Correspondence

A Proposed First-Order Relativity Test Using Lasers*

The new-found ability to mix together the outputs of two coherent optical sources, such as gas lasers, and obtain stable beats, raises the possibility of a new test for ether drift in which the effect to be measured is proportional to the first power of v/c, where v is the expected ether drift.

The classical Michelson-Morley experiments, using one optical source, measure an effect proportional to v^2/c^2 because of the necessity of returning the light beam to its starting point. The difference in transit times, go and return, is of the form

$$L/(c-v)+L/(c+v)=2L/c \cong (2L/c)(v^2/c^2)$$

where L is the one-way path length. If one could arrange to measure directly the difference in transit times for go and return, this would be in the form

$$L/(c-v)-L/(c+v) \cong (2L/c)(v/c),$$

greater than the Michelson-Morley effect by the factor c/v.

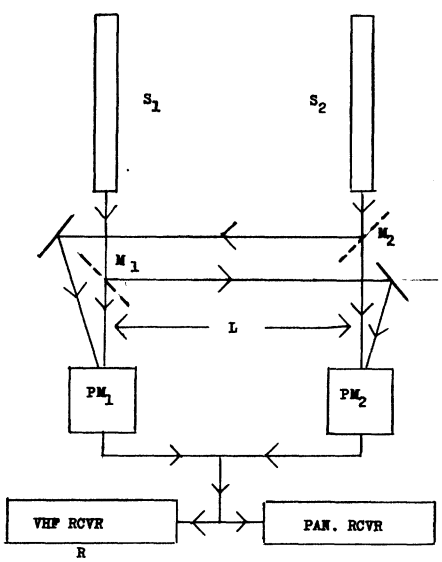

Fig. 1

Referring to Fig. 1, S_1 and S_2 are two CW lasers. Half-silvered mirrors M_1 and M_2 split the two beams, sending one half directly into the photomultipliers PM_1 and PM_2, and the other half over the optical path L. At the ends of L the beam from S_1 is reflected into PM_2, and the beam from S_2 into PM_1. The outputs of PM_1 and PM_2, which contain VHF components due to the mixing of the two optical laser frequencies, are added and their sum is measured by the VHF receiver R. A panoramic receiver is added at this point to facilitate spectrum examination. The two lasers are mounted vertically with the optical path horizontal, and the whole assembly is designed to rotate in the horizontal plane.

For simplicity let us assume that the two lasers are adjusted for one mode of oscillation only, and that they are offset in frequency by 100 mc. There will then be 100-mc components in the outputs of the mixers, and their sum will be selected and measured by the VHF receiver. Let us suppose that in a position of zero ether drift in the direction of L, the path length of one laser is adjusted to make the input to the receiver a maximum, *i.e.*, to assure that the two mixer outputs are in phase at the receiver. Now let us rotate the apparatus through 90° so that L is in the direction of the ether drift v. If this drift is from left to right in Fig. 1, the phase of the waves from S_1 at PM_2 is advanced by the angle $\phi=2\pi(L/\lambda)(v/c)$, where λ is the wavelength, and the phase of the waves from S_2 at PM_1 is retarded by the same angle. Since the phase changes in the optical signals are transferred without loss to the phases of the mixer outputs, there will be a relative change in phase of 2ϕ between the 100-mc signals at the input to the receiver. If the intensities are equal and $\phi=\pi/2$, there will be complete cancellation; intermediate values will be a measure of the phase shift.

Note that the success of this method is due to the ability to measure the relative phase advance and retardation of the optical frequency components at the ends of the optical path by transferring these phase shifts to a greatly lower frequency in the VHF region, so that they may be brought together for comparison with a negligible compensation of the effect.

For a helium-neon laser, with $\lambda=1.153 \times 10^{-4}$ cm, L may be as short as 30 cm for a phase shift of 180° with $v/c=10^{-6}$, corresponding to an ether drift of 0.3 km/sec, the surface velocity of the earth in temperate latitudes due to its rotation. This short length is desirable since it simplifies problems of mechanical rigidity and also makes magnetic shielding possible if changes in magnetic field during rotation turn out to be bothersome.

In the interests of simplicity and lack of fussiness, confocal resonators should undoubtedly be used with the lasers. The presence of many modes in these resonators will produce a complicated spectrum of intermodal beats from the mixers. The panoramic receiver may then be used to select interlaser beats by noting components that are present in the mixer outputs only when both lasers are operating.

It is not anticipated that frequency drift in the lasers will be a problem, provided that the change in the beat frequency will not be so large or rapid as to be unmanageable in the receiver. A slight error will occur in the phase comparison if the frequency of one laser changes during rotation; for a 1-mc shift in one laser the error in the phase angle will be only 0.002 radians. Actually, the apparatus should be small enough to permit fairly rapid rotation.

As has been pointed out by this author,[1] the Michelson-Gale experiment[2] has its simplest nonrelativistic explanation in terms of an ether that does not rotate with the earth. Since this experiment has been explained by the theory of general relativity, and since the precision of the Michelson-Morley experiments was not great enough to detect a rotational ether drift, it is of considerable importance to try a similar method that does have the capability.

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* Received June 13, 1962.

[1] C. W. Carnahan, "Light and gravitation," Proc. IRE (to be published).

