between the fine and coarse regions, this ratio improves only slightly, to about 1.7.

Incidentally, the best-size computation seems to be a good preprocessing operation for coarseness edge detection. Horizontal edge values for Fig. 4, using size 64 by 64, are shown in Fig. 5. (these values are multiplied by 8 for greater visibility). The coarseness discontinuity is sharply detected.

Next, spot values were computed for Fig. 1 using sizes 5, 9, 17, and 33, where the spot value S_k of size 2^k at a given point is defined as follows. Let A_k be the sum of the gray levels in a $2^{k} + 1$ by $2^{k} + 1$ square centered at the given point. Then

$$S_k = c \left| \frac{A_k}{(2^k + 1)^2} - \frac{A_{k+1}}{(2^{k+1} + 1)^2} \right|$$

where c is a constant scale factor (c = 3.1 in the examples). These spot values for Fig. 1 are shown in Fig. 6. The best sizes are shown in Fig. 7. For upper and lower portions of Fig. 1, each 44 rows by 152 columns, the numbers of points having each best size are

Size	5	9	17	33
Upper	3099	2380	977	232
Lower	1843	2030	1749	1066

yielding average best sizes of 8.1 and 12.8, respectively, for a ratio of about 1.6, still about the same as obtained earlier. The 64 by 64 horizontal edges in Fig. 7 are shown in Fig. 8; the results are not as good as in Fig. 5.

Finally, spot values were computed using sizes 5, 9, 13, 17, 21, 25, 29, and 33; this yielded the following numbers of points having each best size.

Size	5	9	13	17	21	25	29	33	
Upper	2805	2039	845	470	250	110	93	76	
Lower	1652	1537	1067	792	523	538	304	275	

The average best sizes are 8.65 and 12.7, slightly worse than before, but still about 1.5 in ratio.

For comparison purposes, Fig. 9 (taken from [5]) shows the results of applying the coarseness edge detection method of [3]-[5] to Fig. 1. A local property was first computed at every point; this was a threshold "gradient" defined by

$$g(x,y) = 1, \quad \text{if } |x_{ij} - x_{i+1,j+1}| + |x_{i+1,j} - x_{i,j+1}| > t$$

= 0, otherwise

where the x are gray levels. Horizontal 64 by 64 edges were then found in the resulting picture. As Fig. 9 shows, similar results, not as good as those in Figs. 5 or 8, are obtained in this way for several values of t.

It should be mentioned that the edge detection schemes described in [3] and [4] were implemented in PAX [8], and were very costly in computer time; a typical run on a 108 by 108 point picture required several minutes of Univac 1108 time. The implementation described in [5] is faster by about two orders of magnitude. For more information about the improved implementation see [9].

IV. CONCLUSION

The results obtained are disappointing and leave many questions unanswered. Since the actual scale ratio is 2.3:1, why do all the methods tried here consistently yield ratios of about 1.6: 1? Is it possible that there is an inherent bias in using discrete detector sizes rather than a more continuous range of sizes? Would the results be different if digitally rescaled images were used rather than images that were photographed at different scales and then independently digitized?¹ The authors hope that publication of the present correspondence will lead to further investigation of the coarseness measurement problem. and hopefully, to the development of more satisfactory measures.

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¹ These questions were raised by one of the referees, whose detailed comments on these and other points led to several major changes in this correspondence.

Fuzzy Chains

ABRAHAM KANDEL AND LAWRENCE YELOWITZ

Abstract-Motivated by the ineffectiveness of classical mathematical techniques in dealing with imprecision in some real life systems, an investigation is made of fuzzy chains from x_1 to x_t , which are simply sequences of elements of the fuzzy set X. A suitable notation is used to represent a primitive connection matrix, and a procedure is given to convert this matrix to the fuzzy transmission matrix for the system. This procedure is a generalization to fuzzy algebra of a procedure to compute the transitive closure of a binary matrix, and it is very efficient, involving only a single scan over the matrix. A proof of correctness of the procedure is given. It should be noted that the imprecision involved stems not from randomness but from a lack of sharp transition from membership in a class to nonmembership in it. Various properties of the matrices involved in such representations are investigated and illustrated.

