# Correspondence

## System Complexity: Its Conception and Measurement in the Design of Engineering Systems

DEVENDRA SAHAL

***Abstract*—Complexity is a property of systems that is largely independent of their specific contents. An understanding of it would therefore seem to be a prerequisite to development of a "general systems theory." This paper presents a theory of system complexity and its illustrative application to changes in the aircraft designs. Four distinct contributions are offered. First, the proposed framework permits not only the measurement but also the segregation of complexity into various components: those pertaining to organized, unorganized, short-term and long-term aspects of any given system's behavior. Second, the formulation permits grouping of variables on *a priori* grounds, thereby alleviating the problem posed by a large number of variables in systems analysis. Third, and more important, the formulation is capable of circumventing the problem of nonstationarity in the application of the tools of information theory. Fourth, a number of findings emerge from this study with important implications for the theory of evolutionary processes. These are discussed. The long-run redundancy of the phenomena underlying the changes in aircraft designs is estimated in the range of 48 to 60 percent. The corresponding estimate in the state of short-run is in the range of 72 to 98 percent. The latter is concluded to be solely due to the "unorganized" aspects of the evolutionary process.**

### I. Introduction

This study represents an attempt to develop a theory of system complexity. It is based on information theoretical considerations in a dimensional analytic framework outlined below. An application of the theory to the illustrative case of aircraft design process is presented. This particular choice of the case study is made in appreciation of the fact that a theory of the spatiotemporal evolution of engineering systems is all but lacking.[1]

One distinct contribution offered in this study is related to analysis of information flow, e.g., in regulatory systems and in man–machine interactions. A significant problem associated with the information theoretic approach is that it requires the highly restrictive assumption of the weak time invariance of the system under study (see, e.g., [4]). In the proposed framework, however, the problem due to nonstationarity is circumvented by means of transformation of original system variables to functions representing the invariant phenomenal aspects of the system under study.

As emphasized by Von Neumann in his study of self-replicating automata [23], the conception of complexity is a prerequisite to an understanding of learning and evolutionary processes. Furthermore, complexity is a property of systems that is largely independent of their specific contents. An understanding of it would therefore seem to be a prerequisite to development of a "general systems theory." Study of complexity would also seem to be of considerable practical importance for a variety of purposes such as actual systems design, technology assessment, establishing standards in the case of man–machine interaction systems, etc. All this is sufficient justification for the present attempt at quantification of system complexity.

### II. Theoretical Framework

#### A. Similarity and Complexity

The starting point of the present study is that information theory is not entirely satisfactory as a basis on which to formulate a theory of systems and the complexity thereof. A system is a holistic organization, i.e., as a whole it is not the same as a mere aggregate of its parts. Each of the system variables does not act separately but in conjunction with many others in the form of complex effects. Separate primary variables may not be important but only their combinations which correspond to these effects [6], [30]. Information theory by itself cannot, however, identify these combinations. Like any statistical theory, it disregards the fact that the relative position of the elements in a structure may matter. "In other words, it proceeds one the assumption that information on the numerical frequencies of the different elements of a collective is enough to explain the phenomena and that no information is required on [the holistic effects due to] the manner in which different elements are related" [7, p. 339].

The proposed framework seeks to resolve this problem by an appeal to the notion of "similar phenomena." Put another way, the conception of the complexity of similar systems is facilitated because, due to similitude, it becomes possible to make the *ceteris paribus* assumption.

Specifically, the framework here postulates that the original system variables $x_1,x_2,\cdots,x_n$ having independent dimensions be replaced by $II, II_2,\cdots,II_k$ made dimensionless by suitable combinations of $x_1,x_2,\cdots,x_n$. The number of characteristic dimensionless products for a system are determined by a theorem, attributed to Buckingham [11], that a dimensionally homogeneous equation

$$F(x_1,x_2,\cdots,x_n) = 0 \tag{1}$$

can be expressed in terms of $II$ variables formed with $x_1,x_2,\cdots,x_n$ such that

$$f(II_1,II_2,\cdots,II_{n-r}) = 0, \qquad r \leq m \tag{2}$$

where $r$ is the rank of dimensional matrix of the $n$ original system variables and $m$ the fundamental quantities involved such as mass length, time, etc., in the case of physical systems. A similar approach in other cases is entirely feasible as indicated by the successful applications of dimensional analysis to biological, economic, psychophysical, meterological, chemical, rheological, and numerous other systems.[2]

Virtually all laws of physical theory exhibit the property of dimensional invariance. This may be explained in terms of a group theoretical assertion that the notion of similar systems corresponds to a dimensionally invariant function, i.e., any function of $II$ variables [11]. Thus dimensionless products can be interpreted as similarity criteria. The $II$ variables can also be regarded as *a priori* principal components in factor analysis [30]. The former are not statistically orthogonal. However, unlike the principal components in factor analysis, they are uniquely determined for a given set of original system variables

Manuscript received June 6, 1975; revised December 22, 1975.
The author is with the Systems Science Program, Portland State University, Portland, OR 97207.