[2] A. A. Michelson and H. G. Gale, *Astrophys. J.*, vol. 61, p. 140, 1925; also, G. Joos, "Theoretical Physics," Hafner Publishing Company, Inc., New York, N. Y., 2nd ed., p. 472, 1950.

A Light Source Modulated at Microwave Frequencies*

When a gallium arsenide p-n junction is biased in the forward direction, radiative band-to-band recombination is observed.[1] Since minority-carrier lifetimes of the order of 10^{-10} sec are readily obtained in GaAs, one may expect that the recombination radiation can be modulated at Gc rates. This communication reports a verification that efficient generation of light modulated at microwave frequencies is possible.

The current through a GaAs diode increases very rapidly when it is forward biased with an increasing voltage nearly equal to the energy gap (about 1.5 volts). Under this bias condition, the current consists of tunnel-assisted radiative band-to-band recombination in the space-charge region of the p-n junction.[2] This radiation occurs in a narrow spectral band in the near infrared (0.84 μ at 77°K). The intensity of the light output first increases very rapidly (more than linearly) with current and then linearly. In the linear range the process is extremely efficient. A quantum efficiency of 0.50 to 1.00 photons/electron has been obtained. However, with the geometry used in our experiment only about 1 per cent of the radiation comes out of the specimen. The over-all power efficiency of the light source is also somewhat reduced by a small ohmic loss due to the internal resistance of the diode.

The following measurements were made with a diode fabricated by alloying a tin dot

* Received June 2, 1962.

[1] J. I. Pankove and M. Massoulié, "Injection luminescence from GaAs," *Bull. Am. Phys. Soc.*, vol. 7, p. 88; January, 1962.

[2] J. I. Pankove, "Tunneling assisted photon emission in Gallium Arsenide p-n junctions," to be published.

to p-type GaAs having a hole concentration of 2.5×10^{18} cm^{-3}. The diode was mounted in series with a 50-ohm resistor at the end of a 50-ohm coaxial cable connected to a signal generator. The diode end of the cable was inserted in a Dewar filled with liquid nitrogen (Fig. 1). The radiation was collected through the two windows of the Dewar by a lens and focused onto a photomultiplier (RCA 7102) having an S-1 spectral response. The output of the photomultiplier was displayed on an oscilloscope. Fig. 2 shows the detection of 200-Mc modulation as displayed on a sampling oscilloscope. A dc bias was inserted in series with the generator to operate the diode in the light-emitting mode. The noise is believed to originate in the photomultiplier.

In its nonlinear range, the radiation from the diode is also modulated at harmonics of the driving frequency. This is illustrated in Fig. 3 where the upper curve (d) is a 6-Mc driving signal, and the lower curve (c), the photomultiplier output. (a) is the zero level for the photomultiplier output. The diode being insufficiently biased to give a linear light output, as the signal swings about the dc level (b), the light output is not symmetrical during the brightening and dimming half-cycles. The distortion of the driving voltage is due to the changing load impedance as the diode conductance increases.

The frequency limitation of our measurements is due to the transit time dispersion of electrons in the photomultiplier. Hence, an operating frequency of 200 Mc is not the upper limit for the diode. The RC limitation of this diode is of the order of 10 Gc.

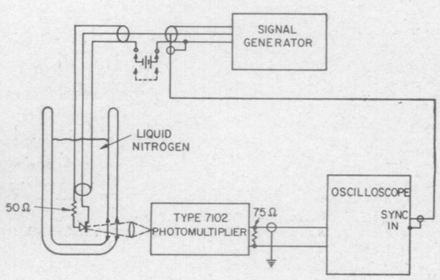

Fig. 1—Schematic diagram of test setup.

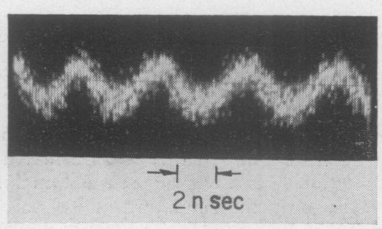

Fig. 2—Detection of an optical signal modulated at 200 Mc as displayed by a pulse sampling oscilloscope.

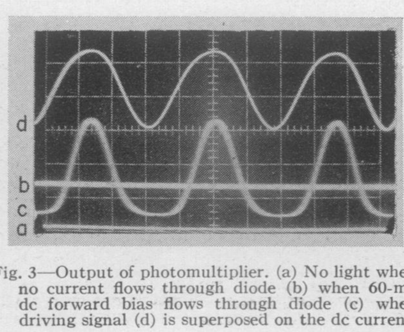

Fig. 3—Output of photomultiplier. (a) No light when no current flows through diode (b) when 60-ma dc forward bias flows through diode (c) when driving signal (d) is superposed on the dc current.

As was stated above, only about 1 per cent of the radiation leaves the specimen through the surface opposite the p-n junction. This light comes out in a 2π-steradian solid angle. An improvement of one to two orders of magnitude in light collection can be obtained by shaping the specimen into a Weierstrass sphere.[3]

We wish to thank G. B. Herzog for valuable suggestions.

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[3] P. Aigrain and C. Benoit-a-la-Guillaume, "Emission Infrarouge du Germanium," *J. de Phys. et Radium*, vol. 17, pp. 709–711; August–September, 1956.