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Fig. 1. Graph G_f corresponding to fuzzy system.

I. INTRODUCTION

Ever since Zadeh [1] introduced the idea of fuzzy set theory by utilizing the concept of membership grade, a number of researchers have been concerned with the properties and applications of fuzzy sets, [2]-[26].

Essentially, fuzziness is a representation of imprecision that stems from a grouping of elements into classes that do not have sharply defined boundaries. Since certain aspects of reality always escape most mathematical models, the strictly binary approach to the treatment of physical phenomena is not always adequate to describe systems in the real world. Real world constraints, such as complexity, ill defined situations, and transition states are reflected upon the various attributes of our models.

Because of these constraints the attributes of the system variables often emerge from an elusive fuzziness, a readjustment to context, or an effect of human imprecision, as usually appears in modeling of "soft" sciences, such as sociology, psychology, natural languages, and pattern description.

Since systems that are either ill-defined or describe transitional behavior do not have a precise quantitative analysis, some graphical approach to represent these systems is needed. It is in this sense that fuzzy logic analysis, through the use of fuzzy chains, might enable us to process decision relevant information by using approximate relations to a primary set of precise data. This approach might be of use in areas such as decision processes linguistics, sequential systems analysis, system modeling approximation, and many more. Some problem oriented examples, which we made no attempt in the present correspondence to investigate are 1) graph representation of combinational and sequential systems during transition, namely, investigation of hazards by means of fuzzy chains; 2) classification of patterns and cluster analysis through the description of fuzzy matrices and graphs; 3) approximation of ill-defined transport networks and maximal matching systems by means of fuzzy representations of chains.

II. FUZZY CHAINS

Fuzzy algebra completely specifies the performance of a fuzzy system with *n*-input terminals x_1, \dots, x_n and a single output terminal, where the fuzzy function f is represented by

$$f(x_1,\cdots,x_n) = \xi.$$

Consider, for example, the two-terminal fuzzy system of Fig. 1. The grade membership of the edges of this system are considered as fuzzy functions. It is quite clear that the set of fuzzy n-variable functions is closed under the operations of union, intersection, and complement and that this set forms a distributive lattice. Thus the fuzzy system may be considered an undirected finite graph, the edges of which are designated by the generators of the distributive lattice.

Definition 1: If Φ is a two-terminal fuzzy system constructed from edge-type elements x_1, x_2, \dots, x_n , then the *fuzzy transmission* function (FTF) of Φ , F_{Φ} , is defined as the union of the closed chains between the terminals of the network. For the system of

$$\rho = \frac{A}{D} \begin{bmatrix} A & B & C & D \\ 1 & 0 & y & x \\ 0 & 1 & w & z \\ y & w & 1 & \bar{x} \\ x & z & \bar{x} & 1 \end{bmatrix}$$

Fig. 2. Primitive connection matrix of Fig. 1.

Fig. 1, the FTF of Φ is given by

$$F_{\Phi} = xz + x\bar{x}w + yw + \bar{x}yz$$

when concatenation represents min and + represents max operations. Formally, the FTF of a two-terminal fuzzy chain network is obtained as follows.

- 1) Determine all irredundant input-output chains.
- 2) For each chain in 1) form the intersection of the corresponding edge grade-memberships in order to obtain the grade-membership of the chain.
- 3) Form the union of all chain grade-memberships obtained in 2).

Definition 2: The dual of a fuzzy function F, written F^{D} . is inductively defined as follows.

- 1) If $F = f_j$, then $F^D = f_j$, for $j = 1, 2, \dots, n$. 2) If A, B, and C are fuzzy functions and A = B + C, then $A^D = B^D C^D.$
- 3) If A, B, and C are fuzzy functions and A = BC, then $A^D = B^D + C^D.$
- 4) If A and B are fuzzy functions and $A = \overline{B}$, then $A^{D} = (\overline{B^{D}})$.

