Correspondence

Image Enhancement Using Smoothing with Fuzzy Sets

SANKAR K. PAL AND ROBERT A. KING

Abstract—**A model for grey-tone image enhancement using the concept of fuzzy sets is suggested. It involves primary enhancement, smoothing, and then final enhancement. The algorithm for both the primary and final enhancements includes the extraction of fuzzy properties corresponding to pixels and then successive applications of the fuzzy operator "contrast intensifier" on the property plane. The three different smoothing techniques considered in the experiment are defocussing, averaging, and max-min rule over the neighbors of a pixel. The reduction of the "index of fuzziness" and "entropy" for different enhanced outputs (corresponding to different values of fuzzifiers) is demonstrated for an English script input. Enhanced output as obtained by histogram modification technique is also presented for comparison.**

I. Introduction

The theory of fuzzy set [1], [2] provides a suitable algorithm in analyzing complex systems and decision processes when the pattern indeterminacy is due to inherent variability and/or vagueness (fuzziness) rather than randomness. Since a grey tone picture possesses some ambiguity within pixels due to the possible multivalued levels of brightness, it is justified to apply the concept and logic of fuzzy set rather than ordinary set theory to an image processing problem. Keeping this in mind, an image can be considered as an array of fuzzy singletons [1], [2] each with a membership function denoting the degree of having some brightness level.

The methods so far developed for image enhancement may be categorized into two broad classes [3]–[6], namely, frequency domain methods and spatial domain methods. The technique in the first category depends on modifying the Fourier transform of an image, whereas in spatial domain methods the direct manipulation of the pixel is adopted. Some fairly simple and yet powerful processing approaches are seen to be formulated in the spatial domain [3], [4]. It is to be mentioned here that all these techniques are problem oriented. When an image is processed for visual 'interpretation,' it is ultimately up to the viewers to judge its quality for a specific application. The process of evaluation of image quality therefore becomes a subjective one.

In this correspondence we present a model (Fig. 1) consisting of primary and final enhancement for a grey tone image using fuzzy algorithm along with smoothing operations. The procedure involves a primary enhancement of an image by the block E followed by a smoothing through S and a subsequent enhancement by a second use of block E. The fuzzy contrast intensification (INT) operator is taken as a tool for both the primary and final enhancements in the fuzzy property domain. This domain is extracted from the spatial domain using fuzzifiers [7], [8] which play the role of creating different amounts of ambiguity in the

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Fig. 1. Block diagram of the enhancement model.

property plane. The function of an image smoother (block S) as introduced after the primary enhancement is to blur the image and the blurred image is then reprocessed by another enhancement block E. The ultimate object of using a second E block is to have further improvement in image quality. The performance of the model for different values of the fuzzifiers is demonstrated on a picture of handwritten English recursive script when the defocussing, averaging, and max-min techniques are used separately in the smoothing algorithm. The results are compared with those obtained using histogram modification technique [4].

The "index of fuzziness" [9], [10] which reflects a kind of quantitative measure of an image quality is measured for each output and is compared with that of "entropy" [11]. The system CDC 6400/6500 was used for numerical analysis.

II. Fuzzy Set and the Concept of Enhancement

A fuzzy set (A) with its finite number of supports $x_1, x_2, \cdots, x_n$ in the inverse of discourse U is defined as

$$A = \{(\mu_A(x_i), \quad x_i)\} \tag{1a}$$

or, in union form,

$$A = \bigcup_i \mu_i / x_i, \quad i = 1, 2, \cdots, n \tag{1b}$$

where the membership function $\mu_A(x_i)$ having positive values in the interval (0, 1) denotes the degree to which an event x_i may be a member of A. This characteristic function can be viewed as a weighting coefficient which reflects the ambiguity (fuzziness) in A. A fuzzy singleton is a fuzzy set which has only one supporting point. If $\mu_A(x_i) = 0.5$, x_i is said to be the crossover point in A.

Similarly, the property p defined on an event x_i is a function $p(x_i)$ which can have values only in the interval (0, 1). A set of these functions which assigns the degree of possessing some property p by the event x_i constitutes what is called a property set [12].

A. Image Definition

With the concept of fuzzy set, an image X of $M \times N$ dimension and L levels can be considered as an array of fuzzy singletons, each with a value of membership function denoting the degree of having brightness relative to some brightness level l, $l = 0, 1, 2, \cdots, L - 1$. In the notion of fuzzy set, we may therefore write

$$X = \bigcup_m \bigcup_n p_{mn} / x_{mn}, \qquad m = 1, 2, \cdots, M; \quad n = 1, 2, \cdots, N \tag{2}$$

where p_{mn}/x_{mn}, $(0 \leq p_{mn} \leq 1)$ represents the grade of possessing some property p_{mn} by the (m, n)th pixel x_{mn}. This fuzzy property p_{mn} may be defined in a number of ways with respect to any brightness level depending on the problems at hand. In our experiment, we have defined (as shown in (6)) it with respect to the maximum level $L - 1$.

B. Contrast Intensification and Enhancement in Property Plane

The contrast intensification operator (INT) on a fuzzy set A generates another fuzzy set $A' = \text{INT}(A)$, the membership func-

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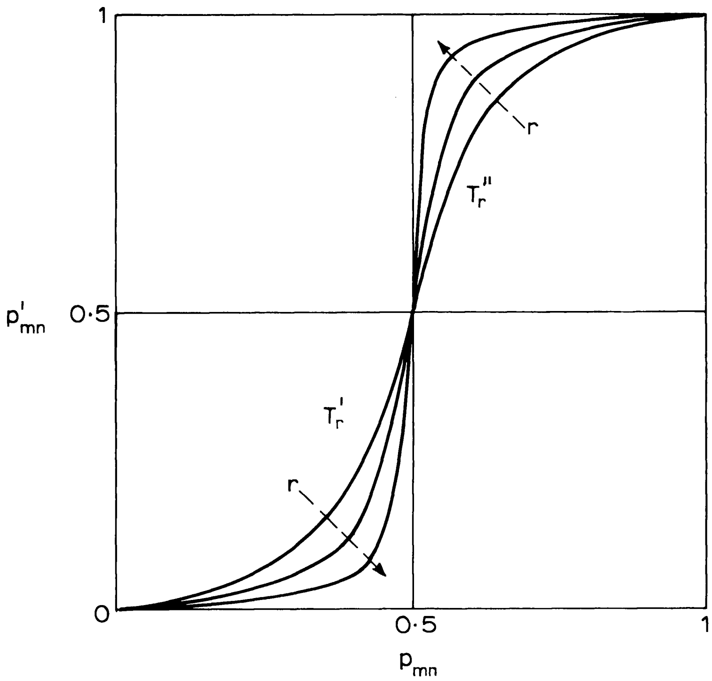


Fig. 2. INT transformation function for contrast enhancement in property plane.

tion of which is

$$\mu_{A'}(x) = \mu_{\mathrm{INT}(A)}(x)$$

$$= \begin{cases} 2[\mu_A(x)]^2, & 0 \leqslant \mu_A(x) \leqslant 0.5 \quad \text{(3a)} \\ \left[1 - 2(1 - \mu_A(x))^2\right], & 0.5 \leqslant \mu_A(x) \leqslant 1. \quad \text{(3b)} \end{cases}$$

This operation reduces the fuzziness of a set A by increasing the values of $\mu_A(x)$ which are above 0.5 and decreasing those which are below it. Let us now define operation (3) by a transformation T_1 of the membership function $\mu(x)$.