Three-Dimensional Parametric Interactions of Waves and Quasi-Particles*

The parametric interaction in a cavity can be regarded as point interactions. The investigations of traveling-wave parametric interactions by Tien,[1] among others, extended the understanding to one-dimensional interactions. The purpose of this note is to show that the concept of these parametric interactions can be further generalized to two and three dimensions, and interpreted as scattering of coherent waves or quantized fields of quasi-particles. These concepts apply not only to electromagnetic waves, but also to interactions involving elastic waves, spin waves, plasma waves, etc. As quantized fields, parametric interactions can be interpreted as the annihilation or creation of photons, phonons, magnons, plasmons, etc.

In order to demonstrate the possibility of achieving parametric interaction of traveling waves in three-dimensional media, we choose a moving coordinate system which is moving at an arbitrary velocity $\boldsymbol{v}$. Let the frequencies of the original traveling waves be ω_p, ω_i, and ω_s for the pump, idler, and signal, respectively. The corresponding Doppler frequencies in the moving coordinates become ω_p', ω_i' and ω_s'. Then

$$\begin{aligned} \omega_p' &= \omega_p - \boldsymbol{\beta}_p \cdot \boldsymbol{v} \\ \omega_i' &= \omega_i - \boldsymbol{\beta}_i \cdot \boldsymbol{v} \\ \omega_s' &= \omega_s - \boldsymbol{\beta}_s \cdot \boldsymbol{v}, \end{aligned} \tag{1}$$

where $\boldsymbol{\beta}_p$, $\boldsymbol{\beta}_i$, and $\boldsymbol{\beta}_s$ are the phase constants of the three traveling waves. $\boldsymbol{\beta}$ and $\boldsymbol{v}$ are vector quantities. Parametric interaction becomes possible if the pump frequency is equal to the sum of the idling and signal frequencies, and if the relationship holds for any arbitrary velocity of the moving coordinate system. That means we have

$$\omega_p = \omega_i + \omega_s, \tag{2}$$

and require that

$$\omega_p' = \omega_i' + \omega_s'. \tag{3}$$

From the above three equations we get

$$\boldsymbol{\beta}_p = \boldsymbol{\beta}_i + \boldsymbol{\beta}_s. \tag{4}$$

Eq. (4) is the condition on the phase constants. Conversely if (2) and (4) hold, (3) becomes valid and parametric interaction is possible. Thus, (2) and (4) are the selection rules to be satisfied for traveling-wave parametric interactions.

For periodic structures such as crystals,[2] (4) is equivalent to

$$\boldsymbol{\beta}_p = \boldsymbol{\beta}_i + \boldsymbol{\beta}_s + 2\pi \boldsymbol{g}, \tag{4a}$$

where $\boldsymbol{g}$ is a lattice vector in the reciprocal lattice. When $\boldsymbol{g}$ is not zero, the interaction corresponds to the so-called "umklapp" process in solids.[3]

In the one-dimensional case, the phase constants can be regarded as scalars with either positive or negative signs. Then (2) and (4) are reduced to Tien's equations.[1] In a nonlinear medium it is possible to achieve either forward or backward traveling-wave parametric amplifications. The significance of the umklapp process has been demonstrated in backward traveling-wave parametric amplifiers.[4]

Eqs. (2) and (4) can be expressed as

$$\hbar\omega_p = \hbar\omega_i + \hbar\omega_s, \tag{5}$$

$$\hbar\boldsymbol{\beta}_p = \hbar\boldsymbol{\beta}_i + \hbar\boldsymbol{\beta}_s, \tag{6}$$

and

$$\hbar\boldsymbol{\beta}_p = \hbar\boldsymbol{\beta}_i + \hbar\boldsymbol{\beta}_s + h\boldsymbol{g}, \tag{6a}$$

where $\hbar$ is Planck's constant, h, divided by 2π.

Eqs. (5), (6) and (6a) can be regarded as the particle aspect of traveling-wave interactions. Eq. (5) indicates the conservation of energy, (6) the conservation of momentum, and (6a) the conservation of crystal momentum. It is expected that (5), (6), or (6a) will be satisfied in any scattering processes. Thus, parametric interactions due to scattering of the coherent quantized fields of quasi-particles can conceivably be achieved.

The above discussion can be readily extended to frequency mixing, harmonic generation and parametric interactions involving multiple frequencies.[5] The selection rules, corresponding to the conservation laws, can be generalized as

$$\sum_i E_i = \sum_s E_s \tag{7}$$

* Received February 19, 1962. This work was supported in part by U. S. Signal Corps Contract No. DA-36-039-SC-87209.

[1] P. K. Tien, "Parametric amplification and frequency mixing in propagating circuits," *J. Appl. Phys.*, vol. 29, pp. 1347–1357; September, 1958.

[2] L. Brillouin, "Wave Propagation in Periodic Structures," Dover Publications, Inc., New York, N. Y., p. 137; 1953.

[3] R. E. Peierls, "Quantum Theory of Solids," Oxford University Press, Oxford, England, p. 41; 1955.

[4] H. Hsu, "Backward traveling-wave parametric amplifier," in "Microwave Tubes," J. Wosnik, Ed., Academic Press, New York, N. Y., pp. 342–345; 1961.

