A Note on Magnetic Circuit Calculations

Abstract—**A technique that is similar to load-line analysis of a vacuum tube amplifier for solving for the flux in a magnetic circuit is given.**

Most textbooks devoted to the basic principles of electrical engineering illustrate a trial and error solution for the flux in a simple magnetic circuit excited by a constant current. A graphical solution is briefly alluded to in Smith [1], Craven [2], and Fitzgerald, Higginbotham; and Grabel [3], and is not suggested at all in a recent reference [4]. Attention is called to a graphical solution of the simple series magnetic circuit, which is closely related to load-line analysis of a simple vacuum tube amplifier.

Consider the magnetic circuit shown in Fig. 1, in which the core is composed of an alloy whose *B-H* curve is shown in Fig. 2. The ferromagnetic material is assumed to have a uniform cross-sectional area A, a mean path length L_m, and an air gap of length L_a. Flux fringing at the gap is assumed to be negligible and leakage fluxes are not considered. The equivalent electrical circuit of the magnetic circuit of Fig. 1 is shown in Fig. 3.

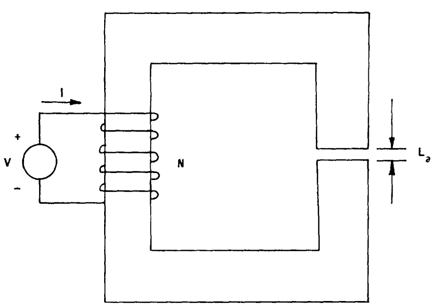

Fig. 1. The magnetic circuit.

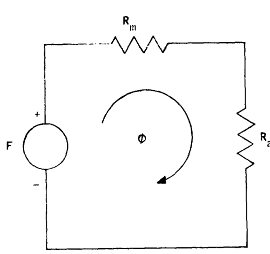

Fig. 3. Electrical circuit.

From Fig. 3 it follows that

$$F = \phi R_a + F_m, \tag{1}$$

where

$F = NI$ = total MMF supplied
R_a = air gap reluctance
ϕ = magnetic flux
F_m = MMF of the ferromagnetic material.

Solving (1) for ϕ gives

$$\phi = -\frac{1}{R_a}F_m + \frac{F}{R_a}.$$

Substituting

$$\phi = B_m A, \qquad F = NI, \qquad R_a = L_a/\mu_0 A,$$

and

$$F_m = H_m L_m,$$

the above equation may be rewritten in the form

$$B_m = -(\mu_0 L_m/L_a)H_m + \mu_0 NI/L_a. \tag{2}$$

Equation (2) is a straight line in the variables B_m and H_m. This equation can be plotted on the *B-H* curve of the material, with a horizontal intercept of NI/L_m and a vertical intercept of $\mu_0 NI/L_a$. The intersection of this load line with the *B-H* curve of the material, as shown in Fig. 4, gives the resulting magnetic flux density from which the flux may be easily computed. (If no air gap is present, the load line is simply a vertical straight line through the point $H_m = NI/L_m$.)

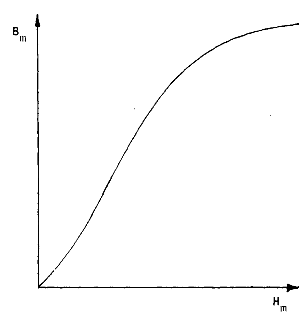

Fig. 2. The *B-H* curve.

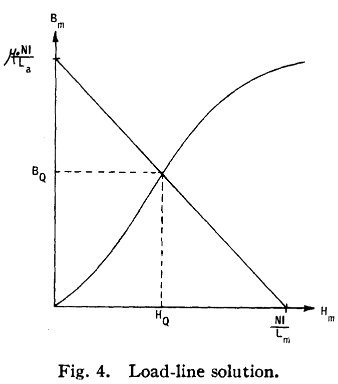

Fig. 4. Load-line solution.

Robert J. Krueger
Electrical Department
College of Applied Science and Engineering
University of Wisconsin-Milwaukee
Milwaukee, Wis. 53201

References

[1] R. J. Smith, *Circuits Devices and Systems*. New York: Wiley, 1966.
[2] A. T. Craven, *Electricity and Magnetism for Electrical Engineers*. Reading, Mass.: Addison-Wesley, 1962.
[3] A. E. Fitzgerald, D. E. Higginbotham, and A. Grabel, *Basic Electrical Engineering*. New York: McGraw-Hill, 1967.
[4] V. Del Toro, *Principles of Electrical Engineering*. Englewood Cliffs, N. J.: Prentice-Hall, 1965.