In general, each p_{mn} in X (2) may be modified to p'_{mn} to enhance the image X in the property domain by a transformation function T_r where

$$p'_{mn} = T_r(p_{mn}) = \begin{cases} T'_r(p_{mn}), & 0 \leqslant p_{mn} \leqslant 0.5 \quad \text{(4a)} \\ T''_r(p_{mn}), & 0.5 \leqslant p_{mn} \leqslant 1 \quad \text{(4b)} \end{cases}$$

$$r = 1, 2 \cdots.$$

The transformation function T_r is defined as successive applications of T_1 by the recursive relationship

$$T_s(p_{mn}) = T_1\{T_{s-1}(p_{mn})\}, \qquad s = 1, 2, \cdots \tag{5}$$

and $T_1(p_{mn})$ represents the operator INT defined in (3) in our problem.

This is shown graphically in Fig. 2. As r increases, the curve tends to be steeper because of the successive application of INT. In the limiting case, as $r \to \infty$, T_r produces a two-level (binary) image. It is to be noted here that corresponding to a particular operation of T' one can use any of the multiple operations of T'' and vice versa to attain a desired amount of enhancement.

C. Property Plane and Fuzzification

All the operations described above are restricted to the fuzzy property plane. To enter this domain from the spatial x_{mn} plane, we define an expression of form similar to that defined by one of the authors in speech recognition [7], [8]:

$$p_{mn} = G(x_{mn}) = \left[1 + \frac{(x_{\max} - x_{mn})}{F_d}\right]^{-F_e},$$

$$m = 1, 2, \cdots, M; \; n = 1, 2, \cdots, N \tag{6}$$

where $x_{\max}$ denotes the maximum grey level $(L - 1)$ desired, F_e and F_d denote the exponential and denominational fuzzifiers, respectively. These fuzzifiers have the effect of altering ambiguity in the p plane. As will be shown later, the values of these two positive constants are determined by the crossover point (x_c, for which $p_{x_c} = G(x_c) = 0.5$) in the enhancement operation.

Equation (6) shows that $p_{mn} \to 1$ as $(x_{\max} - x_{mn}) \to 0$ and decreases as $(x_{\max} - x_{mn})$ increases. In other words, the fuzzy property p_{mn} as defined here denotes the degree of possessing maximum brightness level $x_{\max}$ by the (m, n)th pixel $x_{mn} \cdot p_{mn} = 1$ denotes light and $p_{mn} = 0$ dark.

It is to be noted from (6) that for $x_{mn} = 0$, p_{mn} has a finite positive value α, say where

$$\alpha = \left(1 + \frac{x_{\max}}{F_d}\right)^{-F_e}. \tag{7}$$

So the p_{mn} plane becomes restricted in the interval $[\alpha, 1]$ instead of $[0, 1]$. After enhancement, the enhanced p'_{mn} plane may contain some regions where $p'_{mn} < \alpha$ due to the transformation T'. The algorithm includes a provision for constraining all the $p'_{mn} < \alpha$ values to α so that the inverse transformation

$$x'_{mn} = G^{-1}(p'_{mn}), \quad \alpha \leqslant p'_{mn} \leqslant 1 \tag{8}$$

will allow those corresponding x'_{mn} values to have zero grey level. Of course, one can change α to some other value depending on the contrast or background level desired.

Furthermore, since there are only $L(0, 1, 2, \cdots, L - 1)$ equally spaced allowed levels in an image, each of the transformed x'_{mn} values must be assigned to its closest valid level to result in an enhanced image X'.

D. Selection of F_e and F_d

From the enhancement operation it is noted that we have to select a suitable crossover point x_c from the image plane so that all the $x_{mn} \gtrless x_c$ in spatial domain would possess values $p_{mn} \gtrless 0.5$ in property domain. The successive use of INT operator would then intensify the contrast by increasing the values of $p_{mn} > 0.5$ and decreasing those $p_{mn} < 0.5$.

Suppose we want to put the threshold of enhancement operation between the levels l and $l + 1$ so that after the enhancement operation, all the $x_{mn} \geqslant (l + 1)/\leqslant l$ would possess increased/decreased levels. Then we consider,

$$x_c = l + 0.5$$

$$p_{x_c} = 0.5$$

and the value of F_d for a specific F_e can correspondingly be determined from (6). The higher the value of F_e, the greater will be the rate of increase/decrease of p_{mn} values after/before x_c and hence the lower is the value of r to attain a desired amount of enhancement.

For example, if $l = 9$ then $x_c = 9.5$ and with $x_{\max} = 31$, and for $F_e = 1$ and 2, we obtain from (6) that F_d is 21.5 and 52, respectively. The corresponding p_{mn} values corresponding to $x_{mn} = \cdots, 7, 8, 9, 10, 11, 12 \cdots$, are $\cdots$, 0.473, 0.483, 0.494, 0.506, 0.518, 0.531, $\cdots$ and $\cdots$, 0.468, 0.481, 0.494, 0.507, 0.522, 0.536, $\cdots$ for $F_e = 1$ and 2; respectively. The rate of increase/decrease of p_{mn} values after/before the crossover point is seen to be higher for values of $F_e = 2$ than 1. The enhancement of the contrast for a specific value of r would therefore be better for $F_e = 2$.

E. Elements of the Enhancement Block 'E'

The elements constituting the primary and final enhancement blocks E (in Fig. 1) are shown in Fig. 3.

