

## Book Reviews

**Optimal Filtering**—Brian D. O. Anderson and John B. Moore (Englewood Cliffs, NJ: Prentice-Hall, 1979, 357 pp.). *Reviewed by Mansour Eslami, Department of Electrical Engineering, State University of New York, Stony Brook, NY 11790.*

Nowadays the applications of Kalman filtering are not limited only to some advanced space technology problems and/or to some other communication signal analysis. In fact, the ready availability of extensive computing facilities (on-line or off-line), enhances the application and/or implementation of the Kalman filter such that many nonengineering systems problems utilize this “newly” developed “tool” to construct from data resulting from a measurement on a noisy corrupted environment, an estimation of the state of the system used in designing control of the dynamics of the corresponding encountered system. Correspondingly, extensive research and numerous papers and books on Kalman filtering and its applications show that the Kalman filter is used not only by engineers, but by a wider category that includes several different kinds of applied sciences. This extended application of the Kalman filter is the reason that the appearance of this book is of importance.

The level and the goals of the authors of the present book are well evidenced in the following quotation from the two-page preface, “This book is a graduate-level text which goes beyond and augments the undergraduate experience engineering students might have in signal processing; particularly, communication systems and digital filtering theory. The material covered in this book is written for students in the fields of control and communications and relevant to students in such diverse areas as statistics, economics, bioengineering, and operations research. The subject matter requires the student to work with linear system theory results and elementary concepts in stochastic processes which are generally assumed at graduate-level. However, this book is appropriate at the senior-year undergraduate level for students with background in these areas.”

The reviewer believes that, indeed, the covered theory is truly well documented, and the subject matter is very important to students in the various interdisciplinary areas of engineering and the social sciences. Further, it is the reviewer's thinking that a major contribution of the present textbook is to discuss and/or to present the theory and applications of Kalman filtering in a unified and organized fashion which enfold major contributions by a substantial number of researchers of this topic. This presentation is such that it renders learning “fun” for beginners, and extends the comprehensions of those who have a previous knowledge of the subject. Among the excellent features of the book are the several illustrative examples given in each of its many sections. These include some “real world” engineering problems: a pollution estimation, a stream-flow model identification, demodulation and detection schemes, determination of power-system states, aerospace industry applications, and many others. These examples clarify and well-illustrate the scope of the applications and the meaning of the given theory though there is room for more analytical examples that would facilitate mastering the covered theory. A comprehensive set of student exercises and an excellent list of references pertinent to the level of the text are yet other desirable features.

Theory is largely accurately developed. The figures and illustrations are well delineated, the typography is excellent and readily apparent stylistic errors of the nature of misspelling and deletion or misplacing of mathematical symbols seem few. The double procedures used in numbering the figures; by chapter, by section, and by the figure's number and the equations and/or theorems numbered; only by section and then the equation's number, thus deleting the chapter, seemed distracting, as also was the nonpunctuation attending the equations.

A concise review of the contents of the book follows. Chapter 1—The introduction (eight pages)—covers the fundamental aspects of signal filtering theory and includes a discussion of the historical development of this theory. An outline of the book is given next. It is emphasized, herein,

that the theory in this book is developed for a basic filter with the following characteristics: 1) operation in discrete-time, 2) optimality, 3) linearity, and 4) finite dimensionality.

Chapter 2—“Filtering, linear systems, and estimation” (27 pages)—entails discussion of various concepts of filtering, smoothing, and prediction. The authors first initiate some discussion of the nature of filtering problems: the physical meanings, constraints, and roles in a dynamic system. Next, they present the basic linear system with noisy input and output for which a design is carried out, and, finally, they give some specific ways of trying to use noisy measurement data to infer estimates of the way the internal variable in a system may be behaving.