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Conservation of Energy in the Ideal Capacitor Discharge

Abstract—**The search for the best answer to the question of the final resting place for the energy in an ideal capacitor discharge is completely satisfied by considering the first law of thermodynamics. The energy is transferred to the internal energy of the materials and is conserved. A simple energy-band model is used to illustrate the physical aspects of the problem.**

Levine's recent discussion[1] on the fate of the energy in an ideal capacitor discharge is interesting, not so much because of the interjection of a nonconservative system, but rather the model that diminishes the hand-waving argument of radiation. I would like to offer a better explanation which satisfies the anti-nonconservation concept, but which is probably less satisfactory due to its subtlety.

The first law of thermodynamics states the principle of conservation of energy. In the absence of heat transfer into or out of the system, any rearrangement in the energies involves every transfer between work done on or by the system and gain or loss of internal energy. Consider Levine's model, which does not permit the energy to be radiated, and assume it to be in thermal equilibrium with its surroundings. There are two systems that will be considered below. The first is that total capacitor and its connecting link, and the second is the materials that make up the capacitor and its connecting link.

Suppose that an external force, e.g., a battery, is applied to the capacitor. Then the work done on the capacitor system in transferring charge from one plate to the other is converted to the internal stored energy of the capacitor system. Now when the shorting link is connected, the work done by the capacitor system in dischargng is transferred to the internal energy of the materials that make up the capacitor. The best explanation for the lost energy then is that it has been transferred to internal energy. To complete the discussion, although the above statement completes the argument, it should be stated that the internal system is no longer in equilibrium with its surroundings and that the excess internal energy will be transferred as heat by whatever means are available. Since no discussion of this kind is complete without a pedagogical example, consider the following ideal system.

Fig. 1 represents the energy-band diagram for the two plates, which are assumed to be identical, prior to charging. The interconnecting link will be assumed to be an ideal conductor. When the capacitor is charged, a small number of occupied energy states of plate 1 are emptied and the charge transferred to plate 2. The average energy difference between the charge on the two plates is indicated in Fig. 2, where eV repre-

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[1] R. C. Levine, "Apparent nonconservation of energy in the discharge of an ideal capacitor," *IEEE Trans. Education*, vol. E-10, pp. 197–202, December 1967.

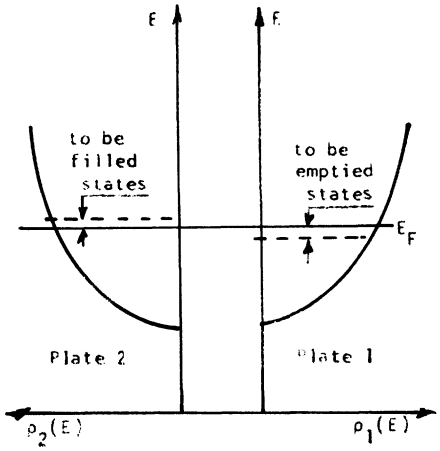

Fig. 1. Equilibrium energy-band diagram showing states from which and to which charge will be transferred.

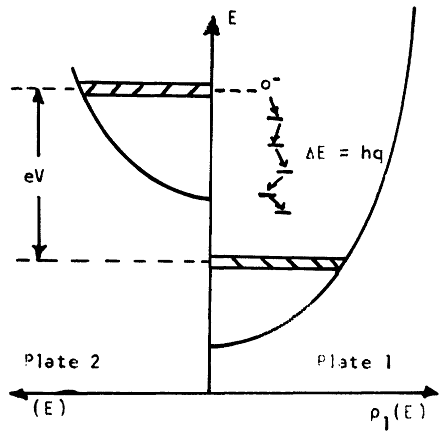

Fig. 2. Energy-band diagram for energized capacitor, which indicates energy transfer to internal energy during discharge.

sents the energy through which each charge has been transferred. When the plates are connected, each transferred charge makes a horizontal transition from plate 2 to plate 1. After transferring, the electron seeks the lowest empty energy state and, in decaying to it, transfers its extra energy to atoms of the plate, thereby increasing the thermal energy of the plate. The plate, in turn, will lose this energy by convection, conduction, or radiation, until it is in equilibrium with its surroundings. The completely satisfying explanation for the final resting place of the lost energy is that it is converted to internal energy.

William B. Berry
Dept. of Elec. Engrg.
University of Notre Dame
Notre Dame, Ind.