The function $G(x_{mn})$ as defined by (6) uses two fuzzifiers F_e and F_d to extract the fuzzy property p_{mn} for the (m, n)th pixel x_{mn} of an $M \times N$ input image array X. The transformation

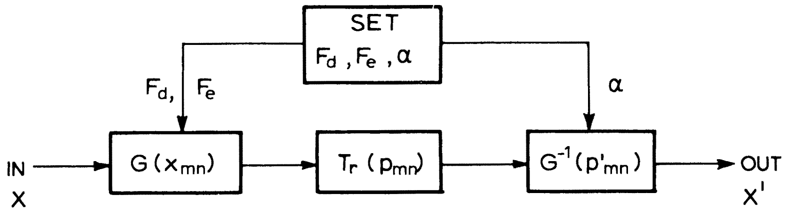


Fig. 3. Elements of block E Fig. 1.

function $T_r(p_{mn})$ serves the role of enhancement in property plane using r successive use of the fuzzy INT operator. This is explained by (3) to (5). The enhanced p' domain after being inversely transformed by $G^{-1}(p'_{mn})$ (8) produces the corresponding enhancement image X' in spatial domain.

III. Index of Fuzziness and Entropy

The index of fuzziness of a set A having n supporting points is defined as [10]

$$\gamma(A) = \frac{2}{n^k} d(A, \tilde{A}) \tag{9}$$

where $d(A, \tilde{A})$ denotes the distance between fuzzy set A and its nearest ordinary set $\tilde{A}$. The set $\tilde{A}$ is such that $\mu_{\tilde{A}}(x_i) = 0$ if $\mu_A(x_i) \leq 0.5$ and 1 for $\mu_A(x_i) > 0.5$. The positive constant k appears in order to make $\gamma(A)$ lie between 0 and 1 and its value depends on the type of distance function used. For example, $k = 1$ for a generalized Hamming distance whereas $k = 0.5$ for an Euclidean distance. The corresponding indices of fuzziness are called the linear index of fuzziness $\gamma_l(A)$ and the quadratic index of fuzziness $\gamma_q(A)$. Considering 'd' to be a generalized Hamming distance we have

$$d(A, \tilde{A}) = \sum_i |\mu_A(x_i) - \mu_{\tilde{A}}(x_i)| = \sum_i \mu_{A\cap\bar{A}}(x_i) \tag{10}$$

and

$$\gamma_l(A) = \frac{2}{n} \sum_i \mu_{A\cap\bar{A}}(x_i), \qquad i = 1, 2, \cdots, n \tag{11a}$$

where $A \cap \bar{A}$ is the intersection between fuzzy set A and its complement $\bar{A}$. $\mu_{A\cap\bar{A}}(x_i)$ denotes the grade of membership of x_i to such a fuzzy set $A \cap \bar{A}$ and is defined as

$$\begin{aligned}\mu_{A\cap\bar{A}}(x_i) &= \min\{\mu_A(x_i), \mu_{\bar{A}}(x_i)\}, \qquad \text{for all } i \\ &= \min\{\mu_A(x_i), (1 - \mu_A(x_i))\}, \qquad \text{for all } i.\end{aligned} \tag{11b}$$

Extending (11) in a two-dimensional image plane we may write

$$\begin{aligned}\gamma_l(X) &= \frac{2}{MN} \sum_m \sum_n \mu_{X\cap\bar{X}}(x_{mn}), \\ m &= 1, 2, \cdots M;\ n = 1, 2, \cdots, N.\end{aligned} \tag{12a}$$

Equation (12a) defines the amount of fuzziness present in the property plane of an image X. μ corresponds to p_{mn}. $X \cap \bar{X}$ is the intersection between fuzzy image planes $X = \{p_{mn}/x_{mn}\}$ and $\bar{X} = \{(1 - p_{mn})/x_{mn}\}$, the complement of X. $\mu_{X\cap\bar{X}}(x_{mn})$ denotes the degree of membership of (m, n)th pixel x_{mn} to such a fuzzy property plane $X \cap \bar{X}$ so that

$$\begin{aligned}\mu_{X\cap\bar{X}}(x_{mn}) &= p_{mn} \cap \bar{p}_{mn} \\ &= \min\{p_{mn}, (1 - p_{mn})\}, \qquad \text{for all } (m, n).\end{aligned} \tag{12b}$$

The entropy of a fuzzy set A having n supporting points as defined by De Luca and Termini (11) is

$$H(A) = \frac{1}{n \ln 2} \sum_i \mathrm{sn}(\mu_A(x_i)), \qquad i = 1, 2, \cdots, n \tag{13a}$$

with the Shannon's function

$$\mathrm{sn}(\mu_A(x_i)) = -\mu_A(x_i) \ln \mu_A(x_i) - (1 - \mu_A(x_i)) \ln (1 - \mu_A(x_i)). \tag{13b}$$

Extending (13) in a two-dimensional image plane we have

$$H(X) = \frac{1}{MN \ln 2} \sum_m \sum_n \mathrm{sn}(\mu_X(x_{mn})) \tag{14a}$$

where

$$\begin{aligned}\mathrm{sn}(\mu_X(x_{mn})) &= -\mu_X(x_{mn}) \ln \mu_X(x_{mn}) - (1 - \mu_X(x_{mn})) \ln (1 - \mu_X(x_{mn})), \\ m &= 1, 2, \cdots, M;\ n = 1, 2, \cdots, N.\end{aligned} \tag{14b}$$

The term $H(X)$, $0 \leq H(X) \leq 1$, measures the ambiguity in X on the basis of the well-known property of Shannon's function sn (μ)—montonically increasing in the interval $(0, 0{\cdot}5)$ and monotonically decreasing in $(0{\cdot}5, 1)$ with a maximum (= unity) at $\mu = 0{\cdot}5$—in the fuzzy property plane of X.

IV. Smoothing Algorithm

The idea of the smoothing is based on the property that image points which are spatially close to each other tend to possess nearly equal grey levels. Let us now explain three smoothing algorithms which have been tested in S block of Fig. 1.

A. Defocussing

The (m, n)th smoothed pixel intensity in the first method is defined as

$$x'_{mn} = a_0 x_{mn} + a_1 \sum_{Q_1} x_{ij} + a_2 \sum_{Q_2} x_{ij} + \cdots + a_s \sum_{Q_s} x_{ij} \tag{15a}$$

where

$$\begin{aligned}&a_0 + N_1 a_1 + N_2 a_2 + \cdots + N_s a_s = 1, \qquad 1 > a_1 > a_2 \cdots a_s > 0, \\ &(i, j) \neq (m, n),\ m = 1, 2, \cdots, M \text{ and } n = 1, 2, \cdots, N\end{aligned} \tag{15b}$$

x_{mn} represents the (m, n)th pixel intensity of the primary enhanced image. Q_1 denotes a set of N_1 coordinates (i, j) which are on or within a circle of radius R_1 centered at (but excluding) the point (m, n). Q_s denotes a set of N_s coordinates (i, j) which are on or within a circle of radius R_s centered at (m, n)th point but which do not fall into Q_{s-1}. For example, $Q = \{(m, n+1), (m, n-1), (m+1, n), (m-1, n)\}$ is the set of coordinates which are on/within a circle of radius one unit from a point (m, n).

