of the three was removed from the list. This reduces the volume of data to be stored.

4) The execution time of the various steps of the program was recorded by the UNIX monitor with the following conclusions. The detection of the endpoints (i.e., the operation equivalent to thresholding) took between 60 and 70 percent of the total processing time. The boundary tracing algorithm between 6 and 12 percent of that time. The colinearity test 1–2 percent, input 5–6 percent, output 6–10 percent with the balance devoted to overhead operations.

5) In order to achieve further compaction a polygonal approximation of each closed boundary was found using the split-and-merge algorithm [11]. Instead of such approximations the boundaries could have been mapped in octal strings using the Freeman chain code [3], [13], [14].

The output of the algorithm could be used for a number of goals.

1) Faults in the wiring could be detected either from the chain code or the polygonal approximations. The first approach has been followed by Jarvis who used regular expressions to test for "bumps" [6]. In polygonal approximations faults are either small holes or *two small sides forming a sharp angle.*

2) Verification of the wiring is reduced to the matching of the LAG to a model graph. This is a straightforward operation, because the direction of the scanning is known and, therefore, both graphs are described in similar ways.

3) Description of the boards can be obtained by a syntactical parsing [4], [13]. This would be useful for deriving production models out of boards which have been wired experimentally during the design of a new device.

Although the algorithm of Table I assumes well defined region features, historically, this methodology was used first for gray scale pictures [10]. In such cases its input can be thought of as resulting from the use of a one-dimensional segmentation algorithm followed by optimization of the breakpoint location [10], [11]. Then the "profile" of the brightness function will be encoded as

$$(x_k, A_k), \qquad k = 1, 2, \cdots, n \tag{1}$$

where each A_k is a set of descriptors for the interval (x_{k-1}, x_k). For example, A_k may be an array of coefficients of a polynomial approximation. For simple run length encoding A_k is a constant. If thresholding has been used, then A_k contains only a binary variable a_k denoting above or below threshold (note that in this case we need only know A_1). It is not necessary to specify A_k any more except to assume that one can tell whether two descriptors are alike and in particular that there exists a predicate

$$P(A_k, A_j) = \text{true if } A_k \text{ is like } A_j. \tag{2}$$

P is evaluated over two adjacent nodes of the LAG. In such cases one must trace a boundary not only by examination of the segment overlap criteria but also by evaluation of the predicate $P(A_k, A_j)$. Examples of this approach can be found elsewhere [2], [10], [13]. Our experience has been that the application of the method is more appropriate for high contrast pictures, even if a formalism for the general case is available.

ACKNOWLEDGMENT

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REFERENCES

[1] O. P. Buneman, "A grammar for the topological analysis of plane figures," *Machine Intelligence* 5. New York: American Elsevier, 1970, pp. 383–393.

[2] H. Y. Feng and T. Pavlidis, "The generation of polygonal outlines of objects from gray level pictures," *IEEE Trans. Circuits Syst.*, vol. CAS-22, pp. 427–439, May 1975.

[3] H. Freeman, "Computer processing of line-drawing images," *ACM Comput. Surveys*, vol. 6, pp. 57–97, Mar. 1974.

[4] K. S. Fu, *Syntactic Methods in Pattern Recognition*. New York: Academic, 1974.

[5] F. Harary, *Graph Theory*. Reading, MA: Addison-Wesley, 1969, p. 103.

[6] J. F. Jarvis, "Regular expressions as a feature selection language for pattern recognition," *Proc. Joint Int. Pattern Recognition Conf.*, 1976, pp. 189–192.

[7] D. E. Knuth, *Fundamental Algorithms*, vol. 1. Reading, MA: Addison-Wesley, 1968, pp. 330–331.

[8] B. Moayer and K. S. Fu, "A tree system approach for fingerprint pattern recognition," *IEEE Trans. Comput.*, vol. C-25, pp. 262–274, 1976.

[9] M. S. Murphy and L. M. Silverman, "Image model representation and line-by-line recursive restoration," in *Proc. IEEE Conf. Decision and Control*, 1976, pp. 601–606.

[10] T. Pavlidis, "Segmentation of pictures and maps through functional approximation," *Computer Graphics and Image Processing*, vol. 1, pp. 360–372, 1972.

[11] T. Pavlidis and S. L. Horowitz, "Segmentation of plane curves," *IEEE Trans. Comput.*, vol. C-23, pp. 860–870, Aug. 1974.

[12] T. Pavlidis and K. Steiglitz, "The automatic counting of asbestos fibers in air samples," in *Proc. Third Int. Joint Conf. Pattern Recognition*, 1976, pp. 789–792.

[13] T. Pavlidis, *Structural Pattern Recognition*. Berlin, Heidelberg, New York: Springer-Verlag, 1977.

[14] A. Rosenfeld and A. C. Kak, *Digital Picture Processing*. New York: Academic, 1976.

[15] J. W. Woods, "Markov image modeling," in *Proc. IEEE Conf. Decision and Control*, 1976, pp. 596–600.

Computer Identification of Bullets

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Abstract—**Computer techniques are presented which analyze the surface markings on bullets to identify those fired from the same gun. The backscattered electron signal of a scanning electron microscope is used to form images of small sections of the bullet surface. This information is quantified to give a local verification signature which includes measures of position, amplitude, and width for all significant striations in the section.**

Comparison of two bullets is performed by comparing all corresponding local signatures, land with land, and groove with groove, in sequence. Matches in striation position, width, and amplitude between each corresponding signature pair are represented as probabilities that at least as good a match could occur at random. Independence is assumed between sections, so the total probability of verification error is represented by the product of all local probabilities.