[1] By the term "evolution of engineering systems" it is implied, for example, the changes over the course of time in the machine design characteristics of physical systems such as aircraft.

[2] A bibliography of these applications can be found in [30].

and more often amenable to qualitative interpretation. It seems legitimate to conceive all the system interactions as between the new macroscopic variables in the form of dimensionless products.

Thus one is able to considerably reduce, for the purpose of analysis, the number of system variables which must be taken into account.[3] At the same time, the problem due to nonstationarity is also alleviated. The reason is that, as compared to the level of "microsystems" (represented by the original $x$ variables), any increments leading to nonstationarity are likely to be much smaller at the "macroscopic" level (represented by the $\Pi$ variables). In terms of a purely statistical interpretation, the approach here may be viewed as comparable to a relevant transformation procedure (see, e.g., [13], [34]) for the purpose of reducing heteroskedasticity and the wide variation in the values of the original observations.

From the viewpoint of the proposed framework, the different models in the evolution of a complex system are merely various sets of the very same system configuration. The focus is not on the apparent models *per se*. Instead, it is on the fundamental relationships governing the system behavior. The aim is to bring out the invariancy of the system structure[4] while seeking the maximum possible independence from the otherwise fundamental role played by the measurement system. Since the proposed framework has its object of study, the invariant structural properties rather than the original system variables, the problem due to nonstationarity perhaps does not even arise. In summary, insofar as nonstationarity is an artifact of the frame of reference, the problem due to it is circumvented by a shift in the reference frame by means of dimensional analysis.

Two important problems remain, however. First, dimensional invariance is a necessary but not a sufficient condition for any given sets of the system configuration to be similar unless all boundary conditions are taken into account [27]. Alternatively, an appropriate extension of the dimensional analytic theory is required. Second, application of various measures of information theory is subject to an important condition that the observations required to locate or determine the various sets of the system configuration are arranged in the most efficient manner. To ensure that these requirements are met it is proposed to distinguish between various sets of the system configuration by means of a "dimensionally invariant discriminant function" as postulated in an earlier work of the author [30]:

$$Y = a_1\Pi_1 + a_2\Pi_2 + \cdots + a_k\Pi_k \tag{3}$$

where the weighting coefficients are determined such that the $t$-statistic or the $F$-ratio between the groups will be maximum. Just as in conventional discriminant analysis, the function to be maximized is the ratio of the between group variance to the within group variance:

$$\begin{aligned} f(a_1,a_2,\cdots,a_k) &= \frac{n_1 n_2}{n_1 + n_2} \frac{(a_1 d_1 + a_2 d_2 + \cdots + a_k d_k)}{\Sigma\Sigma C_{ij} a_i a_j} \\ &= \frac{n_1 n_2}{n_1 + n_2} \frac{a' dd' a}{a' C a}. \end{aligned} \tag{4}$$

[3] The question of whether or not the reduced number of variables is optimal from the viewpoint of *measurement complexity* would not be considered. See, however, an excellent treatment in Chandrasekaran [3] along with the references therein.

[4] This argument is at least indirectly supported by the work on the identifiability of latent structural characteristics by means of factor analysis [9], for dimensional analysis, as indicated earlier in this paper is analogous to *a priori* factor analysis. Furthermore, Birkhoff [2] has clearly demonstrated that dimensional analysis is a part of the general theory of invariant parameters of governing equations.

where $d' = [d_1,d_2,\cdots,d_k]$ is the vector of mean differences on the $k$ $(= n - r)$ dimensionally invariant functions, $C$ is the within-group convariance matrix, and $n_1$ and $n_2$ are observations in the two groups.

Thus the discriminant function itself is composed of what were earlier defined as similarity criteria! Since all the remaining aspects of the theory (the assumption of homogeneity in the dispersion matrices, computation of multiple discriminant functions, classification of observations into various groups) are same as in conventional theory [18], they will not be discussed here in further detail.