This smoothing algorithm is therefore a kind of defocussing technique using a linear nonrecursive filter, where a part of the intensity of a pixel is being distributed to its neighbors. The amount of energy transmitted to a neighbor decreases as its distance from the pixel in question increases. a_0 represents the fraction retained by a pixel after transmission of part of its energy to neighbors. The set $a = \{a_0, a_1, a_2, \cdots, a_s\}$ as seen from this algorithm, plays an important role in smoothing an image and the choice of its values is problem oriented.

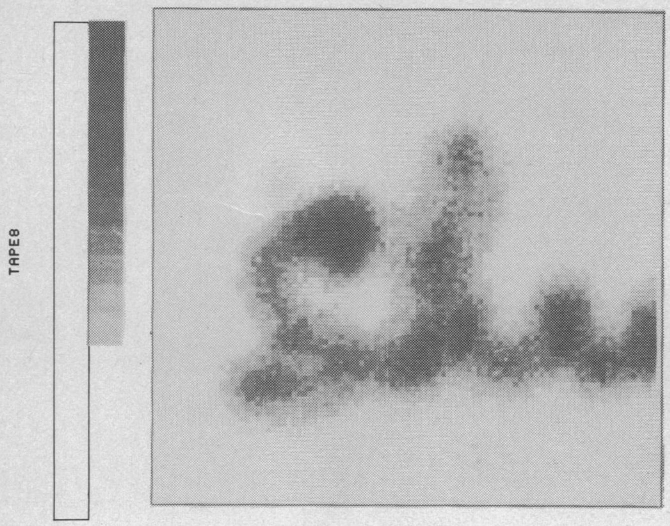


Fig. 4. Input picture.

B. Averaging

The second method is based on averaging the intensities within neighbors and is usually used to remove "pepper and salt" noise. The smoothed (m, n)th pixel intensity is

$$x'_{mn} = \frac{1}{N_1} \sum_{Q_1} x_{ij}, \qquad (i, j) \neq (m, n), \quad (i, j) \in Q_1. \quad (16)$$

This is a special case of defocussing technique (15) with $a_0 = a_2 = a_3 = \cdots = a_s = 0$. For a given radius, the blurring effect produced by neighborhood averaging can also be reduced by using a threshold procedure [3], [4] where the (m, n)th intensity is changed only if its difference from neighborhood values exceeds a specific nonnegative threshold.

C. Max–Min Rule

Equations (15) and (16) are formulated using collective properties of pixels. The third method on the other hand, uses q successive applications of "min" and then "max" operators [13] within neighbors such that the smoothed grey level value of (m, n)th pixel is

$$x'_{mn} = \max_{Q_1}{}^{q} \min_{Q_1}{}^{q} \{x_{ij}\},$$
$$(i, j) \neq (m, n), \ (i, j) \in Q_1, \ q = 1, 2, \cdots. \quad (17)$$

All the smoothing algorithms described above blur the image by attenuating the high spatial frequency components associated with edges and other abrupt changes in grey levels. The higher the values of Q_s, Q_1, and q, the greater is the degree of blurring.

V. Enhancement by Histogram Equalization

If s_l and n_l denote the value of lth gray level and the number of times the lth level has appeared in the image X and n_t is the total number $(M \times N)$ of pixels in X, then the probability of the lth level in X is

$$P(s_l) = \frac{n_l}{n_t}, \qquad l = 0, 1, 2, \cdots, L - 1. \quad (18)$$

Now we apply a transformation function [4]

$$s'_l = T(s_l) = (L-1) \sum_j \frac{n_j}{n_t} = (L-1) \sum_j P(s_j),$$
$$j = 0, 1, 2, \cdots, l \quad (19)$$

which is equal to the cumulative distribution of s_l and we will have the modified values s'_l which is mapped from an original level s_l. Since only L equally spaced discrete levels are allowed in this case, each of the transformed values s'_l was assigned to its closest valid level.

A plot of $P(s'_l)$, the probability of the lth level in enhanced image X', versus s'_l would give the resulting equalized histogram. This implies an increase in the dynamic range of the pixels which can have a considerable effect in the appearance of an image. A detailed discussion about histogram modification techniques is available in [4].

VI. Implementation and Results

Fig. 4 shows an input picture of handwritten script (Shu) which is to be processed with the enhancement model described above. The digitized version of the image of this picture is represented by a 96×99 array where each pixel can have one of the $32(0, 1, 2, \cdots, 9, A, B, \cdots, V)$ grey levels. Thus in our algorithm $M = 96$, $N = 99$, $x_{\max} = L - 1 = 31$.

Some primary enhanced pictures [9] obtained using the operator INT(INT) alone as an enhancement tool ($r = 2$) are demonstrated in Fig. 5. F_e was kept constant at a value of 2. The value of F_d was 45, 43, and 40 for the Figs. 5(a), 5(b), and 5(c), respectively so that the corresponding threshold lay between the grey level C and D, D and E, and E and F. The change in enhancement between Figs. 5(b) and 5(c) is seen to be insignificant. Use of $F_d = 40$ made the output overcorrected and thinner.

Consider the picture of Fig. 5(b) as an input to the smoother. The smoothing algorithm (15) included $R_1 = 1$ unit, i.e., $N_1 = 4$ and $a_0 = 0.4$ so that

$$N_1 a_1 = 0.6 \quad \text{or} \quad a_1 = 0.15.$$

The final enhanced outputs of this smoothed image are shown in Fig. 6 for three different sets of fuzzifiers. $T_2 \equiv$ INT(INT) was considered as an enhancement tool with $F_e = 2$ throughout. The threshold in T_2 operation was placed between the levels 8 and 9 (Fig. 6(a)), 9 and A (Fig. 6(b)), and A and B (Fig. 6(c)) and corresponding values of F_d were 55, 52, and 49.5. Thus the value of α becomes 0.4091, 0.3926, and 0.3782 in the respective cases.

Figs. 7 and 8 correspond to the final outputs when the averaging technique and max(min) rule (16) and (17) within four neighbors ($R_1 = 1$ unit and $q = 1$) were used in the smoother. The crossover points in T_2 operation and the values of the fuzzifiers were considered to be the same as in the three cases of Fig. 6.

The other parameters remaining constant, as in Fig. (6) the output corresponding to $T_3 \equiv$ INT(INT(INT)) operator is demonstrated in Fig. 9. These results (for $F_d = 52$ and $F_e = 2$, corresponding to each of the three smoothed images) are shown as an illustration of system performance resulting from the successive use of the fuzzy INT operator.