H. Hsu and S. Wanuga, "The wide tuning range of backward traveling-wave parametric amplifiers," Proc. IRE (*Correspondence*), vol. 49, pp. 1339–1340; August, 1961.

[5] See, for example, H. Hsu, "Multiple frequency parametric devices," *Rept. of NSIA-ARDC Conf. on Molecular Electronics*, November 13–14, 1958, Washington, D. C., pp. 81–85, 1958; *Digest of Solid-State Circuits Conf.*, February 12–13, 1959, pp. 12–13, 1959.

and

$$\sum_i \beta_i = \sum_s \beta_s \tag{8}$$

or

$$\sum_i \beta_i = \sum_s \beta_s + 2\pi g, \tag{8a}$$

where E_i and E_s are the energies of the incident and scattered quasi-particles or traveling waves, β_i and β_s are the corresponding phase constants. Eq. (8) applies to continuous media, and (8a) to periodic media. Eqs. (7) and (8a) can be identified as the Bragg Law in the special case of direct scattering involving only one incident wave and one scattered wave.

It should be pointed out that the above selection rules can be calculated quantum mechanically from the interaction Hamiltonian in various collision processes. But the concept of three-dimensional parametric interactions was not evident in the formal treatment because the propagation of coherent quantized fields of quasi-particles was not believed possible earlier. With the recent development of the optical maser and the successful propagation of coherent phonons, the concept of three-dimensional parametric interaction may become important in the study of solid-state physics and quantum-field theories. Furthermore, the recent successful generation of optical harmonics utilizing the nonlinearity in the electric susceptibility of piezoelectric crystals,[6] is in fact a demonstration of the three-dimensional parametric interaction of coherent photons. Similar parametric interactions should be possible for phonons, magnons, etc. There appears to be unlimited variety in the potential development of new devices.

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[6] P. A. Franken, A. E. Hill, C. W. Peters and G. Weinriech, "Generation of optical harmonics," *Phys. Rev. Lett.*, vol. 7, pp. 118–119; August 15, 1961.

Comments on "Relativistic Beam-Wave Interaction"*

The theoretical results in Rowe's paper[1] relate to the interaction which takes place between an axial electric field and an electron beam. In the system he considers, it appears that if the circuit potential gradient in the z direction is finite then the forcing field falls to zero when its phase velocity is equal to the velocity of light. His equation (7) is:

$$E_{cz} = -(1 - k_0^2)\frac{\partial V_c}{\partial z}.$$

The application of this equation leads to the appearance of a factor

$$\left[1 - \frac{k_e^2}{(1 + Cb)^2}\right]$$

in the circuit force parts of (21) and the small-signal determinantal equation (28).

In many of the circuits used in high-power traveling-wave amplifiers, however, the axial forcing field is finite when the phase velocity of the fundamental is equal to the velocity of light. The same is true of the structures used in linear accelerators which are designed to give large axial fields at this phase velocity.[2]

One may consider the finite field as being due to the presence of space harmonics. Together with the fundamental these satisfy the condition $E_{\tan} = 0$ at all the conducting boundaries. To do this it is not necessary for them individually to be equal to zero. For example, if we consider a tubular structure with a section as shown in Fig. 1, then whilst $E_z = 0$ over the metal surface from A to B, a finite field may exist between B and C. To satisfy this condition E_z at this radius must be composed of a fundamental, and space harmonics of finite amplitude. These will have different radial propagation constants inside this radius and hence the field pattern along the axis will be of a different form. When the fundamental has a phase velocity equal to the velocity of light its radial propagation constant is zero and hence its amplitude is unchanged.

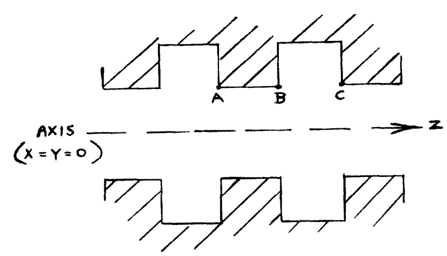

Fig. 1.

As an example of a circuit which would satisfy Rowe's theory we may consider a helix inside a conducting tube. To obtain a phase velocity equal to the velocity of light the helix must be stretched out until it becomes a straight wire. The system then becomes a coaxial line which propagates a TEM wave and E_z is zero at all points inside the circuit.

The cause of the anomaly appears to be in the fact that if V_c is a potential chosen so that E_{cz} is given by equation (7), it should not be identified with the voltage on an equivalent transmission line as is done by Pierce.[3] Since V_c tends to infinity as k_0 tends to unity, it is not really a very convenient variable. Pierce defines a different potential such that $E_{cz} = \Gamma V$, but this is not essential to the theory which can be carried through in terms of E_{cz} directly. The relativity corrections which have to be applied to the space-charge field can, for the small-signal theory, be most conveniently accounted for by working in terms of ω_q, which is the appropriately corrected effective plasma frequency.[4] When these modifications are made the small-signal, small C, determinantal equation becomes

$$\delta^2 = (1 - k_e^2)^{3/2} \cdot \left\{\frac{(1 + jC\delta)^2(1 + C(b - jd))}{(-b + jd + j\delta)(1 + \frac{1}{2}C(b - jd + j\delta))}\right\} - \left\{\frac{\omega_q}{\omega \cdot C}\right\}^2$$

where

$$C^3 = \frac{E^2}{\beta^2 P} \cdot \frac{I_0}{4V_0}.$$

The value of $E^2/\beta^2 P$ used being that for the fundamental component of the electric field.