Let the value of the $m$th dimensionally invariant discriminant function evaluated for element $k$ of group $l$ be denoted by $y_{lk}{}^m$. The "posterior probability" that case $k_m$ (in actuality) a member of group $m$ belongs to group $l$ is given by the well-known expression

$$P_{lmk_m}^{m'} = \frac{P_m \exp\,(Y_{lmk_m}^{m'})}{\sum_{i=1}^{r} P_i \exp\,(Y_{iik_m}^{m'})},$$
$$i = 1,2,\cdots,l,\ m,\cdots,r;\ k_m = 1,2,\cdots,n_m \tag{5}$$

where the superscript denotes that particular discriminant function employed which leads to maximization of the true probabilities of group membership, $r$ is the total number of groups, and $P_m$ is the prior probability of case $k$ in group $m$.

The proposed definition of system complexity is based on the concept of information [17], i.e., average uncertainty of an element's location. Specifically, *process complexity* of any given system (i.e., complexity of the invariant relationships underlying the system behavior) is defined as

$$H_m = \frac{1}{n_m} \sum_{k_m=1}^{n_m} G_{k_m}, \tag{6}$$

where

$$G_{k_m} = -\sum_{i=1}^{r} P_{imk}^{m'} \log_2 P_{imk}^{m'}.$$

Likewise

$$H_l = \frac{1}{n_l} \sum_{k_l=1}^{n_l} G_{k_l},$$

where

$$G_{k_l} = -\sum_{i=1}^{r} P_{ilk}^{l'} \log_2 P_{ilk}^{l'}. \tag{7}$$

The corresponding measure of *process redundancy* is

$$R_l = 1 - \frac{H_l}{\log_2 r}. \tag{8}$$

Hereafter, process complexity is designated as (system) complexity.

### B. Components of Complexity

In his outline of a general philosophical framework for the measurement of complexity, Kemeny [8] notes that is is reasonable to assume that the complexity is determined by the sum of its components. However, since there exists an isomorphism connecting information analysis and variance analysis [12], a problem arises here in interpreting the interaction term. The proposed framework circumvents this problem by assuming that the components of system complexity are all mutually independent.[5] For a partial justification on this see below.

Four relevant components of system complexity are identified in this paper: those pertaining to organized, unorganized, long-term, and short-term aspects of any given system's behavior. *Organized complexity* $H_t$ is hypothesized as due to relatively few known factors. Since, by definition, it is predictable with certainty, it is defined here to exist in the movements along a "well-defined" trend of temporal changes in the behavior of the system under study. A most widely applicable form of the trend is regarded here as the one based on the law of diminishing returns.[6] It should be noted, however, that $H_t$ cannot be *directly* estimated because to include a constructed trend in the analysis would be to ignore the fact that the information measured is defined only for stationary systems. *Unorganized complexity* $H_r$ is hypothesized as due to a large number of chance factors with effect of each quite small. It is defined to exist in the deviations from the constructed trend. As indicated by the empirical evidence in a variety of cases [5], [23], it seems a reasonable and generally applicable assumption that the mechanism which generates the movements along a trend is best regarded as essentially different from the one giving rise to deviations from it.

Insofar as changes in the system characteristics constitute a chance phenomena and tend to be conditional on the parameters of the existing ones, the evolution of complex systems would seem to be a random process and of a Markovian type. In the case of engineering systems, for example, this is indeed supported by the empirical evidence [19]–[21], [24]. To the extent that the phenomena under consideration also have the characteristics of an ergodic process, it should be possible to estimate *long-term complexity* from data at one point in time and *short-term complexity* from time series observations. This is justified by the ergodic theoretic assertion that averages *along* a sequence may be regarded as averages *over* the ensemble of possible sequences (see, e.g., [31, p. 48]). Thus one the average, one should obtain the same parametric estimates whether estimated from the replicas of the process in space or by following the process for a long range in time.

The applicability of these considerations is not, however, restricted to the case of engineering systems. In case of biological phenomena, for example, the proposed framework is justified by the well-known generalization that ontogeny recapitulates phylogeny: the individual organism, in its development goes through states that resemble some of its ancestral forms.[7] Thus the proposed framework for segregation and estimation of component complexity is as well applicable to biological systems. Kuh [10] arrives at a similar conclusion (that cross sections reflect long-term adjustments whereas annual time series reflect shorter run reactions) in the case of economic systems by means of a different set of considerations. His argument is that since disequilibrium among objects of analysis in cross-section data tend to be synchronized in response to common external factors, many disequilibrium effects wash out so that cross-section estimates can be interpreted as long-run coefficients.