The edges in Figs. 6–9 as compared to Fig. 5 are seen to be more smoothed and some of the thinned or missing pixels (especially for S) are also found to be recovered. With the decrease in the value of F_d (i.e., increasing the crossover point) the output becomes more corrected and thin. Use of $r = 3$ (T_3 operation) as compared to $r = 2$ only makes an increase/decrease in intensity value of each of the pixels that is after/before the crossover point. The quality of picture is not altered.

Experiments were also conducted for some other values of a_0, a_s, and Q_s, but the output performance was not satisfactory. For example, for defocusing we considered $a_0 = 0.2$, 0.4, and 0.6 in (15) for each of the three different sets of radii namely, 1) $R_1 = \sqrt{2}$, 2) $R_1 = 1$ and $R_2 = 2$, and 3) $R_1 = \sqrt{2}$ and $R_2 = 2\sqrt{2}$. For averaging and max–min rule (16) and (17) we had used $R_1 = \sqrt{2}$ and 2 separately. The energy distribution corresponding to all these parameters was seen to make such a modification in pixel values that some relevant information (e.g., the white patches, which should exist in the lower whorl of S) got lost.

Fig. 10 shows an output obtained by histogram equalization technique. This is included for comparison of the performance of

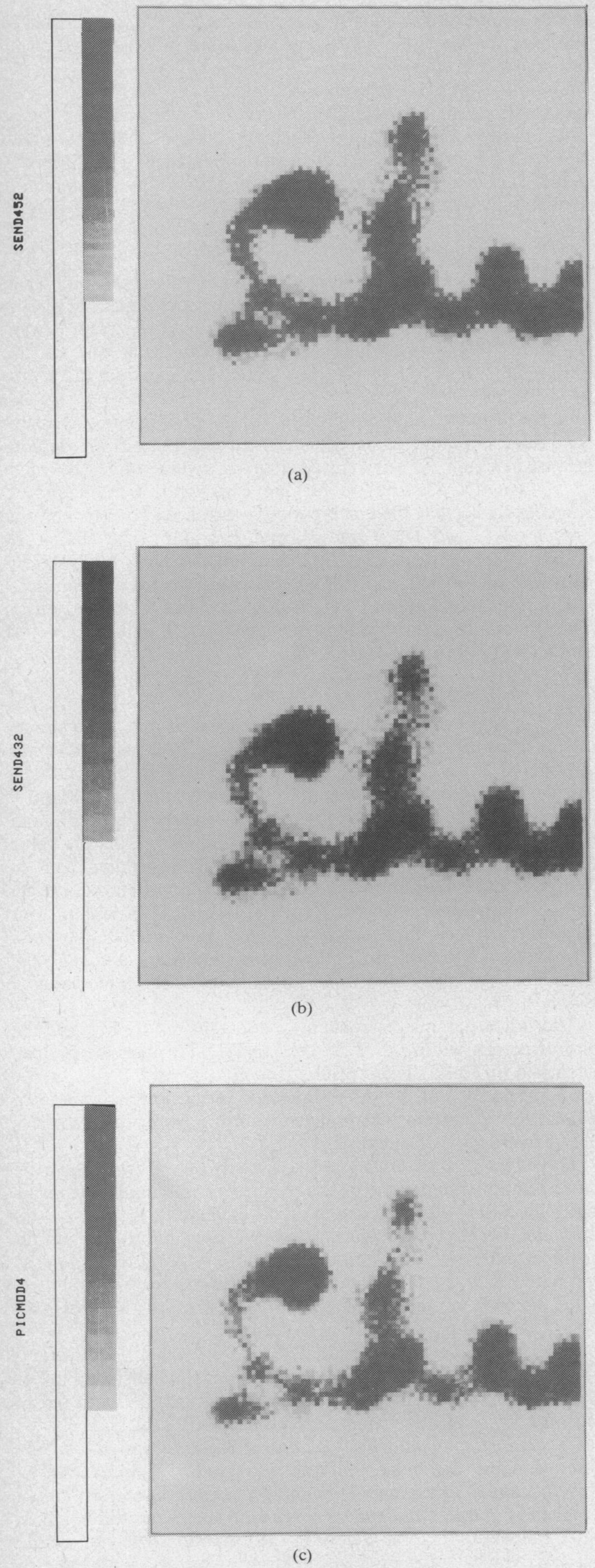


Fig. 5. Primary enhanced output. (a) $F_e = 2$, $F_d = 45$, $r = 2$. (b) $F_e = 2$, $F_d = 43$, $r = 2$. (c) $F_e = 2$, $F_d = 40$, $r = 2$.

