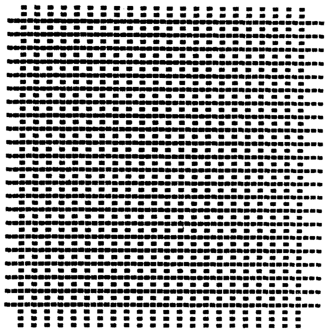

Fig. 12. Test pattern in which no element is deleted and the processing time/iteration is maximum.

those contour elements with $CN = 6$. A further refinement of the algorithm could concern the detection of the end points only in correspondence of the most significant regions of S^0. To obtain this, the end points test (stages 6 and 8) is initially inhibited for a number of steps, the value of which depends on the nature of the figure to be analyzed. Alternatively, other pruning techniques, for instance [12], can be adopted after the transformation has been performed. An analogous reasoning holds for the isolated points. Although such points are not deleted by our algorithm, e.g., one square is thinned down to a single point, it is advisable to consider them as deletable points whenever it is necessary to discriminates, on the same picture, elongated from nonelongated figures. Alternatively, the removal of the isolated points can be performed only initially for a given number of steps. This process, together with the inhibited marking of the end points, discriminates the figures on the basis of their sizes, and it is well-suited for disregarding the noise possibly present on the background. Isolated points deletion is obtained by modifying stages 6 and 8 and by removing those points for which $\Sigma = 0$.

A mention should also be reserved to the metric employed to make $\bar{S}^k$ propagating over S^k. We considered that, at every step, all the contour elements of S^k were candidates to the deletion, so implying that the propagation occurs according to the 8-metric. In turn, a differently shaped H can be obtained, whenever S^0 is not convex, by considering as candidates only those elements of S^k which are 4-adjacent to $\bar{S}^k$. In this case, i.e., when a propagation according to the 4-metric is taken into account, the arcs of H may be characterized by a greater curvature. This fact can be easily verified by comparing the 4-distance transform and the 8-distance transform [4] of S^0. From a computational point of view, more steps are required to remove all the deletable points, but each of them is performed in a shorter time since stages 8 and 9 are omitted.

A computer program implementing the proposed algorithm has been tested on more than one hundred input figures. Also nonstrictly elongated figures (refer to Fig. 11) have been examined in order to evaluate the independence of H on the small contour variations (noise, weak convexities) present in S^0. The image acquisition device described in [13] has been used to provide 50×50 binary digital figures and the processing has been performed on a minicomputer HP 21MX. The time required at every iteration step has been found to depend on the number of the actual contour elements and it ranges from a maximum of about 40 s in the case of Fig. 12, where 1628 contour elements are examined and no one is deleted, to an average of 6 s, for the alphanumerics. Some printouts, regarding this last case, are shown in Fig. 13. For the well shapedness reasons mentioned above, in these examples the central elements of the T-junctions have not been deleted.

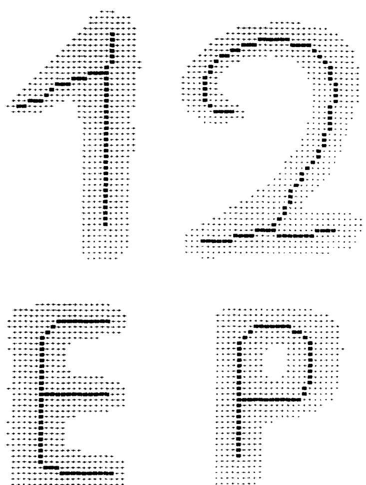

Fig. 13. Some examples showing the performance of the algorithm.

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REFERENCES

[1] A. Rosenfeld and A. C. Kak, *Digital Picture Processing*. New York: Academic, 1976.

[2] S. Levialdi, "On shrinking binary picture patterns," *Comm. ACM*, vol. 15, pp. 7–10, 1972.

[3] A. Rosenfeld, "Arcs and curves in digital pictures," *J. ACM*, vol. 20, pp. 81–87, 1973.

[4] A. Rosenfeld and J. L. Pfaltz, "Sequential operations in digital picture processing," *J. ACM*, vol. 13, pp. 471–494, 1966.

[5] O. Philbrick, "Shape description with the medial axis transformation," in *Pictorial Pattern Recognition*, G. C. Cheng, D. K. Pollock, R. S. Ledley, and A. Rosenfeld, Eds. Washington, D.C.: Thompson, pp. 395–407, 1968.

[6] U. Montanari, "A method for obtaining skeletons using a quasi-euclidean distance," *J. ACM*, vol. 15, pp. 600–624, Oct. 1968.

[7] H. Blum, "A transformation for extracting new descriptors of shape," in *Models for the Perception of Speech and Visual Form*, W. Wathen-Dunn, Ed. Cambridge, MA: MIT Press, 1967, pp. 362–380.

[8] A. Rosenfeld, "Connectivity in digital pictures," *J. ACM*, vol. 17, pp. 146–160, 1970.

[9] ——, "A characterization of parallel thinning algorithms," *Inform. Contr.*, vol. 29, pp. 286–291, Nov. 1975.

[10] J. F. O'Callaghan and J. Loveday, "Quantitative measurements of soil cracking patterns," *Pattern Recognition*, vol. 5, pp. 83–98, 1973.

[11] C. J. Hilditch, "Linear skeletons from square cupboards," in *Machine Intelligence*, IV, B. Meltzer and D. Michie, Eds. Edinburgh: Edinburgh Univ. Press, 1969, pp. 403–420.

[12] A. Rosenfeld and L. S. Davis, "A note on thinning," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-25, pp. 226–228, Mar. 1976.

[13] V. Franchina, S. Levialdi, and A. Pirri, "An image acquisition device for digital processing," in *Proc. Conf. on Computer Assisted Scanning*, Padua, pp. 300–320, Apr. 1976.