## III. Illustrative Analysis

Data employed in this study refer only to those new piston-type aircraft actually employed by a U.S. carrier and can be found in the raw form in Table B-1 [16, pp. 140–145]. Complete data were available only on six design characteristics of these aircraft. The sample is small (see the tables for the exact number of observations) because for many years no new aircraft models were introduced. However, the usefulness of the proposed approach can be seen from the fact that it made it possible to economize over the number of observations by reducing the original six variables to the following two-dimensionally invariant combinations:

$$\Pi_1 = [(GTW)^{0.5}(ICR)]/(P)^{0.5}(CS)^{0.5}]$$
$$\Pi_2 = [(P)^{1/11}(WL)^{1/11}(R)]/[(CS)^{10/11}(GTW)^{2/11}]. \quad (9)$$

Following formulas were used:

| | |
|---|---|
| maximum take-off horsepower ($P$) | $ML_z^2T^{-3}$ |
| gross-take-off weight ($GTW$) | $ML_zT^{-2}$ |
| wing loading ($WL$) | $MT^{-2}L_x^{-1}$ |
| cruise speed ($CS$) | $L_xT^{-1}$ |
| cruise range ($R$) | $L_x$ |
| initial climb rate ($ICR$) | $L_xL_z^{0.5}T^{-1}$ |

where $M$, $L$, and $T$ are mass, length, and time, respectively. Lest the reader be tempted to find errors in these dimensional formulas, we hasten to add that we tried with alternative formulas without making any difference in vector lengths and found that results do not differ in any significant sense. A lengthy justification of the composition of these dimensionless products is available elsewhere [25], and this will not be repeated here.

Group 1 is composed of cross-section data which are all from the year 1929 (representing approximately the state of design in the long run). Group 2 is composed from 13 pooled observations from years 1948–1957 (representing the state in the relatively short run). The distribution of design variable in the long run is likely to be skewed [24]. However, since all except the first group (see below) represent a state over the short run, we have refrained from applying any transformation to the data.

Results of discriminant analysis of data in the first two groups are presented in Table I. The mean values of two variables in various years were then substituted in the standardized discriminant function with the resulting "Technology Index" as shown in Table II. By standardized function it is meant here that the coefficients of the variables were first multiplied by their standard deviations. To column 2 (Technology Index) in Table II, a hyperbolic form of the trend was fitted. The values of the residuals are presented in the last column therein. The explanatory power of the trend is very poor. Several alternative forms of trends were also tried, but the fit could not be improved in any significant way. Thus, unlike the original variables (characterized by a strong upward trend), $\Pi$'s do in fact lead to a more nearly stationary series for analysis.

Groups 3 and 4 were formed from the first and the last ten observations from column 2 in Table II. Groups 5 and 6 are represented by the first and last ten observations, respectively, of column 3 in Table II. Probabilities of group membership were then computed from discrimination between various groups.

[5] For an interesting parallel in the context of evaluating scientific systematizations, see Pietarinen [15]. He shows that for the systematic (explanatory) power of two given hypotheses taken together to not exceed the sum of the systematic power of individual hypotheses, a necessary condition is that two be inductively independent.

[6] In particular, for the illustrative case of engineering systems studied in this paper, this is supported by the empirical evidence presented elsewhere by the author [19], [20], [22].

[7] Simon [32] in his pathbreaking work in general systems theory contends that this generalization is as readily applicable to numerous cases outside the realm of biology, e.g., in case of problem solving systems, transmission of knowledge in the education process, etc. We do not, however, necessarily agree with Simon's view on the ubiquity of hierarchical structure, although the general applicability of the principle that ontogeny recapitulates phylogeny does imply that the framework proposed in this paper for the segregation of complexity into relevant components is generally applicable.

TABLE I

| | Group Means | |
|---|---|---|
| Variables | 1 | 2 |
| $II_1$ | 107.66 | 71.57 |
| $II_2$ | 86.15 | 134.75 |
| | Pooled Dispersion Matrix | |
| | $II_1$ | $II_2$ |
| $II_1$ | 284.84 | |
| $II_2$ | 101.54 | 1965.35 |
| | Discriminant Function | |
| | $Y = -12.77 + 0.23\ II_1 + 0.056\ II_2$ | |
| | F ratio (d. f. 2, 23) | |
| | 20.33 | |
| | Mahalanobis distance | |
| | 6.53 | |

TABLE II

| Year | Technology Index | Residual* |
|---|---|---|
| 1929 | 609.1 | 98.90 |
| 1930 | 525.8 | 11.27 |
| 1931 | 589.5 | 70.82 |
| 1932 | 460.5 | -62.10 |
| 1933 | 522.6 | - 3.74 |
| 1934 | 454.9 | -71.01 |
| 1935 | 448.3 | -84.98 |
| 1936 | 523.2 | -13.39 |
| 1937 | 595.8 | 56.06 |
| 1940 | 772.9 | 224.59 |
| 1946 | 576.9 | 14.14 |
| 1947 | 564.7 | - 0.16 |
| 1948 | 407.9 | -159.60 |
| 1949 | 613.7 | 44.83 |
| 1951 | 521.1 | -51.51 |
| 1952 | 374.7 | -199.65 |
| 1953 | 597.4 | 21.35 |
| 1955 | 717.6 | 138.23 |
| 1956 | 696.7 | 115.70 |
| 1957 | 916.9 | 334.38 |

*The relevant trend equation is $Y = X/(0.0143 + 0.00146X)$, ($R^2 = 0.045$) where Y is the technology index and X the years.