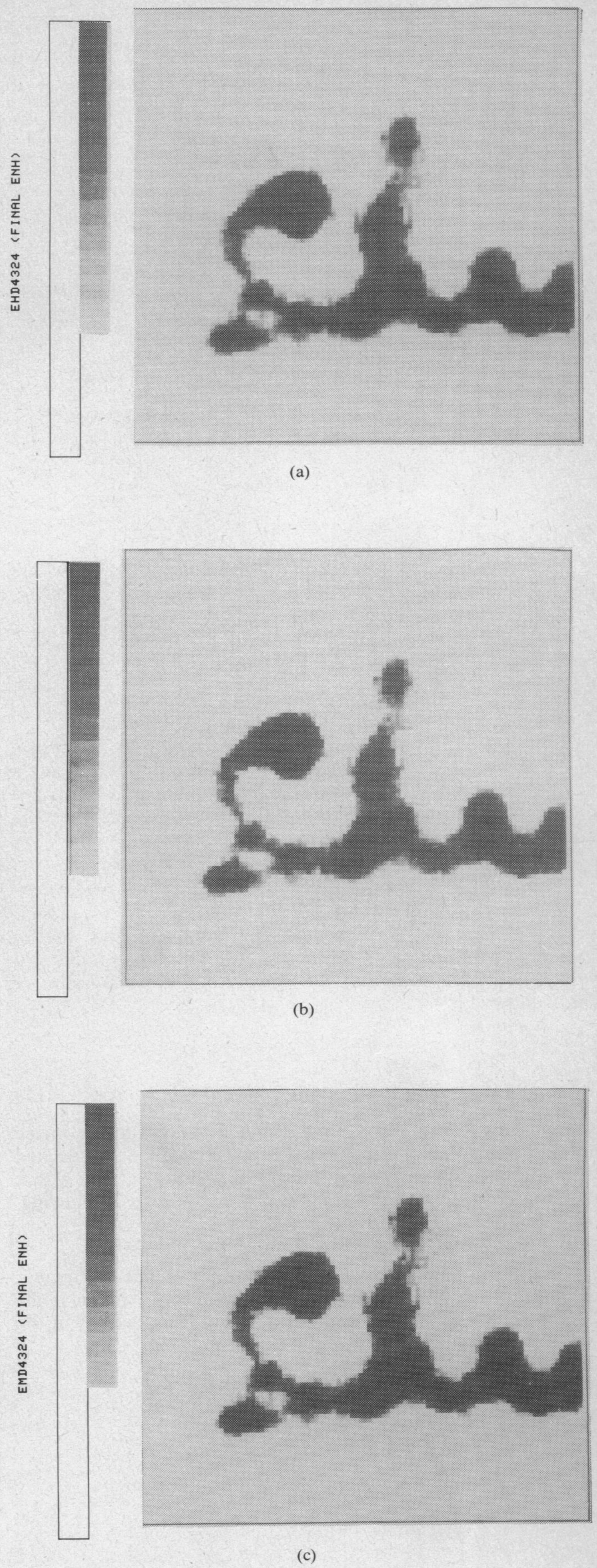


Fig. 6. Final enhanced output using (15). (a) $F_e = 2$, $F_d = 55$, $r = 2$. (b) $F_e = 2$, $F_d = 52$, $r = 2$. (c) $F_e = 2$, $F_d = 49.5$, $r = 2$.

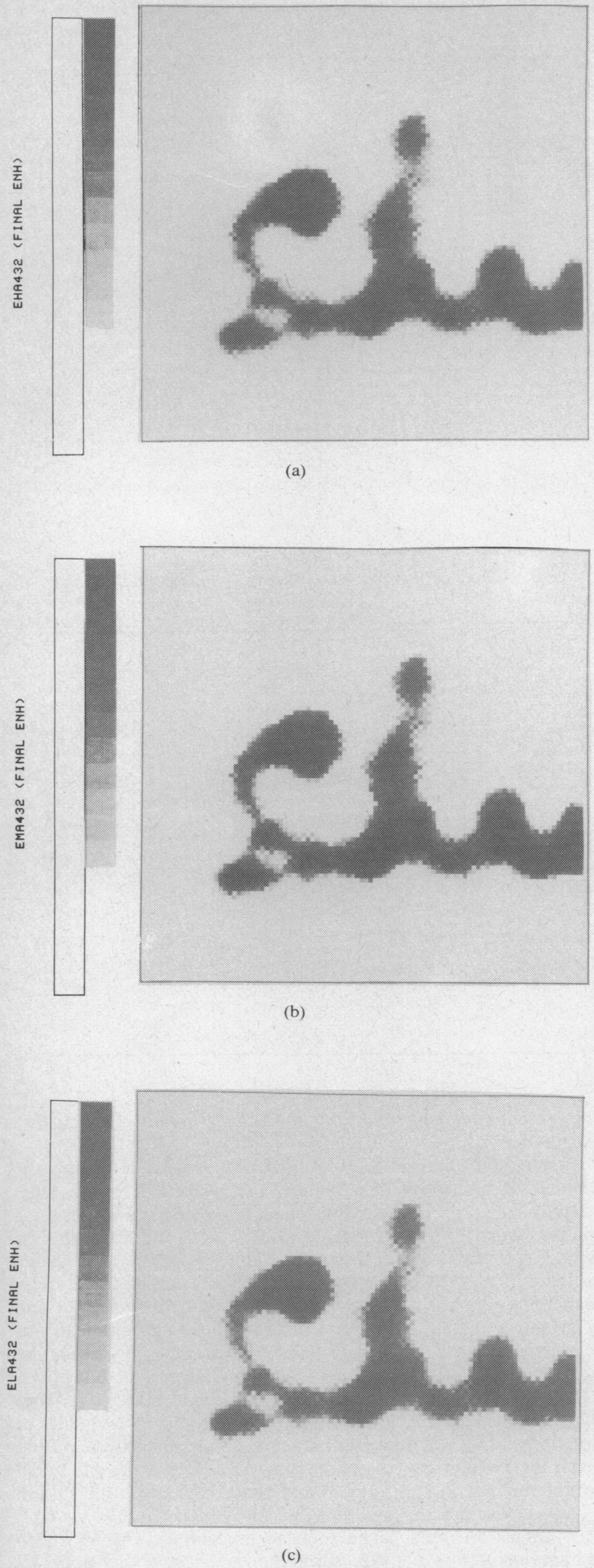


Fig. 7. Final enhanced output using (16). (a) $F_e = 2$, $F_d = 55$, $r = 2$. (b) $F_e = 2$, $F_d = 52$, $r = 2$. (c) $F_e = 2$, $F_d = 49.5$, $r = 2$.