Because the striation information extracted is essentially that used by ballistics experts, the computer techniques closely parallel standard ballistics performance but are able to produce an objective measure of verification. The local nature of the match, furthermore, allows verification of deformed and fragmented bullets. In addition, using only the stronger striations minimizes the effects of signature changes due to gun barrel wear and corrosion.

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I. INTRODUCTION

When a bullet is fired from a gun it is forced to travel along slightly spiraled grooves (rifling) in the gun barrel. Because the lead bullet is much softer than the barrel, the barrel lands (raised portions between grooves) cut into the bullet, leaving a reverse set of grooves and lands around the bullet. In addition, minute imperfections in the gun barrel introduced in its manufacture leave fine accidental marks on the bullet surface. These marks appear as microscopic striations, straight lines parallel to the land and groove edges [1]. For the last half century, police have been using this topographical information to identify a firearm from bullets fired from it. The major class characteristics of bullet diameter, number of lands and grooves, and direction of twist are used to identify the weapon as to manufacturer and model. The striation patterns are then used to identify the actual weapon to the exclusion of all others [2]. There is, however, no established criterion for verifying that two bullets were fired from the same gun. Thus the decision of whether or not two bullets match is left to an examiner's opinion.

Surface analyzing techniques have been applied to the examination of bullets in hopes of automating the procedure, easing the work load, and placing verification decisions on firm, objective ground. The topography of a bullet has been recorded by a fine stylus riding on its surface as it was rotated [3], [4]. Because this technique assumed a generally cylindrical shape to the bullet, it could not be used for bullets badly deformed or fragmented by contact with hard surfaces. Such bullets are common evidence specimens and have been referred to by Detective A. Johnson of the New York City Police Department as the "stumbling block" of all current attempts to automate bullet matching [5]. Finally, the stylus method had to be abandoned because it scratched the soft lead bullet and thus corrupted evidence.

The aim of the work presented here was to develop and test a method of extracting the significant striation information from bullets and quantifying this information to provide an objective measure of verification. Allowance has been made for normal striation variations with repeated weapon firings. The approach taken parallels standard ballistics practice in that it examines small portions of the bullet separately and thus can be used with deformed and fragmented bullets.

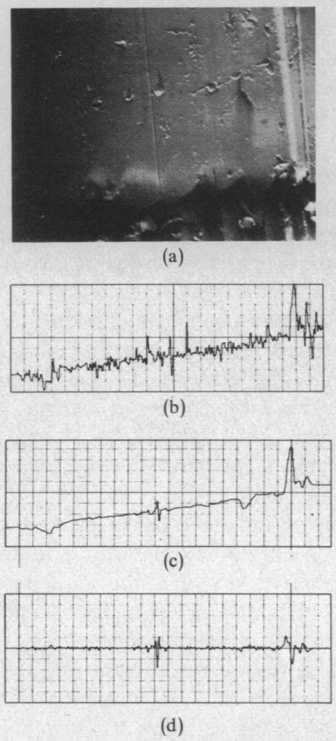
The techniques developed were tested on 13 bullets fired from 4 different 0.38-caliber revolvers.

II. SURFACE ANALYSIS

A. Extraction of Striation Information

In order to allow for severe deformation and fragmentation of the bullet, its surface was examined in small sections. Each land and groove was examined separately and only about one third of its length was used. Since the most significant striations run the full length of a land or groove, the middle third was generally representative of the whole. Occasionally, another portion of the land or groove was used because it had a stronger striation pattern.

Each such section was examined using a scanning electron microscope (SEM), which forms an image similar to a high quality optical image by scanning a fine beam of electrons across the bullet surface in a raster pattern. The backscattered electrons reflected from the surface are detected by a collector whose amplified output drives a cathode ray tube rastered in synchronization with the incident beam. The image is comparable to an optical image viewed at the electron gun position with illumination at the collector position. The SEM, however, has the advantages of greater depth of focus and an electronic signal which is convenient to use as computer input.



(a)

(b)

(c)

(d)

Fig. 1. Striation extraction from a land. (a) SEM image. (b) Single scan. (c) Average of 768 scans. (d) Derivative of average scan.

The first step in extracting striation information from the SEM image was to suppress the signal due to random surface anomalies such as craters and other local marks. This was accomplished by digitizing the scan signals taken from the collector amplifier and averaging them in a direction along the striation. The average of from 700 to 1000 scans gave a significant increase in signal-to-noise ratio (SNR). In order to further reduce any residual noise in the average scan, smoothing was applied in which a parabola was fitted by least squares to small segments of the trace. The central point of the segment was then replaced by the corresponding parabola value. A large amount of smoothing (13-point segments) was used to enhance gross features of the land and groove edges. Moderate smoothing (9-point segments) was used to highlight wide striations and the unsmoothed average trace was used for narrow striations.

The derivative of the smoothed scan (i.e., the slope of the parabola at the central point of the fitted segment) was used as a high-pass filter to remove the low frequencies in the signal due to a shadowing effect over the curved bullet surface. The derivative also emphasized the higher frequency striation information (Fig. 1).

B. Quantization of Striation Information

Using the smoothed average scan, all significant striations were detected and their important features measured. The three most important striation features used by ballistics examiners are 1) position across the land or groove width, 2) width, and 3) amplitude.