The relation between a fuzzy function F and its dual F^{D} is given by the following theorem.

Theorem 1 [23]: If F is a fuzzy function constructed from x_i , for $i = 1, \dots, n$, and F^D is its dual written as $F^D(x_1, \dots, x_n)$, then $F^{D}(x_{1}, \dots, x_{n}) = \overline{F}(\overline{x}_{1}, \dots, \overline{x}_{n}).$ Corollary 1: If F_{1} and F_{2} are fuzzy functions, and $F_{1} \equiv F_{2}$,

then $F_1^D \equiv F_2^D$.

We say that a fuzzy function F_A is self-dual if and only if $F_A = F_A^{D}$. Thus a self-dual expression for the FTF of Φ can be obtained as follows.

- 1) Determine all minimal cut-sets separating the two terminals.
- 2) For each cut-set in 1), form the union of the corresponding edge grade-memberships to obtain the grade membership of the cut-set.
- 3) Form the intersection of all cut-set grade-membership obtained in 2).

The self-dual expression for Fig. 1 is, therefore, $F_{\Phi} = (x + y)$. $(\bar{x} + y + z)(x + \bar{x} + w)(z + w)$. This expression can be derived from the previous one by the absorption law and the distributive law.

We can also derive the FTF of Φ by a suitable fuzzy matrix theory. Fig. 2 shows the primitive connection matrix ρ corresponding to Fig. 1. To completely analyze the fuzzy system, one would desire a $k \times k$ matrix of which the *ij* entry is the fuzzy transmission function of the system with terminals *i* and *j*. This suggests the definition of a fuzzy transmission matrix and the examination of some properties of fuzzy matrices.

In [5] and [26], relation matrices have been discussed and several examples of the relation matrices of some similarity relations have been demonstrated. It is clear that the graph representation of a fuzzy system bears a similarity to the relation matrix discussed by Zadeh and applied by him to the investigation of fuzzy algorithms. It is claimed that the conceptual framework developed in [26] best describes the systems that are too complex or too ill-defined to admit of precise quantitative analysis. It should be noted, however, that there exists a very important source of imprecision in systems, and this is *transition behavior*. Transition of a system or of a specific element in a system can be best described and analyzed by means of a fuzzy description. This is true for hazard detection in combinational systems [24] or for transition models in such fields as economics, management sciences, artificial intellegence, physics, and linguistics. The fuzzy transition is best represented by a fuzzy chain or a fuzzy path on the graph representing the system.

Let x_1, x_2, \dots, x_k be k points in the fuzzy set X with $\mu(x_i, x_j)$ being the grade-membership describing the transition from x_i to x_j , $1 \le i, j \le k$. A sequence $S = (x_r, \dots, x_t)$ will be said to be a fuzzy chain from x_r to x_t , where $1 \le r, t \le k$, and it is said to have the strength of its weakest link.

For any primitive connection matrix ρ we define the characteristic fuzzy matrix or fuzzy transmission matrix $\psi(\rho) = [x_{ij}]$ such that x_{ij} is the fuzzy transmission function of the twoterminal system connecting vertex *i* to *j*. It is clear that $\psi(\rho)$ is a symmetric matrix, since the graph is an undirected one, and thus $x_{ij} = x_{ji}$, for all *i*, *j* and $x_{ii} = 1$, for all *i*.

Theorem 2: Let ρ be a square fuzzy transmission matrix of order *n*. Then there exists an integer $q \le n - 1$ such that $\rho^q = \rho^{q+1} = \cdots = \psi(\rho)$.