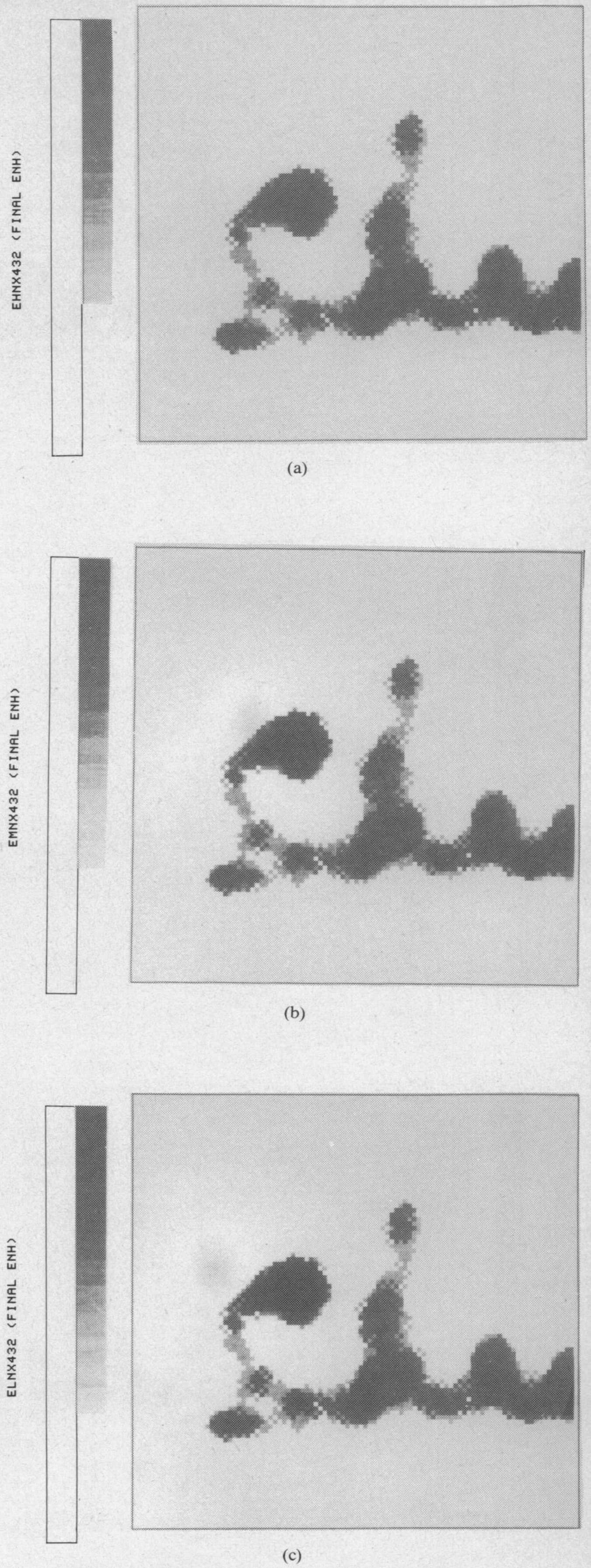
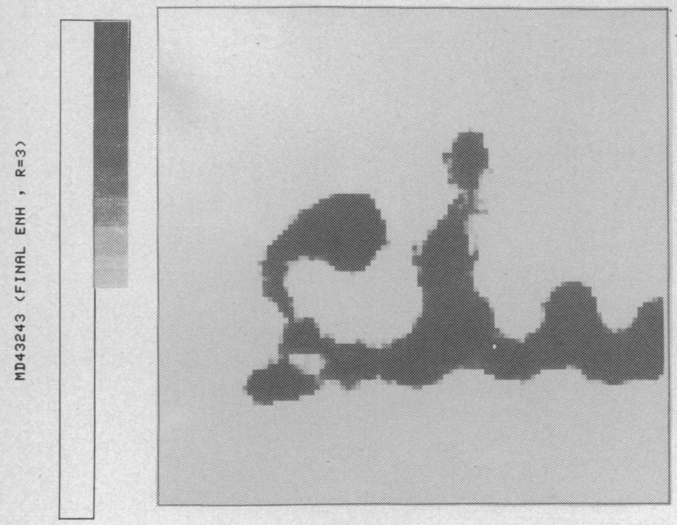
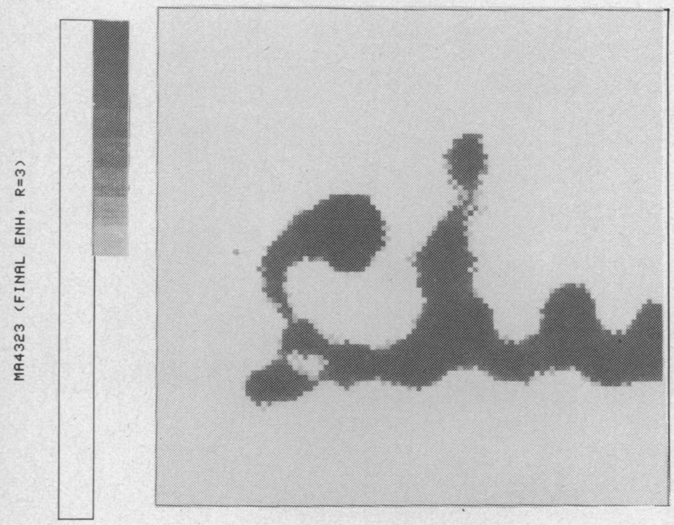


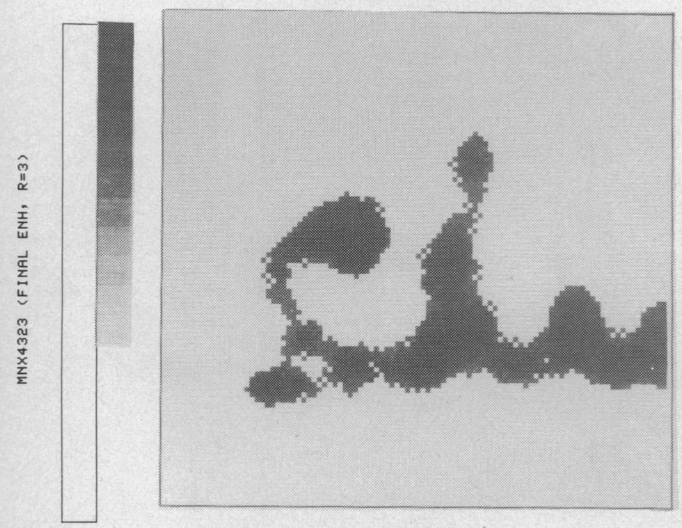
Fig. 8. Final enhanced output using (17). (a) $F_e = 2$, $F_d = 55$, $r = 2$. (b) $F_e = 2$, $F_d = 52$, $r = 2$. (c) $F_e = 2$, $F_d = 49.5$, $r = 2$.



(a)



(b)



(c)

Fig. 9. Final enhanced output for $F_e = 2$, $F_d = 52$, and $r = 3$. (a) Using (15). (b) Using (16). (c) Using (17).

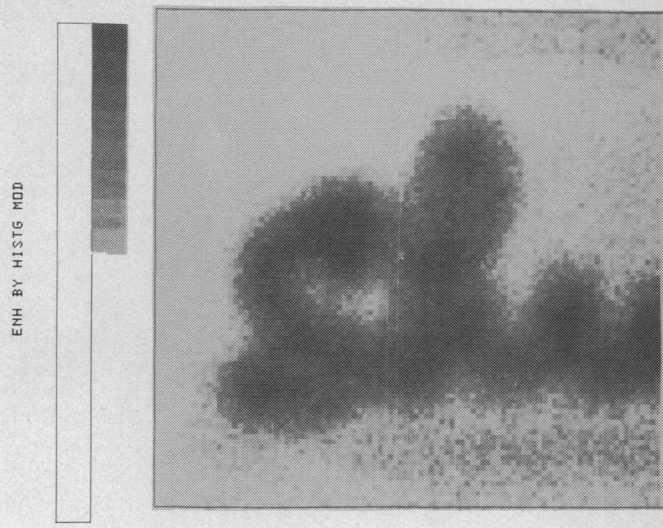


Fig. 10. Enhanced output using histogram modification technique.