This can further be simplified by writing

$$\delta' = p_e\delta, \quad b' = p_e b, \quad d' = p_e d, \quad C' = C/p_e.$$

It then becomes

$$\delta'^2 = \left\{\frac{(1 + jC'\delta')^2(1 + C'(b' - jd'))}{(-b' + jd' + j\delta')(1 + \frac{1}{2}C'(b' - jd' + j\delta'))}\right\} - \left\{\frac{\omega_q}{\omega C'}\right\}^2.$$

Published data on the roots of this equation can be used,[5] the space-charge parameter being approximately

$$4Q'C' = (\omega_q/\omega C')^2.$$

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[4] M. Chodorow, E. L. Ginzton, I. R. Neilson, and S. Sonkin, "Design and performance of a high-power pulsed klystron," PROC. IRE, vol. 41, pp. 1584–1602; November, 1953.

[5] C. K. Birdsall and G. R. Brewer, "Normalized Propagation Constants for a Traveling Wave Tube for Finite Values of C," Hughes Aircraft Co. Culver City, Calif., Tech. Memos. Nos. 331 and 396; October 1953 and June, 1955.

Author's Reply[6]

The relativistic device analysis developed by the author in "Relativistic Beam-Wave Interactions"[1] is a one-dimensional analysis in which only the fundamental RF charge density in the beam is considered in calculating the circuit RF voltage amplitude and phase. Harmonics of the fundamental charge density are however considered in computing the space-charge field in the force equation. Boundary conditions, of course, do not enter into a one-dimensional problem.

Fundamental to the analysis is the assumption that only one space-harmonic component of the circuit field has appreciable interaction with the beam. In a two-dimensional problem wherein one takes all space harmonics of the total field into account including the complete energy storage circuit, each individual space-harmonic field

* Received April 23, 1962.

[1] J. E. Rowe, "Relativistic beam-wave interaction," PROC. IRE, vol. 50, pp. 170–177; February, 1962.

[2] R. B. R-Shersby-Harvie, "Travelling wave linear accelerators," *Proc. Phys. Soc.*, vol. 61, pp. 255–270; September, 1948.

[3] J. R. Pierce, "Traveling Wave Tubes," D. Van Nostrand Co., Inc., New York, N. Y., ch. 2; 1950.

[6] Received May 9, 1962.

component does not satisfy Maxwell's equations whereas the total field does. In this situation some space-harmonic components will be zero at $k_0=1$ whereas others may not be. The exact solution of the two-dimensional problem plus boundary conditions will indicate this.

For arbitrary periodic structures the axial component of electric field will go to zero at the upper cutoff frequency where the phase velocity is infinite and in general there will be a nonzero axial electric field at $v_p=c$.

In the present analysis the optimum interaction condition occurs when the beam is synchronized with the wave-phase velocity, and since the electron velocity is necessarily less than the velocity of light then both k_e and k_0 are less than unity and there will be a finite (though small) axial electric field. Synchronism between the beam and the wave results in the RF wave appearing as a static wave and it in turn exerts long continued forces on the electrons. As pointed out by Monk and Curnow the helical waveguide satisfies these conditions in that the axial electric field becomes vanishingly small at $v_p \rightarrow c$ and in the limit a TEM wave exists. The presence of other space harmonics may be accounted for in the circuit equations by including space-harmonic terms as additional driving terms. A specific RF structure type would then have to be considered in order to analytically specify the form of the driving terms.

The Lorentz transformation of the potential four-vector indicated in (1) assumes that three components of the vector potential are zero in the moving reference frame (purely electrostatic) which results in (7) and also the final working equations. An alternate transformation to (1) may be made in which the three components of the vector potential are assumed zero (purely electrostatic field) in the laboratory system leads to $E_z=-\partial V_c/\partial z$ which parallels the suggestion made by Monk and Curnow. Both situations are, of course, only approximate since the magnetic energy storage has been neglected. Its importance depends partly on the transverse structure dimensions relative to a free-space wavelength. If the alternate transformation is used the only change is that the factor $[1-k_e^2/(1+Cb)^2]$ is eliminated from (21) and (28). Representative efficiency calculations made using the alternate definition of E_c are shown in Fig. 2, and are compared with the earlier calculations.

In any case it is apparent that as $k_e \rightarrow 1$ the parameter C becomes small and large interaction lengths would be required. It is thus important to carry out beam bunching at low velocities and extract energy in a relativistic section.