The first problem in determining position is to locate the land or groove end points. The left land end and right groove end are "shadowed" and show a drop in intensity level on the scans. The right land end and left groove end are highlighted and show distinctive peaks on the scans. Furthermore, these points of large topographical change produce extreme values in the intensity derivative. Because each bullet was positioned the same way in the SEM, using a clear plastic template for land and groove end

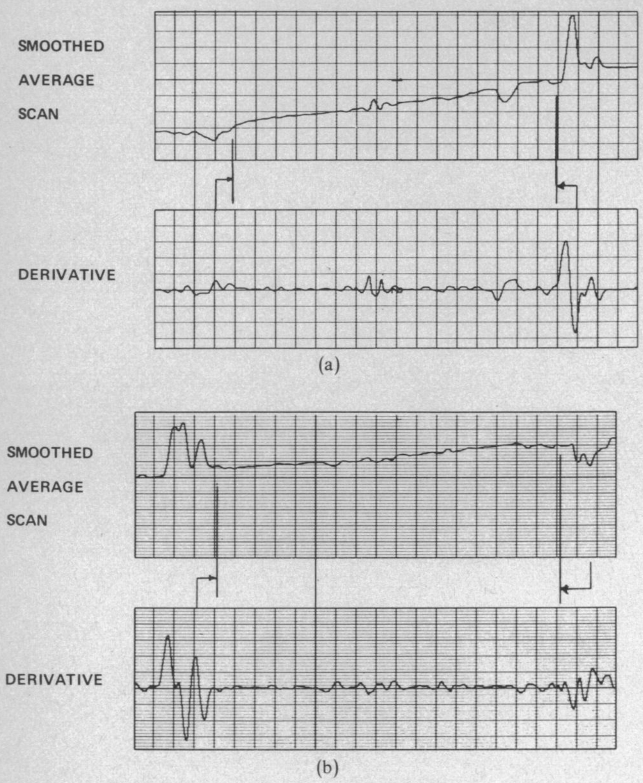


Fig. 2. Land and groove ends. (a) Land smoothed average scan and derivative. (b) Groove smoothed average scan and derivative.

points, the search for these features could be limited to small regions to avoid confusion with extraneous shadows and highlights. In addition, a liberal 13-point smoothing was used to emphasize these gross features over the sharper striation peaks.

The left land end was found by searching a small region defined at the left end of the scan for a minimum in intensity followed by a nominal local maximum in slope. The right land end was found by searching another region at the right end of the trace for the minimum slope. Both locations thus found represented points on the sides of the land. The end points were defined as points fixed distances interior to these to avoid highlights of the sides that might be mistaken for striation peaks (Fig. 2).

Groove ends were identified in an analogous fashion. The left end was defined as a point a fixed distance interior to the maximum slope found in a small region at the left. The right end was a point interior to the minimum intensity in a region at the right.

With the section end points determined, the portion of the average scan between these limits was searched for significant striations. The search was performed twice, once with a moderate 9-point smoothing on the scan and once with no smoothing. The moderate smoothing accentuated wide striations and the unsmoothed scan highlighted narrow striations. For both degrees of smoothing the average absolute derivative between end points was computed and used as a criterion for striation significance. A significant striation was considered to be detected on the scan at any point where the magnitude of a negative derivative peak exceeded 3 times the average derivative and its absolute integral exceeded 8 times the average derivative. The magnitude of the negative derivative minimum represents the sharpness of the intensity excursion from highlight to shadow and is therefore related to the sharpness of the striation. The absolute integral of the negative derivative peak is equivalent to the difference between maximum highlight and shadow levels on the intensity signal and was used as a measure of striation amplitude, since deeper striations tend to have stronger highlight-shadow differences.

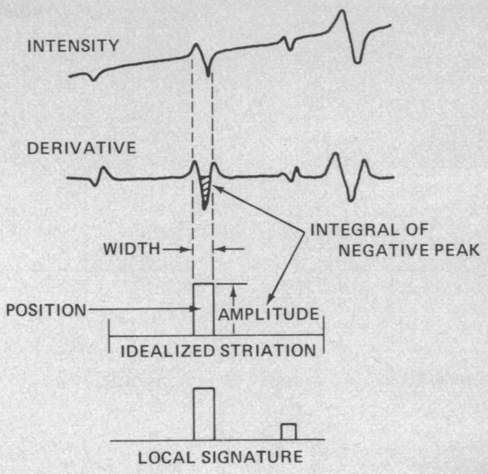


Fig. 3. Idealized striation and local signature. (a) Intensity. (b) Derivative. (c) Idealized striation. (d) Local signature.

The position, amplitude, and width of each significant striation between section end points were measured and normalized (Fig. 3). The position was measured on the smoothed digitized scan as the number of points between the left end point of the section to the negative derivative peak. This value was normalized by dividing by the section width (i.e., the number of points between end points). The amplitude was measured as the absolute integral of the negative peak and was normalized by dividing by the average absolute derivative.

The width was measured as the number of points between local derivative maxima bracketing the negative peak. These points of local maxima were chosen because they represent points marking the start of the highlight region and end of the shadow region on the intensity signal. The width was normalized by dividing by the section width.

All these measurements were obtained for both the unsmoothed and the smoothed scan. Those parameter values corresponding to the case of maximum normalized amplitude were saved as the striation features. Thus for each significant striation in the section, an idealized striation was constructed. All the idealized striations in the section formed a local signature for that section.

The local signature extraction for two lands on two matching bullets is shown in Fig. 4.