Chapter 3—“The discrete-time Kalman filter” (26 pages)—unifies the ideas of the previous chapter and describes the Kalman filter problem and its corresponding derivations through an easy-to-understand approach. The more advanced derivations of the Kalman filter are developed in a later chapter. Apparently this procedure enables the student to grasp the theory and its physical implications more easily than he would otherwise. An interesting section of this chapter is about application of the Kalman filter to a “real world” problem that has been (extensively) solved by using Kalman filtering ideas. This example clearly demonstrates the potential applications of this subject to a number of other such problems.

Chapter 4—“Time-invariant filters” (28 pages)—opens with a discussion on the definition of the time-invariance filter which, in turn, results in the analysis that goes with a time invariance random process which is associated with the system to start. This chapter continues with a study of the stability properties of linear discrete-time systems, the stationary behavior of linear systems, and the time invariance and asymptotic stability of the filter (wherein it is shown that under assumptions of complete detectability and stabilization, the filter is time-invariant and asymptotically stable for an arbitrary initial error covariance). The chapter closes with a section on frequency-domain formulas for the sake of relating the time-invariant optimal filter to the signal process model.

Chapter 5—“Kalman filter properties” (39 pages)—is comprised of a brief historical account of the development of Kalman filtering, a minimum variance and linear minimum variance estimation, an orthogonality and projection theorem (wherein a linear minimum variance estimator is developed in terms of first and second-order statistics), and a formal definition and the applications of innovations sequences. In the next section, the Kalman filter is derived again, (by using the notion of innovations) for a one-step prediction state estimate and the corresponding error covariance. It is proved that the Kalman filter is the best filter among a subset of all linear filters, but it is the best filter among the set of all filters when the noise processes are Gaussian, and it is the best linear filter among the set of all linear filters otherwise. This chapter continues with the establishment of a set of equations for calculations of the true-filtered estimates and to demonstrate a connection between improvement of the classical idea of signal-to-noise ratio and the true filter. Finally, in the last section, the well-known question of when a filter is optimal (i.e., the inverse problems) is discussed.

Chapter 6—“Computational aspects” (36 pages)—alerts the reader to the very important problems which exist in the computational aspects of the Kalman filter applications. These problems stem from various types of modeling and computational errors. Problems such as the kind of errors, either physically (nonexact model) or computationally (round-off errors of further computations on the system model), are of great concern in the design of Kalman filters. In fact, it is well known that the Kalman filter is very sensitive to the system model that has been adopted and (intuitively) as more data becomes available, the performance degradation of the filter becomes more manageable. Correspondingly, in this chapter the effects of certain errors, with their consequences (such as computational divergence and/or sensitivity on the overall performance of the filter) are anticipated and some techniques for eliminating these errors are proposed. Meriting special mention are sections on the following: exponential data weighting (or a filter with a desired degree of stability), wherein it has been

emphasized that information filter equations are occasionally more efficient than covariance filter equations; sequential processing, in which the measurement vector is processed one component at a time and it is shown that this procedure results in some computational advantages over similar methods; square-root filtering, which is essential to acquiring accuracy for many applications with restrictions on processing devices, ensures non-negativity of covariance and information matrices and lowers requirements for computational accuracy by increasing further calculations. In the next section the high measurement noise case, wherein simplified equations are used to calculate the filter gain and performance, is discussed. This chapter ends with the Chandrasekhar-type, doubling, and nonrecursive algorithms which are other ways of proceeding when the signal model is time-invariant and the input and measurement noise are stationary.

Chapter 7—"Smoothing of discrete-time signals" (28 pages)—advances the fundamentals of smoothing technique wherein it is shown that the fixed-point smoothing problem is to some extent a Kalman filtering problem. In the following sections fixed-lag smoothing and its applications is discussed in detail and the chapter ends with a section on fixed-internal smoothing.