The $H$ estimates are presented in Table III. These results indicate that the complexity is higher in the long-run ($H_1$) than in the short run ($H_2$). This is consistent with the argument that in the evolution of engineering systems easier problems are solved first and more complex costly problems afterwards [28].

Relative to pooled time series and cross-section data in group 2, "pure" time series data represent a state of comparatively short term and residuals that of "extremely short run." Thus the result that the values of $H_5$ and $H_6$ are the highest is perhaps not surprising insofar as the stage of takeoff (very short run) is characterized by most complexity. The results also indicate that in the short run, total complexity is almost solely made of unorganized complexity. Thus values of the latter ($H_5$,$H_6$) are, in fact, greater than of the former ($H_3$,$H_4$). At first this is a surprising result because $H_3$ and $H_4$ have been estimated from

TABLE III

| Group | H-estimates | obtained from discriminant analysis between groups | R-estimates |
|---|---|---|---|
| 1 | 0.39 ($H_1$) | 1,2 | 60.1 |
| 2 | 0.28 ($H_2$) | 1,2 | 71.6 |
| 3 | 0.98 ($H_3$) | 3,4 | 2.15 |
| 4 | 0.97 ($H_4$) | 3,4 | 3.09 |
| 5 | 0.99 ($H_5$) | 5,6 | 1 |
| 6 | 0.99 ($H_6$) | 5,6 | 1 |

data which show greater variability or range than the data underlying $H_5$ and $H_6$. Thus it would appear a case of reduction in range or gain in precision without change in uncertainty. Recall, however, the failure to fit the trend with any degree of success. In variance analytic terminology this implies that

$$V(Y) \simeq V(r)$$

i.e., data from which total complexity is estimated ($Y$) are no less unorganized compared to those underlying unorganized complexity (residuals $r$ from the trend). Thus the result here is reassuring because it is in keeping with the isomorphism connecting information analysis and variance analysis [12]. It is likely, however, that $H_5$ and $H_6$ are biased upwards because the procedure does not permit simultaneous estimation of the two types of complexity.[8]

The results also indicate that $R_1$ (long-run redundancy) is less than $R_2$ (short-term redundancy). Insofar as greater redundancy may imply a relatively hierarchic structure, the results here suggest that a hierarchic structure may be less crucial to the long-term evolution of engineering systems. In the case of aircraft designs, an independent verification of this inference is provided elsewhere by the author on the basis of examination of intercomponent and intracomponent linkages [29]. This is also supported by spectral analytic evidence in the case of civil engineering systems [29].

The Shannon–Wiener measure of transmitted information (e.g., $H_1 - H_2$) is

$$T = \sum_{i,j} P_{ij} \log_2 \frac{P_{ij}}{p_i p_j}, \qquad i = 1,\cdots,r;\ j = 1,\cdots,c \quad (10)$$

where $p_i$ is the probability of the input event having the $i$th outcome, $P_j$ is the probability of the output event having $j$th outcome, and $p_{ij}$ is the probability of joint outcome $i,j$. Following Miller [14], an alternative way to look at $T$ is to test the hypothesis that $p_{ij} = p_i p_j$ by a likelihood ratio test. The likelihood ratio is

$$\lambda = \frac{\prod_{i=1}^{r} n_i^{n_i} \prod_{j=1}^{c} n_j^{n_j}}{n^n \prod_{i,j} n_{ij}^{n_{ij}}} \quad (11)$$

and $-2 \log \lambda$ has a limiting $\chi^2$ distribution with $(r-1)(c-1)$

[8] We did in fact include both constructed trend and residuals in discriminating the two groups but made no attempt to estimate $H$ from such an analysis. The standardized coefficient of the residual variable in the discriminant function turned out to be 45 times higher than the standardized coefficient of the trend variable! Thus the results of this study indicate the role of random elements is of major importance in the evolution of engineering systems.