III. VERIFICATION

The verification procedure traditionally used by ballistics experts consists in lining up a land or groove on each of two bullets and looking for striations that line up and possess similar characteristics such as width and apparent depth. When common striations are found on one section, the bullets are rotated together so the examiner can follow the same procedure on sequential sections. As more and more information on matching striations accumulates in the examiner's mind, his opinion of the probability that the two bullets were fired from a common gun develops. In the process, many fine striations and possibly some strong ones that do not match are disregarded if enough prominent ones agree. The important question is "given two bullets with a number of similar markings and some dissimilar ones, what is the probability that as good a match could occur at random?" If the probability is high enough, then the random factors that create and change a gun's signature could cause such a match between two different guns.

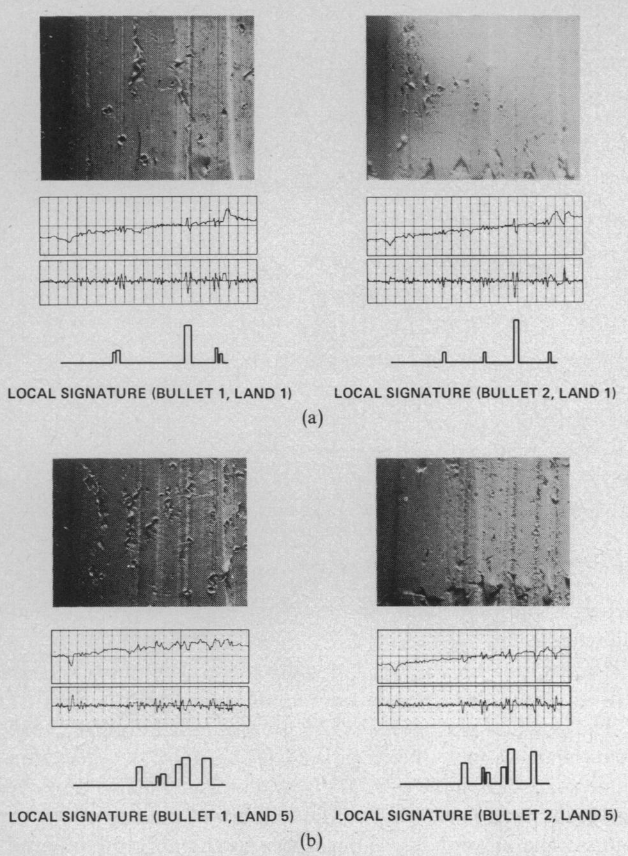


Fig. 4. Signature extraction from matching bullets. (a) Land 1 of bullets 1 and 2. (b) Land 5 of bullets 1 and 2.

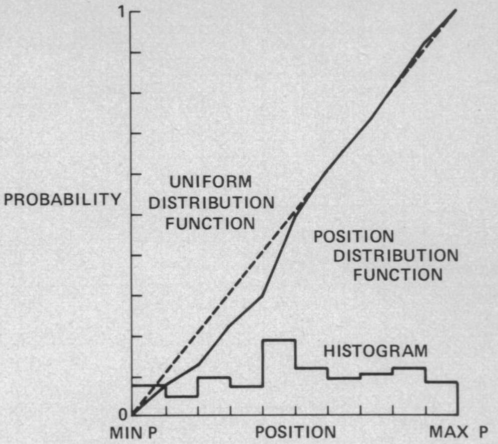
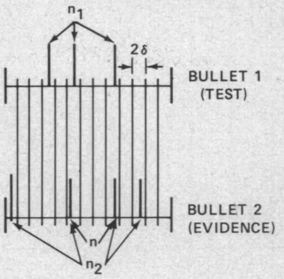


Fig. 5. Position distribution.



EXACTLY K STRIATIONS MATCH –

$$P_K = \frac{\binom{n_2}{K}\binom{m-n_2}{n_1-K}}{\binom{m}{n_1}}$$

AT LEAST n STRIATIONS MATCH –

$$P(P) = 1 - \sum_{K=0}^{n-1} P_K$$

Fig. 6. Position match.

The important elements, then, are how well the striations line up and how similar they are in width and amplitude. Since there is no physical reason to suspect any relation between striation position and physical characteristics, we will assume independence and examine the features separately. First we will determine the probability that those striations matching in position could do so at random and then determine the probability that those that line up could be as well matched in width and amplitude by random processes.

A. Position Match

The following analysis of position match assumes a uniform distribution of striations across the section. This is intuitively justifiable because there is no physical reason for the distribution to favor any location on the section. The histogram and distribution function generated from the data support this (Fig. 5).

Let us make the hypothesis that the two bullets we are comparing were fired from the same gun. Then the probability of error $P(P)$ is the probability that our position match criterion value could occur at random. Given bullet 1 with n_1 striations and bullet 2 with n_2 and n matching in position within a distance δ, we can determine the probability of a match as good or better occurring at random. Let us call bullet 1 the test bullet and bullet 2 the evidence bullet. Without loss of generality, we assume $n_1 \leq n_2$. First we divide the section into intervals (Fig. 6). Since a match is just as good or better anywhere within a distance δ on either side of the test striation, we use an interval increment of 2δ and form $m = 1/2\delta$ equal increments. Now $P(P)$ is just the probability of from n to n_1 striations on the test bullet falling in intervals corresponding to those occupied by the n_2 evidence bullet striations. The probability of n or more striations matching is just 1 minus the sum of probabilities of getting $1, 2, \cdots, n-1$ matches. Call the probability of getting exactly 1 match P_1. Then P_1 equals the probability of 1 match from n_1 on n_2 *and* $n_1 - 1$ striations hitting the blank intervals on bullet 2.