Proof: Let
$$\rho = [p_{ij}]$$
. The *ij* entry of ρ^2 is

$$\sum_{k=1}^{n} p_{ik} p_{kj}$$

and this term has the grade-membership of

$$\max_{k} \left[\min \left(p_{ik}, p_{kj} \right) \right]$$

iff there is a direct path between vertices *i* and *j*, or there is a path from *i* to *j* through one intermediate vertex. Extending this argument to ρ^i it is clear that no path requires more than n - 2 intermediate vertices, since there are only *n* vertices, and internal loops are excluded. Hence, the *ij* entry of ρ^{n-1} has the grademembership of

$$\max_{\text{subterms}} \{ ij \text{ terms of } \rho^{n-1} \}$$

iff *i* and *j* are connected, namely, $\rho^{n-1} = \psi(\rho)$. Q.E.D.

Based on these results $\psi(\rho)$ can be computed by successive multiplication of ρ .

The repeated matrix multiplication makes it unattractive from an efficiency viewpoint. Algorithm 1 achieves the same result and requires only a single scan over the matrix. In fact, Algorithm 1 works correctly on a wider range of input, since it is not required that the diagonal elements of the input matrix equal one. Algorithm 1 is an extension to fuzzy logic of an algorithm of Warshall [28] to compute the transitive closure of a binary matrix.

Algorithm 1:

- 1) Label all vertices by the integers $1, \dots, N$.
- 2) Construct the primitive connection matrix ρ the *ij* entry of which denotes the fuzzy transmission function of the two-terminal fuzzy system connecting vertices *i* and *j* through a direct chain.
- 3) DO K = 1 TO N
- 4) DO I = 1 TO N
- 5) if $\rho(I,K) \neq 0$ then

- 6) DO J = 1 TO N7) $\rho(I,J) = \max(\rho(I,J), \min(\rho(I,K), \rho(K,J)))$
- 8) END
- 9) END
- 10) END

The basic idea is to scan down *column K*, and for each nonzero element encountered (e.g., in row *I*), each element in *row I* (e.g., element $\rho(I,J)$) is possibly improved by comparing $\rho(I,J)$ to min ($\rho(I,K)$, $\rho(K,J)$). A rigorous proof of correctness is achieved by attaching the following inductive assertion A [27] between statements 7) and 8):

$$\rho(I,J) = M(I,J,K)$$

where $M(I,J,K) \triangleq \max \{ \min (\text{all chains from } I \text{ to } J \text{ such that } each$ *intermediate* $element has a label <math>\leq K \} \}$.

Before proving that assertion A is true whenever control leaves step 7), it is noted that the relation $\rho(I,J) = M(I,J,N)$ is the desired relation at the termination of the algorithm, since $M(I,J,N) = \max \{\min (\text{all chains from } I \text{ to } J)\}$. Assertion A is proved by induction on K.

K = 1. The first time A is reached, K has the value one, and path analysis [27] shows that ρ(I,J) = max (ρ₀(I,J), min (ρ₀(I,1), ρ₀(1,K))), where ρ₀ represents the original matrix and the right side of the equation equals M(I,J,1).
 Assume ρ(I,J) = M(I,J,K), 1 ≤ K < N. Show ρ(I,J) = M(I, J, K + 1).

There are two subcases to consider. If M(I, J, K + 1) does not involve element K + 1, then no change is made to the matrix and the desired result is true. If M(I, J, K + 1) does involve element K + 1, then we can guarantee that element K + 1 appears only once, since loops do not increase the max of any chain.

Thus we can break the optimal chain into two subchains $\rho(I, K + 1)$ and $\rho(K + 1, J)$. Since both subchains involve intermediate elements numbered $\leq K$, the inductive hypothesis applies to each subchain and the desired result follows.

It is interesting to note that during the process of computing the characteristic fuzzy matrix, minimization of the fuzzy structures are possible. In general, one can not apply the identities $x \cdot \overline{x} = 0$ and $x + \overline{x} = 1$ to fuzzy expressions, and thus binary techniques of minimization are insufficient. Therefore, more specific methods, directed toward the minimization of fuzzy functions, should be used.

