over, it can be proved that μ' is the completely additive set function defined on the sets of $\bigcup_{n=1}^{\infty} \mathscr{F}_n$. (Prof. S. Watanabe suggested this fact.) Therefore, by Hopf's extension theorem, μ' can be uniquely extended to the measure on $\mathscr{F}_\infty$. ($\mathscr{F}_\infty$ is the least Borel field including $\bigcup_{n=1}^{\infty} \mathscr{F}_n$.) Let the extended measure of μ' be μ. Concerning the integrals of $p_\infty{}^i(i = 1,2)$ and $p_i(n)$, $i = 1,\cdots,r$, we consider the measure μ defined on the sets of Borel field $\mathscr{F}_\infty$.

ACKNOWLEDGMENT

We would like to thank Prof. K. Okugawa, Prof. S. Watanabe, and Prof. H. Akashi for their kind advice.

REFERENCES

[1] I. J. Shapiro and K. S. Narendra, "Use of stochastic automata for parameter self-optimization with multimodal performance criteria," *IEEE Trans. Syst. Sci. Cybern.*, vol. SSC-5, pp. 352–360, Oct. 1969.
[2] R. Viswanathan and K. S. Narendra, "A note on the linear reinforcement scheme for variable-structure stochastic automata," *IEEE Trans. Syst., Man, Cybern.* (Corresp.), vol. SMC-2, pp. 292–294, Apr. 1972.
[3] M. F. Norman, "On the linear model with two absorbing barriers," *J. Math. Psychol.*, vol. 5, pp. 225–241, June 1968.
[4] ——, "Some convergence theorems for stochastic learning models with distance diminishing operators," *J. Math. Psychol.*, vol. 5, pp. 61–101, June 1968.
[5] J. L. Doob, *Stochastic Processes.* New York: Wiley, 1953.

Correction to "A Model-Driven Question-Answering System for Mixed-Initiative Computer-Assisted Construction"

JOHN S. BROWN, RICHARD R. BURTON, AND FRANK ZDYBEL

In the above paper[1] the title should have read "A Model-Driven Question-Answering System for Mixed-Initiative Computer-Assisted Instruction."

The authors are with the University of California, Irvine, Calif. 92664.
[1] J. S. Brown, R. R. Burton, and F. Zdybel, *IEEE Trans. Syst., Man., Cybern.*, vol. SMC-3, pp. 248–257, May 1973.

Book Reviews

Perturbation Methods in Applied Mathematics—Julian D. Cole (Waltham, Mass., Blaisdell, 1968, 260 pp.). *Reviewed by Lawrence E. Levine, Department of Mathematics, Stevens Institute of Technology, Hoboken, N.J. 07030.*

The use of perturbation procedures to find approximate solutions to nonlinear problems is a well-known phenomenon amongst applied mathematicians, physicists, and engineers, and several of these techniques go back at least as far as Poincaré. However, in recent years, renewed interest in procedures for solving equations involving a small parameter, say ε, has been generated, since many physical problems lead to so-called singular perturbation problems, i.e., problems in which the straightforward approach of assuming an asymptotic expansion of the solution in terms of ε and the independent variables does not yield uniformly valid results. The two most powerful techniques developed thus far for handling such problems is the method of matched asymptotic expansions and Lighthill's method of strained coordinates.

Although from the title of this volume one would expect a discussion of both of these methods as well as the more classical techniques for dealing with regular perturbation problems, the reader quickly finds that the text deals mainly with the matching procedure. In fact, aside from a brief treatment of a spring-mass-damping system for the case when the damping coefficient is small in comparison to the mass and spring constant (pp. 4–7), there is no further mention of regular perturbation problems. (Incidentally, this problem itself is not really a regular one since a perturbation solution for all time $t \geq 0$ requires the use of so-called multivariable expansions to avoid the occurrence of secular terms.) This is really a major drawback of the book since the unprepared reader will find it almost impossible either to understand fully or appreciate the power and scope of the matching procedures. This certainly turned out to be the case for the students in a perturbations course which the reviewer has taught twice in the past two years. In fact, almost all these students (some graduate and some undergradute) found the text highly unreadable.