The small-signal, *small-C*, determinantal equation given by Monk and Curnow is not restricted to small C and should contain another factor, $(1+jC\delta)^2$ in the second term, *i.e.*, $4QC(1+jC\delta)^2$ represents the space-charge field. Replacing $4QC^3$ by $(\omega_q/\omega)^2$ assumes that $\omega_q/\omega \ll 1$ which is not always valid in large C devices. In general

$$4QC^3 = \left\{\frac{\dfrac{\omega_q}{\omega}}{1+\dfrac{\omega_q}{\omega}}\right\}^2.$$

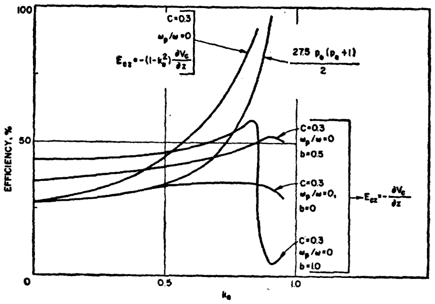

Fig. 2—Efficiency vs relativity factor.

A general relativistic analysis must include the effects of transverse fields since the Lorentz contraction applies differently to axial and transverse fields and in addition the space-charge field components are important in the two directions. The author is presently developing a general two-dimensional nonlinear analysis in which space-harmonic field components are considered.

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Scatterer Echo Area Enhancement*

Various communications systems have been proposed in which the received signal is one scattered from an object irradiated by a distant source.[1,2] It is desired that the power available at the receiver be as large a fraction of the transmitted power as is economically feasible. One method of achieving this is by making the echo area of the scatterer large in the direction of the receiver. In the case of satellites this may be achieved by rotationally stabilized high gain scatterers or isotropic unstabilized scatterers. If it is not desired to use rotationally stabilized satellites, the weight of extremely large isotropic scatterers such as spheres ultimately becomes prohibitive. It would be desirable to further increase the apparent echo area of scatterers without a prohibitive weight increase while retaining most of the advantages of the rotationally unstabilized isotropic scatterer.

A method of enhancing the echo area of a satellite scatterer has been proposed in which a Van Atta array would have amplifiers inserted in the lines connecting the array elements. This approach has the merit of providing reliability through the redundancy of small components as well as requiring only a relatively coarse rotational stabilization of the vehicle. The realization of such an array in which the weight is not prohibitive is a formidable task in miniaturization techniques development. If a simple lightweight technique could be developed for enhancing the echo area of a scatterer, array theory could be used to develop an entire class of pseudopassive scatterers for application to problems of communications and field measurement.

Although the problem is one of realizing an enhanced bistatic scatterer (*i.e.*, a three-object system consisting of source, scatterer and receiver), it is very similar to that of the monostatic scatterer (*i.e.*, the two-object case in which scattering is directed back to the source). In the monostatic case echo area is defined as that area for which the field incident on the scatterer contains sufficient power to produce, by omnidirectional radiation, the same field as is actually backscattered to the source. Hereafter the words "echo area" will be understood to pertain to that defined above for the monostatic case.

The echo area of a scattering object having two closely spaced terminals to which is connected a load impedance, Z_L, has been formulated in terms of the two-port open circuit impedance parameters.[4] The subscript 2 pertains to the variables and parameters of the scatterer and the subscript 1 pertains to those of the radiating object. These objects are assumed to be imbedded in linear, isotropic matter.

The general relationships are specialized to the plane wave case by allowing the source to recede to an infinite distance from the scatterer. The echo area, O, of the scatterer then becomes[5]

$$\sigma = \frac{\lambda^2}{\pi}\left|(Z_{11}-Z_1)-\frac{Z_{12}^2}{Z_{22}+Z_L}\right|^2. \quad (1)$$

The open circuit impedance parameters comply with the standard two-port definitions. Z_{11} and Z_1 must be carefully distinguished. Z_{11} is the driving point impedance at the source with the scatterer terminals open circuited (*i.e.*, $Z_L=\infty$). Z_1 is the driving point impedance at the source with the scatterer removed (*i.e.*, the one-port condition). Z_{11} does not in general equal Z_1, otherwise open circuiting of the scatterer would require a zero echo area even though the two electrically isolated parts of the scatterer are capable of scattering the field.

If (1) is further specialized for the description of short dipole scatterers the term $Z_{11}-Z_1$ becomes appreciably small relative to the remaining term.[6] If attention is confined to this type of elementary scatterer and the echo area of the scatterer is normalized with respect to its echo area with its terminals shorted (*i.e.*, $Z_L=0$), the normalized echo area σ_N becomes

$$G_N \approx \left|\frac{Z_{22}}{Z_{22}+Z_L}\right|^2. \quad (2)$$

A very small dipole resonated with an inductive load may have a quality factor as high as 485 and behaves like a series RLC

* Received June 13, 1962.

[1] J. R. Pierce and R. Hompfner, "Transoceanic communications by means of satellites," Proc. IRE, vol. 47, pp. 372–380; March, 1959.

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circuit near the resonant frequency.[1] Although great cross section enhancement occurs, the narrow bandwidth of the scatterer makes it more suitable as a field measuring device than as a communications relay. A dimensionally resonant dipole would be more suitable as a communications signal scatterer as its equivalent RLC quality factor near resonance approximates 10. At 10 kMc this would represent a 10^3 Mc bandwidth between half-power points.