TABLE I
LINEAR INDEX OF FUZZINESS OF PICTURES FOR DIFFERENT VALUES OF FUZZIFIERS

PICTURE X	LINEAR INDEX OF FUZZINESS $\gamma_1(X)$					
	$F_e=2$			$F_e=3$		
	$F_d=20$	$F_d=30$	$F_d=40$	$F_d=20$	$F_d=30$	$F_d=40$
Fig. 4	0.439	0.643	0.794	0.209	0.370	0.517
Fig. 5(a)	0.391	0.555	0.672	0.197	0.327	0.445
Fig. 5(b)	0.380	0.545	0.669	0.185	0.315	0.431
Fig. 5(c)	0.363	0.532	0.664	0.169	0.297	0.416
Fig. 6(a)	0.346	0.486	0.602	0.187	0.287	0.383
Fig. 6(b)	0.349	0.489	0.606	0.188	0.289	0.386
Fig. 6(c)	0.346	0.487	0.606	0.184	0.286	0.383
Fig. 7(a)	0.348	0.488	0.603	0.189	0.289	0.385
Fig. 7(b)	0.351	0.491	0.607	0.190	0.291	0.388
Fig. 7(c)	0.348	0.490	0.608	0.186	0.287	0.385
Fig. 8(a)	0.344	0.489	0.604	0.190	0.284	0.384
Fig. 8(b)	0.343	0.489	0.605	0.184	0.283	0.382
Fig. 8(c)	0.341	0.486	0.608	0.176	0.281	0.379
Fig. 9(a)	0.313	0.454	0.574	0.158	0.254	0.350
Fig. 9(b)	0.314	0.456	0.575	0.160	0.255	0.352
Fig. 9(c)	0.318	0.456	0.575	0.156	0.257	0.355
Fig. 10	0.586	0.668	0.681	0.430	0.545	0.622

the present system in enhancing an image with that of an existing technique.

Finally, the "linear index of fuzziness" $\gamma_1(X)$ reflecting the amount of ambiguity in a picture was measured for all these outputs by (12a). Table I illustrates the $\gamma_1(X)$ values of these pictures when F_d was considered to be 20, 30, and 40 separately with $F_e = 2$ and 3 in measuring p_{mn} values (μ in (12a)). With the increase in F_d or decrease in F_e, the index value of a picture is seen to be increased. This can be explained considering (6) and (12b). Since the p_{mn} value for a pixel increases as F_d increases or F_e decreases, its $(p_{mn} \cap \bar{p}_{mn})$ value (responsible for measuring $\gamma_1(X)$) would correspondingly increase/decrease for $p_{mn} < 0.5/> 0.5$. Now for all the pictures, it is found that the number of pixels having grey levels lower than the crossover point (as determined by those fuzzifiers) is much greater than those having levels higher than the crossover point. Therefore, there will be an overall increase in $(p_{mn} \cap \bar{p}_{mn})$ and hence $\gamma_1(X)$ with increase in F_d or decrease in F_e.

γ_1 values are seen to be reduced (except for Fig. 10) with enhancement. For Fig. 10, since the enhancement is done by histogram equalization technique, it possesses an almost uniform histogram. As a result, it contains, as compared with the input

TABLE II
ENTROPY OF PICTURES FOR DIFFERENT VALUES OF FUZZIFIERS

PICTURE X	ENTROPY H(X)					
	$F_e=2$			$F_e=3$		
	$F_d=20$	$F_d=30$	$F_d=40$	$F_d=20$	$F_d=30$	$F_d=40$
Fig.4	0.749	0.896	0.963	0.474	0.679	0.812
Fig.6(a)	0.639	0.778	0.854	0.398	0.561	0.682
Fig.6(b)	0.642	0.783	0.860	0.399	0.564	0.686
Fig.6(c)	0.640	0.783	0.862	0.395	0.562	0.685
Fig.7(b)	0.644	0.785	0.862	0.400	0.566	0.688
Fig.8(b)	0.639	0.782	0.861	0.393	0.560	0.683
Fig.10	0.825	0.867	0.864	0.681	0.793	0.842

(Fig. 4), a large number of levels near the crossover points and it is these levels which cause an increase in $(p_{mn} \cap \bar{p}_{mn})$ value. But the case is different for $F_e = 2$ and $F_d = 40$, where the crossover point becomes lower than all the others and the number of pixels having intensity below this point therefore becomes smaller than that in the input picture. The index value is thus decreased. Outputs in Fig. 9 do possess a minimum γ_1 value due to the T_3 operation, which reduces the ambiguity by further increasing/decreasing the property values which are greater/smaller than 0.5.

In a part of the experiment, these γ values were compared with those of "entropy" $H(X)$ (14) of the pictures. Table II shows the H values for some of the images (as typical cases for illustration) with the same values of F_e and F_d as used for $\gamma_1(X)$. The nature of variation of entropy with F_e and F_d is seen to conform to that of the linear index of fuzziness; only the effective values are larger.

VII. CONCLUSION

The concept of the fuzzy set is found to be applied successfully to the problems of grey-tone image enhancement. The addition of a smoothing algorithm between primary and final enhancement operations resulted in an improved performance. The three different smoothing techniques considered here are defocussing, averaging, and max–min rule over the neighbors of a pixel. All these techniques are seen to be almost equally effective (as measured by the amount of fuzziness present) in enhancing the image quality. The performance of this system in enhancing an image is also compared with that of the histogram equalization technique, an existing method and is seen to be much better as far as ambiguity is concerned. The linear index of fuzziness $\gamma_1(X)$ and entropy $H(X)$ of an image reflect a kind of quantitative measure of its quality and are seen to be reduced with enhancement. The amount of ambiguity is found to be minimum when the T_3 rule is adopted in the enhancement algorithm. $H(X)$ provides higher effective values of fuzziness as compared to $\gamma_1(X)$ but the nature of their variation among the different images with respect to F_e and F_d is identical.

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The Weibull Distribution as a Human Performance Descriptor

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***Abstract*—Results which support the contention that the Weibull distribution is a better fit to human task performance times than the Gaussian are shown. A method for estimating the Weibull parameters is shown to be accurate.**

INTRODUCTION

There are many situations in which it is necessary to predict the performance of people doing complex tasks in human–machine systems. Control of vehicles (aircraft, ships, space shuttle, etc.), control of processes (refineries, nuclear power stations, chemical plants, etc.), and communications, command, control, and intelligence (C^3I) situations (tactical or strategic) are some that come to mind which have in common that consequences of incorrect or untimely performances can be disastrous. To be able to predict the likelihood of correct and timely human actions in these and many more mundane situations would assist in the design of systems which would increase the probability of successful operation.

Computer simulation of human–machine systems is one valuable way of gaining insight into the performance of the entire system and the interactions of the human and machine components within the system. Monte Carlo simulations, however, require certain types of data to be input, one of which is information about the distribution of performance times for the individual tasks which comprise the network of interaction. Clearly, the probability distribution used to model individual task performance times is crucial to this method. There is little point in acquiring (at significant expense) accurate data about performance times if it is only used to derive parameters for an

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