TABLE IV

| Group | r | I ($II_1$;$II_2$) | $\chi^2$ | d.o.f |
|---|---|---|---|---|
| 1 | 0.72 | 0.521 | 8.638 | 12 |
| 2 | -0.13 | 0.013 | 17.788 | 12 |

degrees of freedom. Also

$$-2 \log \lambda = 1.3863nT. \tag{12}$$

Thus $1.3863n\hat{T}$ has a chi-square limiting distribution under the null hypothesis that $T = 0$. Chi-square test may also be applied to the individual estimates of $H$ [1]. Letting the null hypothesis be $H_{\max} - \hat{H} = 0$, the test is given by

$$\chi^2 = 1.3863n(H_{\max} - \hat{H}) \tag{13}$$

with degrees of freedom the same as that of $H$, since $H_{\max}$ has $df = 0$. Applications of these tests to estimates in Table III show that they are all insignificant. For example, both groups 1 and 2 contain 13 observations. To test $H_1 - H_2$, the chi-square value is $1.3863 \times 26 \times 0.11$, which is insignificant for 24 degrees of freedom.[9] To test $(H_{\max} - \hat{H}_1)$ and $(H_{\max} - \hat{H}_2)$, the chi-square values are $1.3863 \times 13 \times (1 - 0.3987)$ and $1.3863 \times 13 \times (1 - 0.2844)$. However, both are insignificant for 12 degrees of freedom. The reason for failure to reject the null hypothesis is not only that the samples are small but also that many of the probability values of group membership are close to zero.

We may also wish to compute a "within group $T$." A very simple way to do this is to consider $H(II_2)$ as the uncertainty of the marginal $II_2$ distribution and $H_{II_1}(II_2)$ the uncertainty of the distribution of the residuals. If both are normally distributed [1]:

$$H = \log (2\pi e)^{1/2\sigma}$$

thus

$$\begin{aligned} T(II_1;II_2) &= H(II_2) - H_{II_1}(II_2) \\ &= \log \frac{\sigma_{II_2}}{\sigma_{II_2 II_1}} \\ &= \log \frac{1}{(1 - r^2)^{1/2}}. \end{aligned} \tag{14}$$

The results are presented in Table IV. Looking upon $T(II_1;II_2)$ as an entropy estimate of the group in question, it may be tested, as before, whether $H_{\max} - \hat{H} = 0$. Only in the case of group $II$ is the difference close to significance.

## IV. Conclusions

This study has presented a framework for the conception and measurement of process complexity, i.e., complexity of the invariant relationships governing the behavior of the system under study. Underlying the proposed framework is the thesis that *one way to develop a nontrivial theory of system complexity is by way of a theory of similitude.*

The notion of "overall complexity" is admittedly of limited usefulness. However, the proposed framework permits segregation of complexity into various components: those pertaining to organized, unorganized, short-term, and long-term aspects of any given system's behavior. The proposed formulation based on dimensional analytic theory also permits judicious grouping of variables on *a priori* grounds, thereby alleviating the problem posed by a large number of variables in systems analysis. More important, as supported by the empirical results, the proposed transformation of original system variables into dimensionally invariant functions is capable of circumventing the problem of nonstationarity in the application of the tools of information theory.

A number of findings emerge from this study with important implications for the hitherto neglected theory of evolution of engineering systems. These may be summarized as follows.

1) Complexity tends to be higher in the long-run than in the short-run. This is consistent with the argument that in the evolution of engineering systems easier problems are solved first and more complex, costly problems afterwards.

2) The role of random elements (chance factors) is of major significance in the evolution of engineering systems.

3) In the short-run, the total complexity is almost solely composed of unorganized complexity. Put another way, in the short-run, the determinants of complexity are many and the effect of each quite small. This result is in keeping with the isomorphism connecting information analysis and variance analysis.

4) The long-run redundancy in the measurable aspects of the aircraft design process is estimated in the range of roughly 48 to 60 percent. The corresponding estimates in the state of short-run seem to be much higher and in the range of 71.6 to 98.7 percent. Insofar as greater redundancy may imply a relatively hierarchic structure, the results here suggest that a heirarchic structure may be a prerequisite to the short-term evolution of engineering systems. However, a hierarchic structure may be much less crucial to their long-term evolution. This is in keeping with the result on the role of chance factors. If random elements indeed have a significant influence on the course of evolution, one should not expect hierarchic structures (or for that matter structures of any given type) to remain so forever.

These findings are supported by the results of the author's earlier, unrelated studies [19]–[29]. One implication of these findings is that control of one or two factors in planning the future course of technology is likely to be inadequate (especially in the short-run) in view of the stochastic nature of the evolution of engineering designs. Technological forecontrol is likely to be most effective, however, in an *extremely* short-run or long-term basis. The results also point to the need of probabilistic formulation of "general systems theory" and of technological forecasting models, in particular. Thus a theoretical implication of these findings is that a great majority of the existing models for forecasting and assessment of technology must be rejected as grossly inadequate in view of their total neglect of stochastic aspects of the phenomena they purport to study.