$P_1 \propto (n_2/m)$ represents the probability of 1 of the n_1 striations falling on intervals occupied by the n_2 striations of bullet 2.

$$P_1 \propto \left(\frac{m-n_2}{m-1}\right)\left(\frac{m-n_2-1}{m-2}\right)\cdots\left(\frac{m-n_2-(n_1-2)}{m-(n_1-1)}\right)$$

represents the probability of the $n_1 - 1$ remaining striations falling on the empty intervals of bullet 2. Since we can choose one from the n_1 striations in any way, we must multiply these factors by the combinatorial factor

$$\binom{n_1}{1}$$

to get

$$P_1 = \binom{n_1}{1}\left[\frac{n_2}{m}\right]\left[\left(\frac{m-n_2}{m-1}\right)\left(\frac{m-n_2-1}{m-2}\right)\cdots\left(\frac{m-n_2-n_1+2}{m-n_1+1}\right)\right].$$

Similarly,

$$P_2 = \binom{n_1}{2}\left[\left(\frac{n_2}{m}\right)\left(\frac{n_2-1}{m-1}\right)\right]\left[\left(\frac{m-n_2}{m-2}\right)\left(\frac{m-n_2-1}{m-3}\right)\cdots\left(\frac{m-n_2-n_1+3}{m-n_1+1}\right)\right]$$

and the general term is

$$P_k = \binom{n_1}{k}\left[\left(\frac{n_2}{m}\right)\cdots\left(\frac{n_2-k+1}{m-k+1}\right)\right] \cdot \left[\left(\frac{m-n_2}{m-k}\right)\cdots\left(\frac{m-n_2-n_1+k+1}{m-n_1+1}\right)\right]$$

which can be simplified as follows:

$$P_k = \left(\frac{n_1!}{(n_1-k)!\,k!}\right)[n_2!/(n_2-k)!] \cdot \left[\frac{(m-n_2)!/(m-n_2-n_1+k)!}{m!/(m-n_1)!}\right]$$

$$= \frac{\dfrac{n_2!}{(n_2-k)!\,k!}\;\dfrac{(m-n_2)!}{(m-n_2-n_1+k)!\,(n_1-k)!}}{\dfrac{m!}{n_1!\,(m-n_1)!}}$$

$$= \frac{\binom{n_2}{k}\binom{m-n_2}{n_1-k}}{\binom{m}{n_1}}.$$

This is the number of ways the n_2 intervals on bullet 2 can be combined to accept the k matching striations times the number of ways the empty intervals can be combined to accept the remaining of the n_1 striations, divided by the number of ways all the intervals can be combined to accept all the n_1 striations.

P_k has the important property [6] that

$$\sum_{k=0}^{n_1} P_k = \frac{\binom{m-n_2+n_2}{n_1-k+k}}{\binom{m}{n_1}} = \frac{\binom{m}{n_1}}{\binom{m}{n_1}} = 1$$

which agrees with the fact that the event of getting a match of from 0 to all the n_1 striations is a certainty.

Another important property is the symmetry between n_1 and n_2 which justifies choosing n_1 the smaller. This is shown as follows. Interchanging n_2 and n_1 in P_k, we get

$$\frac{\binom{n_1}{k}\binom{m-n_1}{n_2-k}}{\binom{m}{n_2}}$$

$$= \frac{\dfrac{n_1!}{k!\,(n_1-k)!}\;\dfrac{(m-n_1)!}{(m-n_1-n_2+k)!\,(n_2-k)!}}{\dfrac{m!}{n_2!\,(m-n_2)!}}$$

$$= \frac{\dfrac{n_2!}{(n_2-k)!\,k!}\;\dfrac{(m-n_2)!}{(n_1-k)!\,(m-n_2-n_1+k)!}}{\dfrac{m!}{(m-n_1)!\,n_1!}}$$

$$= \frac{\binom{n_2}{k}\binom{m-n_2}{n_1-k}}{\binom{m}{n_1}} = P_k.$$

To use this position match probability to get a measure of match error we calculate P_k for $k = 0$ to $n - 1$. The probability of at least as good a match at random based on position alone is the probability of at least n striations matching or

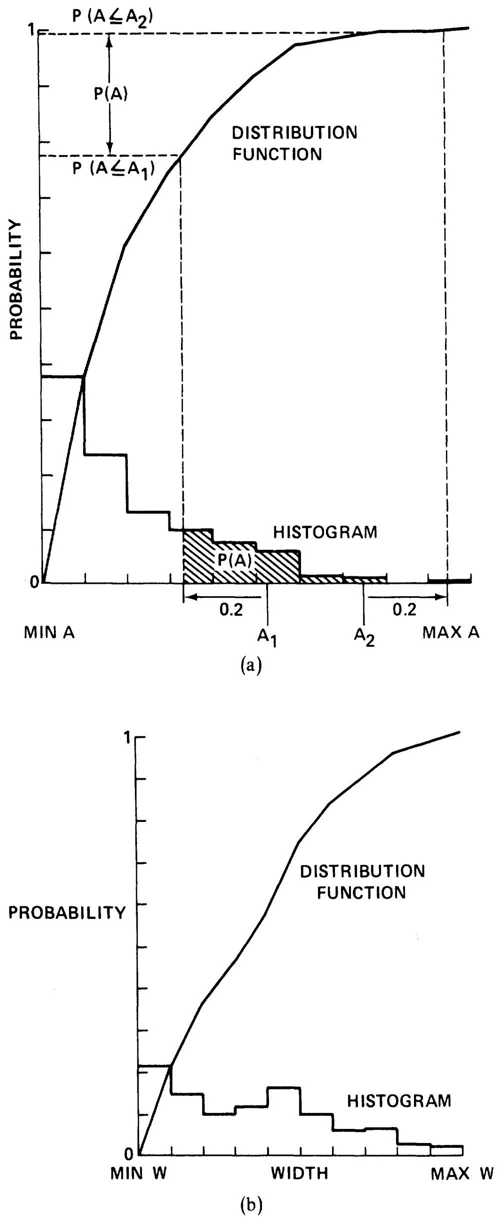


Fig. 7. Amplitude and width distributions. (a) Match on amplitude. (b) Width distribution.