Author's Reply[2]

W. B. Berry has realized the same conclusion reached by C. Goldberg,[3] that the energy originally stored in the electrostatic field of the capacitor is converted into internal kinetic energy of the electrons within the conductor. To amplify upon Berry's description, it can be noted that during continuous current in an ohmic conductor, the electrons transfer to a higher energy level, and then lose some of their energy through inelastic collisions with the lattice of atoms. Thus, there is a continual flux of electrons up and down in energy level in the case of an ohmic conductor, but only an *upward* transfer in a perfect conductor, as Goldberg has noted.

Richard C. Levine
Stevens Inst. of Technology
Hoboken, N. J. 07030

[2] Manuscript received July 8, 1968.
[3] C. Goldberg, "The concept of 'zero resistance,'" *IEEE Trans. Education* (*Correspondence*), vol. E-11, pp. 159–160, June 1968.

Comments on "Apparent Nonconservation of Energy in the Discharge of an Ideal Capacitor"

In the above correspondence,[1] Levine has spent five pages on that old chestnut, the discharge of an ideal capacitor, without getting to what I think is the main point. He correctly asserts that the lossless case can be considered as the limiting case of an RC circuit when the resistance approaches zero. He also implies the correct statement that it can be considered as the limiting case of an LC circuit in which the inductance approaches zero. Interestingly, these two limiting situations lead to different results. His argument about radiation is invalid unless it is considered as still another limiting situation. By neglecting the fringing fields ("consistent with circuit theory," he asserts) he has rendered invalid his conclusion that "there is no power flow at the edges of the cylinder volume and no radiation of energy to the outside world."

It is trivial to state that energy is, in fact, conserved; any energy which is not stored in the capacitors after the transient has been radiated, converted into heat, dissipated as acoustic energy, or gone somewhere else where it could be accounted for by appropriate theory. A somewhat more profound way of stating this is that we believe in the law of the conservation of energy somewhat more strongly than we believe in the "laws" of circuit theory. The most important pedagogic point is that the models of circuit theory are exactly that—models. A linear capacitor without resistance or inductance is a most useful concept, and models based on it have allowed significant understanding of real electrical phenomena, but it is clearly an abstraction and, occasionally, can get us into trouble. When it is combined with an otherwise innocuous perfect switch and resistanceless and inductanceless wires, it does get us into trouble, as in this particular example.

A comparable chestnut, which has been discussed in the literature,[2] concerns the attempt to distinguish between two black boxes, one of which contains a pure resistor and the other a certain series-parallel resonant RLC circuit whose impedance is independent of frequency. The answer comes down to this: if you believe your "linear, lumped, bilateral, passive" models, then the two are indistinguishable; if you do not (for example, if the black boxes are real), then there are any number of tests which can be applied to distinguish between them. Anything else which may be said about this problem is extrinsic pedantry.

For the amusement of those who enjoy "paradoxes" based on the incompatibility of otherwise useful model abstractions, I submit the following from thermodynamics. In Fig. 1, AB and CD are arcs of confocal

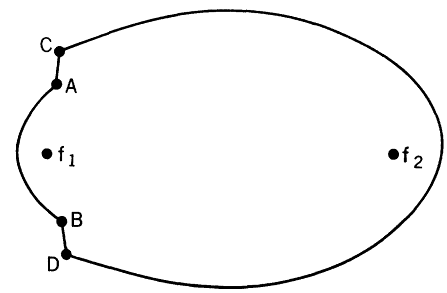

Fig. 1.

ellipses, the foci being f_1 and f_2. BD and AC are arcs of a circle centered on f_2. f_1, B, and D are colinear, as are f_1, A, and C. (Note: this construction is possible.) We now rotate this plane figure around the line f_1f_2 to generate a three-dimensional body of revolution. We coat the inside of it with the usual perfect reflectors, place the usual black bodies at f_1 and f_2, and heat them to some temperature t. All of the radiation from f_1 will now be incident on one of the ellipsoids and will therefore be reflected to and absorbed at f_2. The same will be true of some fraction of the energy radiated from f_2, but a nonzero fraction of the energy from f_2 will be reflected by the spherical surfaces back onto f_2 where it will be absorbed. It follows that the temperature of f_2 will rise above that of f_1 in contradistinction to the Second Law.

Robert E. Machol
School of Business
Northwestern University
Chicago, Ill. 60611

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[1] R. C. Levine, *IEEE Trans. Education*, vol. E-10, pp. 197–202, December 1967.

Author's Reply[3]

Most of the comments, written and oral, which I have received concerning my original paper,[1] fall into two distinct classes. They say either, "Your treatment is under-

[2] See M. T. Lebenbaum, "Earlier essay on black box problem," *Proc. IEEE* (*Correspondence*), vol. 51, p. 864, May 1963, where earlier references are cited and well assessed.
[3] Manuscript received June 27, 1968.