Chapter 8—"Applications in nonlinear filtering" (30 pages)—extends in certain ways the results of Kalman filtering for the linear cases to the nonlinear systems. These extensions are performed by various approximations such as truncation and the corresponding further approximations on the linear model. This result is generally a suboptimal filter for the truncated signal model. However, it is noted that the application of the extended Kalman filter to a specific nonlinear problem requires appropriate simplifications which in general may not be easy to obtain. Therefore, in addition to the experience of the designer, certain trade-offs between the filter performance and the algorithm complexity must be obtained. The chapter continues with a section on a bound optimal filter that basically concerns a class of nonlinear filters giving insight into a linear filter that has unintentional cone-bounded nonlinearities. Finally, this chapter ends with a section on Gaussian sum estimators involving a bank of extended Kalman filters that provide a significant performance improvement over the simple extended Kalman filter. This is a trade-off for more filter complexity and more experience on the part of the designer.

Chapter 9—"Innovations representations, spectral factorization, Wiener and Levinson filtering" (44 pages)—opens with the characteristics of the Kalman filter which correspond to the innovations model. It is shown that the computing of an innovations representation from a stationary covariance, parallels that of a classical minimum phase factorization. The Kalman filter design from covariance data is discussed next. In the remaining portion of this chapter, various kinds of innovations representations and their properties are detailed. The first such property is that with finite initial time, which is "determinable from the covariance data only and is unique." "The input to the innovations model can be determined from the output, and the Kalman filter can estimate the state of the innovations model with zero error, and the Kalman filter innovations sequence is identical with the input noise sequence of the innova-

tions model." The second such problem is that of the stationary innovations representations. Under this section, which is a review of classical frequency-domain spectral factorization, discussions of state-variable innovations representations commencing for both at a finite initial time and in the infinitely remote past for stationary processes and a discussion of autoregressive moving average (ARMA) representations and the Wold decomposition are given. This chapter closes with a section on Wiener filtering (discrete-time) and a section on Levinson filters (which is a computationally simplified version of the Wiener filtering).

Chapter 10—"Parameter identification and adaptive estimation," (21 pages)—describes a more realistic situation of filtering in which the signal models contain unknown parameters and/or the noise statistics are not known. Yet, further, in most "practical" problems the parameters of a process vary slowly in some random manner, and therefore the on-line filtering of such processes requires an adaptive scheme. Correspondingly, the question of extensions of the Kalman filtering, developed previously, to systems (assumed linear) with unknown parameters (that is termed adaptive estimation) is of considerable importance and is a well-deserved separate study. Therefore, in the remaining sections of this chapter, adaptive estimation, parallel to that of extended least squares, for a class of state-space signal models with unknown parameters (provided that the states can be measured and can be estimated, given the parameters) is discussed. It is then shown that by a simultaneous state and parameter estimation, and with the parameter [state] estimator using state [parameter] estimates, rather than the true estimates, very useful adaptive estimators can be constructed.

Chapter 11—"Colored noise and suboptimal reduced order filters" (18 pages)—concludes the derivations of the Kalman filtering with the assumption of colored noise instead of the white noise which has been assumed for measurement and input noise processes throughout the book up to this point. A rather general approach for optimal dealing with this problem is discussed and it is shown that on many occasions, the problem is manageable with the expense of a filter having a higher complexity. Implementation of this filter can then be carried out by suboptimal designs and correspondingly in this chapter, this problem is focused on extensively. In the next three sections of this chapter, the colored noise is considered in turn, as Markov output noise, singular or near-singular output noise, colored input or measurement noise, and the associated filter design is studied. The chapter closes with a suboptimal filter design by model order-reduction.

The text is concluded with four useful appendices: A) (17 pages) "Brief review of results of probability theory", B) (16 pages) "Brief review of some results of matrix theory", C) (7 pages) "Brief review of several major results of linear system theory", and D) (3 pages) "Lyapunov stability." A preface, table of contents, and author and subject indexes round out the book.

The text is well suited for a graduate-level introductory course and as a reference and/or self-study text for the engineer in practice who desires to update his or her knowledge of Kalman filtering and its applications. Overall, the book well combines theory and application of Kalman filtering!