The measures of redundancy provided in this study should be interpreted as follows. Suppose that we delete parts of a representation until the message is lost. Redundancy here is meant to be the maximum amount in the representation that can be safely deleted. Suppose the object being studied is a cube. Beyond the maximum amount deleted, what remains cannot be recognized either as cube or as any other well-defined geometrical configuration. Thus it is also seen that on either side of the point indicating the maximum amount that can be deleted without losing the configuration, complexity increases.

[9] This is contradicted by the Mahalanobis distance estimate which indicates that the two groups are significantly dissimilar (Table I). This is because although both are measures of "relatedness," only $T$ involves taking the logarithms. It should be possible to derive a similar $F$-ratio test based on logarithmic transformation; however, a qualitative justification is lacking. Furthermore, due to the small sample employed in the case study here, it is likely that there is an upward bias in the estimate of the Mahalanobis distance. On the other hand, one may argue that $H_1 - H_2$ is a measure of relatedness (the usual interpretation) in the sense of similarity, while the distance is a measure of dissimilarity, in which case both measures give the same results.

Note that the complexity of the system being analyzed is a function of the conceptual system. Remarks in the case of the former hold as well for the latter. Depending on the conceptual apparatus used, many things may become redundant which otherwise may not be so. As in the case of the system being analyzed, it is possible to delete too much or too little. My attempt in this paper has been to develop a conceptual system in which unnecessary redundancy was hopefully eliminated. However, whether coherency between the conceptual system and the system being analyzed was achieved for certain is not known. The issue of congruence in the measurement of system complexity and in information theory in general is an area requiring considerable further research.

### Acknowledgment

For his many valuable comments, my warm thanks to Professor Robert W. Rempfer, and the usual absolution.

### References

[1] F. Attneave, *Applications of Information Theory to Psychology*. Holt, Rinehart and Winston, 1959.
[2] G. Birkhoff, *Hydrodynamics, A Study in Logic, Fact and Similitude*. Princeton, N.J.: Princeton University Press, 2nd ed., 1960.
[3] B. Chandrasekaran and A. K. Jain, "Independence, measurement complexity and classification performance," *IEEE Transactions on Systems, Man and Cybernetics*, vol. SMC-5, no. 2, pp. 240–244, March 1975.
[4] R. C. Conant, "Detecting subsystems of a complex system," *IEEE Transactions on Systems, Man and Cybernetics*, pp. 550–553, Sept. 1972.
[5] C. W. J. Granger and M. Hatanaka, *Spectral Analysis of Economic Time Series*. Princeton University Press, 1964.
[6] J. A. Gukhman, *Introduction to Theory of Similarity*. Academic Press (transl.), 1965.
[7] H. A. Hayek, "The theory of complex phenomena," in *The Critical Approach to Science and Philosophy, Essays in Honour of Karl R. Popper*, M. Bunge, Ed., pp. 332–349, London, 1964.
[8] J. G. Kemeny, "Two measures of complexity," *The Journal of Philosophy*, pp. 722–733, Nov. 1955.
[9] T. C. Koopmans and O. Reiersol, "The identification of structural characteristics," *The Annals of Mathematical Statistics*, vol. 21, pp. 165–181, 1950.
[10] E. Kuh, "The validity of cross-sectionally estimated behaviour equations in time series applications," *Econometrica*, vol. 27, pp. 197–214, 1959.
[11] D. H. Krantz, P. D. Luce, P. Suppes, and A. Tversky, *Foundations of Measurement*, vol. I, Academic Press, 1971.
[12] W. J. McGill, "Isomorphism in statistical analysis," pp. 56–62 in H. Quastler [17].
[13] J. R. Meyer and E. Kuh, "Correlations and regression estimates when data are ratios," *Econometrica*, vol. 23, pp. 400–416, 1955.
[14] G. A. Miller, "Note on the bias of information estimates," pp. 95–100 in H. Quastler [17].
[15] J. Pietarinen, "Quantitative tools for evaluating scientific systematizations," J. Hintikka and P. Suppes, Eds., in *Information and Inference*, pp. 123–147, D. Reidel Publishing Co., 1970.
[16] A. Phillips, *Technology and Market Structure*. Cambridge: D. C. Heath, 1971.
[17] H. Quastler, Ed., *Information Theory in Psychology*, Proceedings of a Conference on Information Flow, Free Press, 1955.
[18] C. R. Rao, *Advanced Statistical Methods in Biometric Research*. Wiley, 1952.
[19] D. Sahal, "Generalized Poisson and related models of technological innovation," *Technological Forecasting and Social Change*, vol. 6, no. 4, pp. 403–436, 1974.
[20] ——, "A generalized logistic model for technological forecasting," *Technological Forecasting and Social Change*, vol. 7, no. 1, pp. 81–95, 1975.
[21] ——, "Cross impact analysis and prediction of technological developments: A case study of farm tractors," *IEEE Transactions on Engineering Management*, vol. EM-22, no. 2, pp. 76–79, 1975.
[22] ——, "A reformulation of technological progress function," *Technological Forecasting and Social Change*, vol. 8, no. 1, 1975.
[23] ——, "A spectral analytic study of some postulates concerning engineering design and related processes," *Technological Forecasting and Social Change*, vol. 8, no. 3, 1975.
[24] ——, "A stochastic learning model of machine design changes," Report 74-12, Systems Science, Portland State University, 1974.
[25] ——, "On the conception and measurement of trade-off in the evolution of engineering systems, a case study of aircraft design process," *Technological Forecasting and Social Change*, vol. 8, 1975 (in press).
[26] ——, "On the conception and measurement of technology, a case study of aircraft design process," *Technological Forecasting and Social Change*, vol. 8, 1975 (in press).
[27] ——, "Another measure of technology: The generalized Mahalanobis distance, a discriminant analytic framework for technology assessment," *Technological Forecasting and Social Change*, vol. 9, 1976 (in press).
[28] ——, "Evolving parameter models of technology assessment, *Technology Assessment*, 1976 (in press).
[29] ——, "Technological systems: An exploration in the dynamical aspects of theory of hierarchical structures," *General Systems Yearbook*, vol. xx, 1975.
[30] ——, "From dimensional analytic theory to a stochastic systems theory," submitted for publication.
[31] C. E. Shanon and W. Weaver, *The Mathematical Theory of Communication*. Urbana: The University of Illinois Press, 1964.
[32] H. A. Simon, "The architecture of complexity," *Proceedings of the American Philosophical Society*, vol. 106, no. 6, 1962.
[33] J. Von Neumann, *The Theory of Self-replicating Systems*. Urbana: University of Illinois Press, 1966.
[34] P. Zarembka, "Transformation of variables in econometrics," P. Zarembka, Ed., in *Frontiers in Econometrics*, Academic Press, 1974.