The first author has presented [25] a novel method for the minimization of fuzzy functions by extending the concepts of prime implicants and consensus to fuzzy logic. In [25] an algorithm that generates all the fuzzy prime implicants is introduced, and a proof of completeness of the algorithm is given. The minimization technique takes into consideration the refinement of the classical map approach and the properties of fuzzy consensus in the context of fuzzy logic. It is recommended to implement the technique described in [25] for the derivation of the simplified characteristic fuzzy matrix $\psi(\rho)$.

The characteristic fuzzy matrix represents a mean by which the analysis of any finite fuzzy system can be obtained. The analysis technique that has been given is quite general, and the use of matrix techniques leads to efficient computations, particularly in the description of fuzzy sequential procedures such as decision-making and procedures involving sequences of imprecise operations, which can be best represented by graphs and fuzzy chains.

III. CONCLUSION

The applicability of fuzzy algebra to the study of fuzzy chains has been introduced. Program correctness techniques were used to certify the main algorithm.

The main contribution of this note consists of two parts. First, a new conceptual framework for the study of fuzzy systems is provided, facilitating the derivation and stimulating the discovery of various results in applied areas. Second, a proof of correctness of the main algorithm is given. This technique for certifying algorithms shows conclusively that no errors exist, in contrast to the usual technique of testing, which can only show that no errors have been found in a certain number of trial runs [27].

Several problem-oriented examples have been mentioned in the introduction, and it is our hope that the interested reader will be able to find many more applications in his field of interest.

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An Algorithm for Spoken Sentence Recognition and Its Application to the Speech Input-Output System

KATSUHIKO SHIRAI AND HIROMICHI FUJISAWA

Abstract-An algorithm for spoken sentence recognition is described. The problem of sentence recognition is mathematically formulated as an optimization problem with the constraint of sentence structure. It is solved by a dynamic programming technique. The algorithm presented has advantages not only in that the solution is optimal in the Bayesian sense, but also that the effective number of words that affects the recognition score is reduced, the end of a sentence is automatically detected, and a sentence that is logically invalid can be rejected. The algorithm was applied to a practical situation as a speech command recognition and vocal response system. It recognizes speech command sentences and responds in voice to the operator. The vocabulary of conversation between the operator and the machine is limited, but flexibility in the conversational style is allowed. The system that was built utilizes a minicomputer with an eight kiloword memory capacity, a hardware feature extractor for speech recognition, and a hardware speech synthesizer for vocal responses. If a larger computer is available, the system can be enlarged with only minor modifications.

I. INTRODUCTION

Many speech pattern recognition systems have been designed to classify spoken words [1]-[4], but few have been designed so as to treat spoken sentences. Strictly speaking, it is difficult to define what is recognition of a sentence or what is understanding of meaning. However, unless the system responds to a sentence or changes its internal state according to the meaning of the input sentence, it cannot be said that it recognizes the meaning. That is to say, a sentence recognition system is required to be more than a simple classification machine.

In this correspondence a method is presented for the design of a system that recognizes spoken sentences, makes vocal responses, and changes the related state. This method was applied to a conversational system, the Speech Input-Output System (SPIO) of the robot called WABOT-1 (Waseda Robot) [5]. It accepts Japanese spoken command sentences, which are strings of separately spoken words, responds to the meaning of the command in speech, and makes the robot move as commanded.

One of the most important factors in the design of such a system is that the machine and the operator have a common recognition of the situation or the scene that is talked about between them. Therefore, the concept of situation is introduced in terms of "states" as in automata. The state makes a transition after the recognition of an input sentence and simultaneously makes an output. Probable sentences that may appear under a state are limited, and thus the effective number of words (sentences) that affects the recognition score is reduced. Further words in a sentence should be ordered in a restricted way, which is not necessarily grammatical. This is conveniently taken into account by the concept of sentence structure.

In a practical application, the purpose and the ability of the machine is always limited, and the contents of conversation can be finite. It follows that the problem of sentence recognition can be considered on the extension of a classification problem.

Another difficult problem lies in the recognition of the naturally spoken sentences [6]. This stems from the fact that they are continuous and the segmentation becomes necessary. In the

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