In the case of the dimensionally resonant, thin half-wave dipole Z_{22} is real and about 73 ohms. In the purely passive case equation (2) indicates that σ_N would be maximized to unity when $Z_L=0$. Assuming that it were possible to realize a negative real load R_L that approached R_{22}, σ_N could rise to indefinitely large values. For example, if it desired to make σ_N equal 100, R_L of -65.7 ohms would be required. Assuming the equivalent lumped reactance remained constant near resonance the quality factor would have increased from 10 to 100. Under these conditions the half-power bandwidth at 10 kMc would be 100 Mc, a useful communications bandwidth.

Amplification at microwave frequencies using the negative resistance characteristics of tunnel diodes is well known. A slot transmission amplifier using this effect has been demonstrated.[7] Echo area enhancement of dimensionally resonant dipoles loaded by reflection-type tunnel-diode amplifiers has been demonstrated[8] although much work remains in achieving simple, lightweight, regulated biasing power supplies which are insensitive to frequency changes in the desired bandwidth.

In addition to the network problems involved in designing the combined diode and associated bias supply to appear as a pure negative resistance over the microwave bandwidth of interest is the problem of obtaining a long-life, lightweight power supply. The electron-emitting isotopes will provide power for long periods[9] and have promise for this application.

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The Screening Effect of the Ionosphere*

The reflection of CW signals from the vicinity of an artificial earth satellite continues to be of interest. Studies concerning this phenomena are usually carried out at frequencies just above the critical frequency of the ionosphere where the largest effects are expected. However, it is in this exact frequency range that ionospheric screening is important. This phenomenon, although well known, is often overlooked in examining results of any CW reflection studies at diverse geographical locations. For example, the inclusion of ionospheric screening in statistical studies of CW reflection is essential in order to obtain good correlation.

In particular, WWV reflections, in which the transmitter is near Washington, D. C. and the receivers are in Columbus, Ohio, are only possible from induced ionization above the F layer maximum if the transmitter frequency is high enough to penetrate the ionosphere without being essentially "reflected" back to earth. Thus, to penetrate the ionosphere the transmitted frequency, f, must satisfy

$$f > f_c \sec \zeta$$

where

f_c is the critical frequency of the ionosphere, and

ζ is the angle of incidence, as shown in Fig. 1.

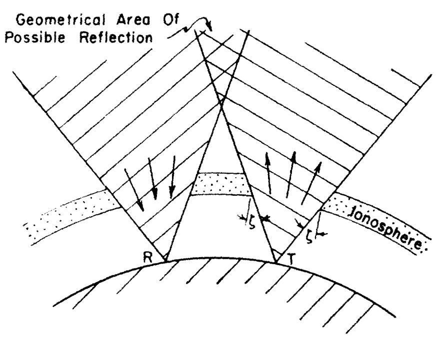

Fig. 1—Screening effect of the ionosphere relative to satellites above the F layer maximum.

For a spherical earth[1]

$$\zeta = \tan^{-1} \frac{\sin \psi/2}{1 + h/R - \cos \psi/2}$$

where

R is the radius of the earth,
ψ is the angle at the center of the earth subtended by the earth radii through the two points of interest, and
h is critical height of reflection in the ionosphere.

If f and f_c are fixed, Fig. 1 shows approximately how penetration of the ionosphere can occur from the standpoint of the transmitter location and how reflected signals at frequency f can repenetrate to the observing station.

These areas of penetration are "ionospheric holes"; their overlap allows the drawing of contours which enclose the maximum geometrical area of possible direct reflection from a satellite, at a fixed ratio of f to f_c, and a fixed satellite height (assumed to be above the F layer maximum). These contours are shown in Figs. 2–4 for three different satellite heights over Washington, D. C. and Columbus, Ohio.

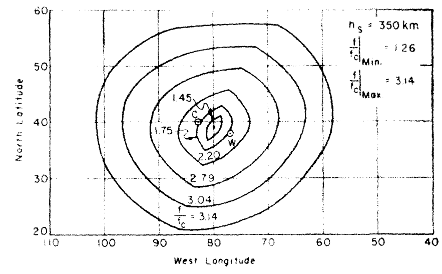

Fig. 2—Geometrical areas of possible reflection at a satellite height of 350 km. (Assumed above F_2 max.)

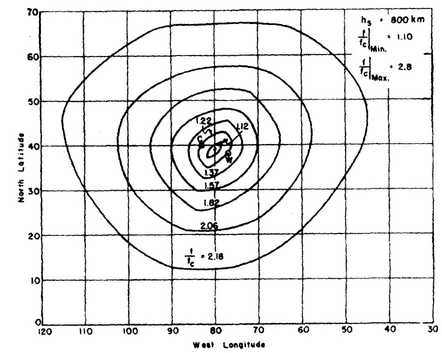

Fig. 3—Geometrical areas of possible reflection at a satellite height of 800 km.

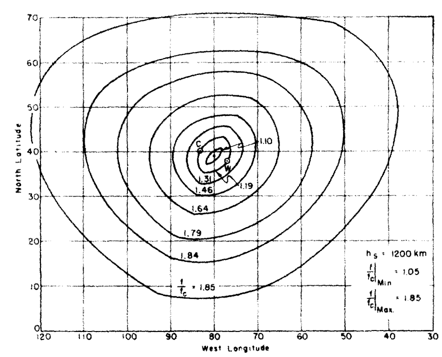

Fig. 4—Geometrical areas of possible reflection at a satellite height of 1200 km.