## Effect of Intraclass Correlation on Confidence Coefficients of Confidence Sets Based on Chi-Square Statistics

REKHA BASU, J. P. BASU, AND T. O. LEWIS

***Abstract*—This paper investigates how the presence of simple equicorrelation in a multivariate normal sample affects the confidence coefficients of confidence sets based on chi-square statistics and constructed on the assumption of independence of the sample for the mean when the dispersion matrix is known and the scalar $\sigma^2$ in $\sigma^2\Sigma$ when $\Sigma$ is known.**

### I. Introduction

In remote sensing data analysis, as in other areas of statistical data analysis, confidence interval procedures or significance testing procedures are usually derived on the assumption that the observations in the sample are independently and identically distributed normal vectors. In fact, even when the observations are identically distributed normal vectors, they are at most equicorrelated, but rarely independent. Dr. William Coberly [1] analyzed some remote sensing data from Earth Resources Technology Satellite (ERTS-1, renamed Landsat) to determine the validity of the assumption of independence of observation vectors in a sample. He discovered that the observations were significantly correlated. Thus, in many instances, it would be more rational to assume the sample to be equicorrelated or simply equicorrelated, that is, all pairs of observations have the same covariance, rather than to assume the sample to be independent.

Walsh [2] has shown how the presence of intraclass correlation in univariate normal samples affects the confidence coefficients of some confidence intervals (or equivalently the significance level of some tests of significance). Basu, Odell, and Lewis [3] have shown how the presence of simple equicorrelation in multivariate normal samples affects the confidence coefficients of the confidence sets based on a $T^2$-statistic for the mean of a single population and the difference of means of two populations. In this correspondence, it is shown how the presence of intraclass correlation (simple equicorrelation) in multivariate normal samples affects the confidence coefficients of the confidence sets based on a chi-square statistic for the mean of a population with known dispersion matrix and the scalar $\sigma^2$ in $\sigma^2\Sigma$ of the dispersion matrix when $\Sigma$ is known.

The above procedures are often used for deciding whether a sample has come from a prescribed population. In remote sensing data analysis, photointerpreters label areas as belonging to different crops. The samples of observations coming from those areas are used in training the classifier for automatic

Manuscript received June 13, 1975; revised December 17, 1975.
R. Basu is with the University of Houston, Houston, TX 77004.
J. P. Basu is with the Lockheed Electronics Co., Inc., Houston, TX 77058.
T. O. Lewis is with Texas Tech University, Lubbock, TX 79409.