$$P(P) = 1 - \sum_{k=0}^{n-1} P_k.$$

B. Amplitude and Width Match

The assumption of uniformity for the distributions of amplitude and width values could not be supported intuitively, nor was it supported by the data (Fig. 7). Instead, distribution functions were used to determine the probability of a striation match within a given tolerance in amplitude and width occurring at random.

Only striations matching in position were used in this analysis. To find $P(A)$, the probability of random match in amplitude, the values of amplitude for the matching striation pair were read from the distribution function by linear interpolation between values computed from the data over 10 intervals. To be conservative and allow for the imprecision in the amplitude measurements, a nominal value of 0.2 of the total amplitude range was added to the

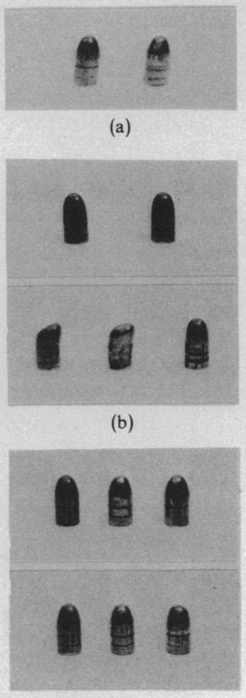

(a)
(b)
(c)

Fig. 8. Test case bullets. (a) Bullets 1 and 2 for Case I. (b) Bullets 3 to 7 for Case II. (c) Bullets 8 to 13 for Case III.

higher of the two values and subtracted from the lower value before the interpolation. The difference between these values equaled the area under the corresponding histogram and represented the probability of getting as close a match or better at random from striations having the same amplitude distribution as the data. The same procedure was used to get $P(W)$, the probability of width match occurring randomly.

C. Independence

For each land and groove we thus determined a probability of match in position for all striations. In addition we measured for each striation a probability of chance match on amplitude and width. Assuming width, position, and amplitude to be independent, and each striation to be independent of all others, we multiplied these values to get the total probability of the local match being due to chance:

$$P_e(\text{section}) = P(P) \prod_{\substack{\text{all} \\ \text{striations}}} P(A)P(W).$$

Furthermore, each land and groove was independent of all others so the total probability was

$$P_e = \prod_{\substack{\text{all} \\ \text{sections}}} P_e(\text{section}).$$

Thus a probability of error in verification could be calculated using the same line of reasoning used by professional examiners but including an objective, numerical criterion.

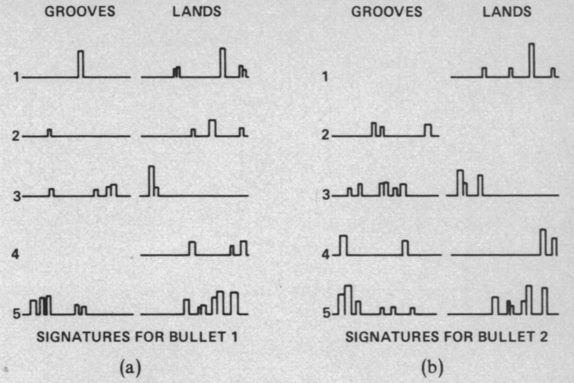


Fig. 9. Bullet signatures for test case I. (a) Signatures for bullet 1. (b) Signatures for bullet 2.

VERIFICATION NUMBER MATRIX

	CASE I		CASE II					CASE III					
	1	2	3	4	5*	6*	7	8	9	10*	11	12	13
1	46	13	0	0	0	0	0	0	1	1	0	0	1
2	13	50	0	1	0	0	0	1	0	0	0	0	0
3	0	0	17	7	8	1	4	0	1	0	0	0	0
4	0	1	7	23	6	3	4	1	1	0	2	1	1
5*	0	0	8	6	21	2	2	1	1	0	1	0	0
6*	0	0	1	3	2	17	2	1	0	0	1	0	0
7	0	0	4	4	2	2	6	0	1	0	0	0	1
8	0	1	0	1	1	1	0	38	0	0	6	1	1
9	1	0	1	1	1	0	1	0	14	0	0	0	1
10*	1	0	0	0	0	0	0	0	0	13	0	3	0
11	0	0	0	2	1	1	0	6	0	0	25	0	0
12	0	0	0	1	0	0	0	1	0	3	0	32	0
13	1	0	0	1	0	0	1	1	1	0	0	0	20

* DEFORMED BULLET

MATCHING BULLETS:

1, 2
3, 4, 5, 6, 7, 8, 11
10, 12
9, 13

Fig. 10. Verification number matrix.

IV. Test Cases

Three test cases were run to evaluate the techniques outlined earlier. A total of 13 bullets fired from four different guns were provided by the Nassau County Police Department for examination (Fig. 8). All guns used were 0.38-caliber Smith and Wesson revolvers firing 0.38 special bullets with the same major class characteristics. The number of lands and grooves was five and the twist was to the right. Land and groove width, depth and pitch angle were not measured. A typical set of signatures for two matching bullets is shown in Fig. 9.