The author could have easily remedied this shortcoming by including an introductory chapter in which several physical examples on the order of those given by M. Van Dyke in Chapters 1 and 2 of *Perturbation Methods in Fluid Mechanics* (New York: Academic Press, 1964) and a few mathematical problems such as those discussed by R. Bellman in Part 1 of *Perturbation Techniques in Mathematics, Physics, and Engineering* (New York: Holt, Rinehart and Winston, 1964). Otherwise, the reader should be warned in the preface of prerequisites tacitly assumed and also of the somewhat limited scope of the book.

The text is divided into five chapters. Chapter 1 is a much too brief introduction to ordering and asymptotic sequences and expansions. Chapter 2 is concerned with singular perturbation problems for ordinary differential equations. The approach, as we have mentioned earlier, is via the method of matched asymptotic expansions in which inner and outer expansions are obtained in two different regions and then matched in an overlap region. The technique is first introduced by studying the linear spring-mass-damping system, which was also used as an introduction to regular perturbation problems, as was mentioned previously, but this time for the case where the mass is small in comparison to the spring constant and the damping coefficient. Despite the statement in the Preface that "physical reasoning is often used as an aid to understanding various problems" and the inclusion of a brief discussion of such physical reasoning, this reviewer feels that in this and other parts of the book, the attempts at giving insight and motivation are far too skimpy. In particular, the intuitive reasoning for obtaining inner and outer expansions of the solution and then matching is much more clearly presented by H. Ashley and M. T. Landahl in Chapter 3 of their book *Aerodynamics of Wings and Bodies* (Reading, Mass.: Addison-Wesley, 1965). With this example as a model, the rest of Chapter 2 deals with several other singular problems including second-order equations with variable coefficients, relaxation oscillations of the van der Pol oscillator, as well as two singular boundary value problems and a singular problem for a fourth-order equation arising in the theory of elastic beams.

Chapter 3 is entitled "Two-Variable Expansion Procedures" and deals with a class of problems for ordinary differential equations for which the (regular) straightforward perturbation technique leads to secular terms and hence an expansion which is not valid for large time. (Strictly speaking, these problems are not singular.) The approach, which has been developed by the author and others, is to consider the solution as a function of two independent time variables and seek an expansion in terms of these. Aside from the fact that the reasoning behind the choice of the particular variables for each problem is not made explicit, this chapter provides several very interesting examples of the use of the two-variable procedure. These include the linear oscillator with small damping and with cubic damping, the approach to a limit cycle of the van der Pol oscillator, the Mathieu equation, and others.

Singular perturbation problems are again discussed in Chapter 4, but this time for various problems involving second-order partial differential equations. The first section gives a rather exhaustive discussion of the different methods used to handle elliptic, hyperbolic, and parabolic equations with constant coefficients. In order to fully understand this discussion the reader needs to be familiar with the theory of boundary value problems, initial value problems, and characteristics. The rest of this chapter presents examples which utilize and serve to illustrate the discussion of the first section. Since the physical problems treated are fluid mechanical in nature, the reader also needs some background in fluid dynamics. (This prerequisite is indicated in the Preface.)

The concluding chapter presents several examples "... in which the main aim is the derivation of approximate equations." These approximate equations are actually the equations satisfied by the first- and second-order terms in the asymptotic expansion. In the treatment of thin airfoil theory at various Mach numbers, several parameters occur between which it is necessary to specify a relation so that a distinguished limit results and the expansion can be determined. This involves holding fixed one parameter in the limit. In the other problems discussed, namely the piston problem of acoustics (compression problem), and small-amplitude waves in shallow water, it is the far-field conditions which provide the distinguished limit.

Thus one sees that the book treats certain classes of problems in perturbation theory very thoroughly and actually brings together for the first time many examples in which these procedures are applicable. This alone is certainly of considerable value. However, this is also a drawback, since the book is not suitable as a text either for a course or for self-education due to the omission of many important topics, such as regular perturbations and Lighthill's method of strained coordinates. As a result, the book can be recommended as a somewhat difficult-to-read reference for those with sufficient background in perturbation theory and fluid dynamics, but should probably not be used as a classroom text.

Digital Computer Process Control—C. L. Smith (Scranton, Pa.: Intext Educational, 1972, 289 pp.). *Reviewed by K. Carter, Union Carbide Corporation, Bound Brook, N.J. 08805.*

This is a well-written book showing an awareness of the needs of industry. It will be valuable to both the beginner and the more advanced industrial control engineer. A description of the many aspects of computer control is given, from the role of the computer to the mathematics of advanced control techniques.

Of particular importance is the chapter describing the hardware and executive software of computer control systems. The understanding of this area is extremely important in any computer project, yet it is overlooked by the majority of texts. The book fills this gap by giving an excellent concise description of the interrelationships of the many components that are involved in a computer system. It would have been ideal if the author had included a section on staffing and implementing a project. As it is, it will be invaluable for an engineer specifying his first system.

Various advanced control techniques which are currently used in industry, such as multivariable, adaptive, and feedforward control, are elucidated in such a manner that the principles involved are readily understood. Adequate examples are given to ease the path from theory to practice for a nonmathematician. In the chapter on control algorithms, a section is devoted to the techniques of tuning, and a mention is made of the importance of using the correct sampling interval.

Other chapters describe with clarity the mathematics of sampled-data systems, on-line identification techniques, and frequency domain considerations. These provide a background for the aforementioned control techniques. A chapter is also devoted to the requirements for optimal control. Although few instances are known of economic applications of optimal control in industry, the author believes it will be an important tool in the future.

In conclusion, this book would be a useful addition to the shelves of a practicing control engineer and would also be suitable for use as a lecture guide.

Probabilistic Programming—S. Vajda (New York: Academic Press, 1972, 127 pp.). *Reviewed by A. P. Bonaert, Instituto Tecnologico, Monterrey, Mexico.*

This book is a survey of properties and different methods to solve the following type of problems: an objective function $c'x$ minimized subject to $Ax \geq b$ and $x \geq 0$ (with b,c,x, as vectors, A as a matrix, and $'$ denoting the transpose), when the elements (or some of them) of A,b,c are linear functions of a set of random variables (represented by a vector t).

The first chapter discusses the convexity and polyhedralicity of two types of sets: 1) the one formed by the values of t yielding a feasible (optimal) solution to the aforementioned problem for a given value of x, and 2) the other formed by the values of x yielding a feasible (optimal) solution for a given value of t. Properties of the intersection of these sets as well as inequalities over the minimum of the objective function are presented when respectively x or t varies.

The determination of decision regions (values of t making a basis feasible and optimal for a given objective function) as well as the influence upon them of the probability density functions of t are considered.

The second chapter (roughly half of the book) deals mainly with the so-called two-stage program and some of its extensions to solve stochastic programs. They are formulated as

$$\min_x E_c(c'x + E_{A,B,b}(\min_y d'y), \qquad \text{subject to } Ax + By = b \text{ and } x, y \geq 0$$

where E is the expectation operator with respect to its subscripts, B is a matrix, and y and d are vectors.

The following cases are treated successively: 1) b random (rather than in detail); 2) A,b random; and 3) A,B,b random (fairly superficially).

The determination of upper and lower bounds to the objective function is also examined (including the case when c is random). The last chapter (roughly a quarter of the book and too short for the wealth of topics it covers) deals with another approach to stochastic programs—the use of chance constraints, such as

$$P(a_i x \geq b_i) \leq z_i \text{ or } P(a_i x \geq b_i) \leq z_i$$

(z_i being a scalar between 0 and 1; and a_i being the ith row of A and b_i an element of b). The possibility to transform such constraints into quantile rules is considered.

Similarly, deterministic equivalents to problems formulated with chance objective are treated (P and V model). One of the cases treated deals with the

$$\max P(\min (c'x + d'y \leq k)), \qquad \text{with } x \text{ subject to } Ax + By = b \text{ and } x, y \geq 0$$