Using the signatures and the feature histograms, verification tests were performed for all possible pairs of the 13 bullets. For each pair all possible matchups of land-on-land and groove-on-groove in sequence around the bullet were made and the lowest probability was saved. To compensate for the five possible matches between each pair, the probability was multiplied by 5. The negative logarithm (base 10) was then taken and its integer value saved as the match criterion or *verification number*. The verification numbers for all matches (including self-matches) are tabulated in matrix form in Fig. 10.

The verification number can be viewed as a measure of how uniquely the bullets match. For example, self-matches have astronomical numbers as high as 50. This can be interpreted as saying that only one in 10^{50} guns will have a signature matching the gun in question as well as it matches itself. Such a high number is due to the fact that every peak is matched exactly in both position, amplitude, and width. The self-match number can be viewed as a measure of the amount of information in the signature. The more sections in the signature and the more peaks in each section, the higher the number.

The verification number of a match between different bullets can be related to the total number of existing guns with the same class characteristics. Since there were on the order of 10 million or 10^7 such guns produced, a verification number greater than 7 will indicate the identity of an evidence weapon.

A. Case I: Bullets 1 and 2

The first test case consisted of two pristine (undeformed) lead bullets fired into cotton wadding from an actual evidence pistol. These bullets had strong markings on several lands and grooves typical of evidence weapons, which are generally not maintained well or cleaned frequently.

Because the gun left so many common marks on both bullets, the verification number is high enough to identify the weapon to the exclusion of all other weapons. Land 5 alone has six strong marks on each bullet, and gives a verification error probability of 5.8×10^{-5}. So, any bullet fragment containing land 5 would be sufficient to identify this gun as a candidate for visual examination.

B. Case II: Bullets 3–7

The second test case consisted of five lead bullets fired from a police revolver. Often evidence bullets will be deformed or scratched by contact with bone or other hard surfaces. Furthermore, bullets with surfaces other than pure lead are often encountered. The bullets chosen for test Case II reflect these peculiarities. Bullets 3 and 4 are pristine bullets as in test Case I; bullet 5 was deformed by firing through a $\frac{1}{4}$ inch wood board; bullets 6 and 7 are lubaloy bullets (lead with a thin copper alloy coating). Bullet 6 was also deformed by firing through a $\frac{1}{4}$ inch board. Because the pistol used was, like police weapons in general, newer and better cared for, it left fewer markings than the evidence revolver in the first test case.

Nonetheless, a good match was made between all the uncoated lead bullets including the deformed bullet shot through the wood board. These results are particularly encouraging in view of the limited number of sections, the small number of major striations and the deformity of bullet 5.

The lubaloy bullets gave less impressive results for several reasons. First, the small number of sections recorded limited the information available to work with. Secondly, the flaking of the copper coating caused detection of spurious peaks, not all of which were edited out. This same flaking is the problem that precluded recording more lands and grooves on the SEM because it obliterated so much of the surface information. Strong striations can be picked up, however, even in the midst of large amounts of flaking as in groove 5 of bullet 7 shown in Fig. 11.

C. Case III: Bullets 8–13

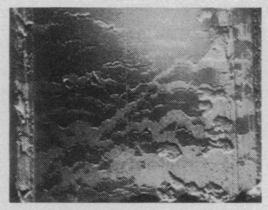
The final case was a blind study in which two bullets from each of three different guns were fired to see if these analysis techniques could match the pairs properly without prior knowledge of which was which.

The matches obtained in the blind study were not as good as those obtained in the previous two cases. Bullets 8 and 11 matched best, bullets 10 and 12 next best, and no verification was obtained between 9 and 13. Subsequent visual comparison by the Nassau County Police rated bullets 8 and 11 matching well, bullets 10 and 12 matching, but poorly. No match could be obtained between bullets 9 and 13.

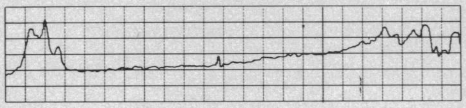
V. Summary and Conclusions

Techniques have been developed to solve the basic problems in automating ballistics examinations. These techniques have been demonstrated on thirteen bullets fired from four different 0.38-caliber Smith and Wesson revolvers with similar class characteristics.

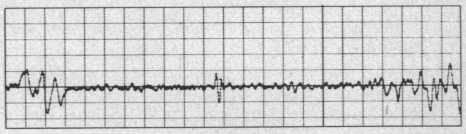
LUBALOY BULLET



GROOVE 5 OF BULLET 7
(a)



AVERAGE SCAN
(b)



DERIVATIVE
(c)

LOCAL SIGNATURE
(d)

Fig. 11. Lubaloy bullet. (a) Groove 5 of bullet 7. (b) Average scan. (c) Derivative of average scan. (d) Local signature.

The scanning electron microscope has been shown to be an effective means of extracting topographical information from the surfaces of bullets. Computer techniques have been developed to compress the SEM data by averaging scans running transverse to the line of the bullet striations. This has been shown to extract essential striation information and suppress irrelevant surface markings. Because of the nature of the SEM signal, it is easily used as computer input and can be computer controlled, providing a basis for automation of the bullet scanning process.

Because the average SEM scan can be quantified, the striation information can be given numerical values. The information is the same as that used qualitatively by ballistics experts but can now be used quantitatively to give an objective measure of verification. This information is striation position, strength, and width, as represented by average image intensity variations across lands and grooves and is quantified in a verification signature.

Because the verification signature is defined on a local level for many small portions of the bullet surface, deformed and fragmented bullets can be examined in the same manner as whole, pristine specimens. The normalizing of striation position by dividing by the land or groove width helps compensate for local deformities.

The local nature of the signature also tends to suppress effects of changes with time in the overall striation pattern, unless all areas of the bullet undergo great change. The time-variance problem is also attacked here by selecting the stronger, more stable, striations for the signatures.

The surface analysis and verification techniques have been demonstrated successfully on the thirteen test bullets, including three deformed bullets and two with a flaking lubaloy coating.

The quantized information used in the verification procedure also provides a basis for classifying bullets in an ordered file which allows efficient search for a limited number of verification candidates. The subject of classification is beyond the scope of this paper and is discussed in [7].

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REFERENCES

[1] J. S. Hatcher, *Textbook of Firearms Investigation.* Planterville, SC: Small Arms Pub., 1935.
[2] G. Burrard, *The Identification of Firearms and Forensic Ballistics.* London: Herbert Jenkins, 1934.
[3] J. E. Davis, *An Introduction to Tool Marks, Firearms and the Striagraph.* Springfield, IL: Charles C. Thomas, 1958.
[4] O. Halsey, Ed., "A U.S. bullet identification system: Impossible or up to electronics?" *Amer. Rifleman,* pp. 37-47, Oct. 1968.
[5] A. Johnson, "Recent developments in bullet search systems," *Amer. Chem. Soc. Symp. Series,* Sept. 1974.
[6] D. E. Knuth, *The Art of Computer Programming,* Vol. 1. Reading, MA: Addison-Wesley, 1968.
[7] G. Y. Gardner, "Computer identification and classification of bullets," Doctoral dissertation, Polytechnic Institute of New York, June 1976.

The Optimum Number of Nodes in a Star-Configured Distributed Computer System

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Abstract—**Having more local processors in a distributed computing system reduces the cost of user communications but requires more small hardware sites and more network communications circuits. Assuming uniformly distributed users and making the simplest tenable assumptions about the economies of scale of local processors and communications, the optimum number of local processors is found. In several interesting special cases (one of which is quadratic economies of scale), the solution is approximated by a very simple form. The analysis is also extended to users in urban concentrations, which tends to give smaller optimum networks.**

I. INTRODUCTION

Interactive computer systems with access from a large area are becoming more common in banks, reservation systems, enquiry/response systems, and information systems of various kinds. These systems are often star-configured with a single central processor and a number of local processors distributed around a serving area. Users access the nearest local processor, which may pass the transaction directly to the center, or may perform some processing on it first. Examples of local processors are data concentrators, local data bases, and central nodes of terminal clusters. Note that the term processor in this context covers not just the CPU, but rather all the hardware and software comprising a processing system.

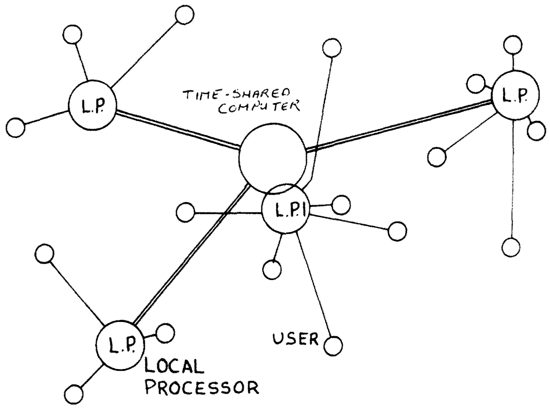

Fig. 1. Time-shared computer with local processors.

In the preliminary configuring of a system one of the most important decisions is the approximate number of processor sites to be provided, and this decision must usually be made without the benefit of a detailed design allocating processors to specific sites, as in [1]. Therefore, this correspondence focuses just on the number of local processors, using simple assumptions as to the distribution of users and sites. It also focuses on star-configured systems as being of widest interest.

This correspondence analyzes the variation of system costs as a function of the number of sites, and as a function of several parameters. The parameters represent the relative importance of different cost components, and the effects of economies of scale in different types of resources. The optimum number of processors is found, and the variation of this optimum number due to changes in six important cost parameters is investigated. The approach is applied to find the optimum number of data concentrators, the impact of new communications pricing policies, and the effect of urban concentrations of users, with results that are not obvious in advance.

The following simplifying assumptions are used in order to focus the cost variation onto N, the number of local processors. With reference to Fig. 1, notice that there is a central processor which is always accessed via one of the N local processors. Therefore, there are N local processors for N user areas.

1) a) Users and local processors are distributed uniformly over an area, or b) users are distributed over a set of urban areas (this assumption is used in Section IV).

2) There are N local processors, one of which consists of the local processor to accommodate users from the area around the central processor.

3) Users are connected to the nearest local processor, by dial-up lines or leased lines.

4) Each local processor is connected directly to the central system; however, LP1 is always connected by a link of effectively zero length, so there are only $N-1$ links of significant length.

5) Users are homogeneous in their use of the system.

6) The central processor assumes a fixed work load for all values of $N \geq 1$, so it is unaffected by N.

The overall cost is \$$C$/year, made up of three components

$$C(N) = C_H(N) + C_U(N) + C_C(N) \tag{1}$$

where

$C_H(N)$ cost of all resources located at local processor sites (including hardware, software, support);

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