At each satellite height, there is a critical ratio $f/f_c|_{\min}$ which is the lowest ratio for which any reflection area is possible. Also, at each height a ratio $f/f_c|_{\max}$ gives the maximum geometrical area of possible reflection; higher ratios will yeild no larger area. It is emphasized that Figs. 1–4 are geometrical pictures and no allowance is made for refraction of the rays. It should be noted that the two stations which are the basis for Figs. 2–4 are close together (≈ 300 miles); however, when the transmitting station and the receiving station are separated by a larger distance $f/f_c|_{\min}$ becomes larger, and, at a particular ratio, the area of possible reflection is smaller. But, most important, in Figs. 2–4, it is seen that the higher the frequency, relative to the critical frequency, the larger the area of expected reflection.

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* Received February 5, 1962.

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A New Criterion for Evaluating the Number of Complex Roots of an Algebraic Equation*

In many physical problems it is very interesting to be able to evaluate the number of real roots of an algebraic equation with real coefficients. The problem may be solved directly or by means of the evaluation of the number of complex root pairs of the same equation. In order to reach this last result we introduce here a new criterion, different from those presented by Budan-Fourier, Sturm, and Segre.

The gist of the present method consists in constructing a polynomial $R(x)$ having as many positive real part zeros as are the complex roots of the equation; then the number of positive real part zeros of $R(x)$ may be evaluated by applying to this polynomial either the Routh-criterion or the Hurwitz-criterion.

Let the equation be

$$P_n(x) = \sum_0^n a_i x^i = 0.$$

A first approach to the solution of the problem might be to choose as polynomial $R(x)$ the polynomial $Q_{2n}(x)$, which is the product of the complex coefficients polynomials $P_n'(x)$ and $P_n''(x)$ whose zeros are obtained by multiplying the zeros of $P_n(x)$ by j and $-j$. In fact $Q_{2n}(x)$ [Fig. 1(a) and (b)] has as many positive real part zeros as there are complex zeros of P_n, but it also has some imaginary zeros (corresponding to the real zeros of P_n), and so the criteria of Routh and of Hurwitz might not be applied in a direct way. Instead of Q_{2n} we have then to construct a polynomial $R_{2n}(x)$ whose zeros are shifted by a little negative real quantity δ from those of Q_{2n} [Fig. 1(c)].

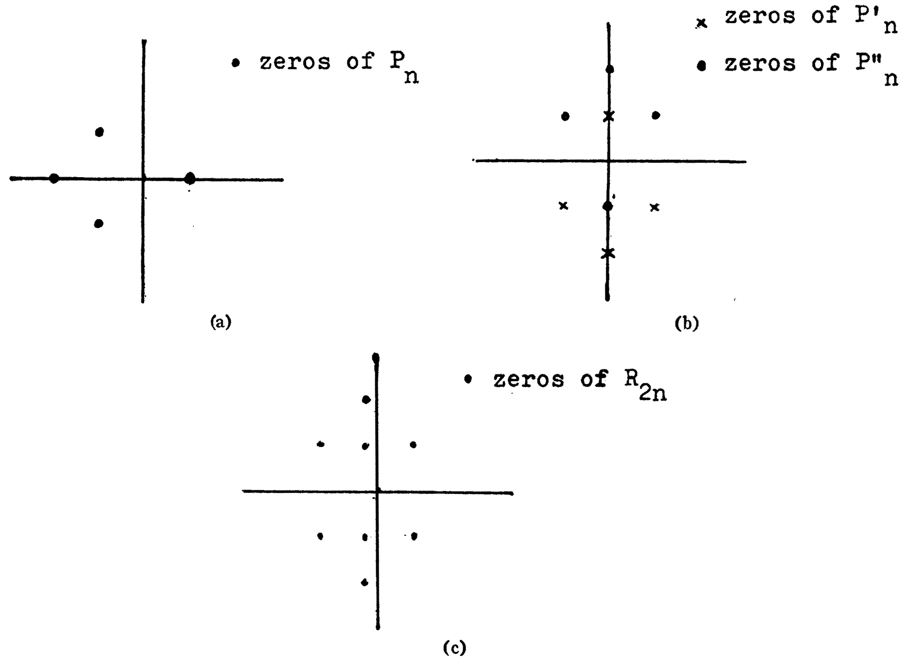

Fig. 1.

It is easy to realize that in the polynomial Q_{2n} the coefficients of the odd powers are equal to zero, and the coefficients b_{2i} of the even powers are

$$b_{2i} = \sum_{0}^{\min(i,n-i)}{}_k \; h a_{i-k} a_{i+k}$$

$$h = \begin{matrix} +1 & \text{if } k = 0 \\ 2(-1)^k & \text{if } k \neq 0 \end{matrix}$$

It is also easy to see that the coefficients c_i of the polynomial R_{2n} are given by the formulas

$$c_{2n} = b_{2n}$$

$$c_i = \left[b_i + \sum_1^{2n-i}{}_k (-1)^k \binom{i+k}{k} b_{i+k} \delta k \right], \quad (i \neq 2n)$$

For the purpose of the application of the criteria of Routh and of Hurwitz, we can choose a value of δ so small that its powers can be neglected. In such a way the criterion of Routh or the criterion of Hurwitz may be applied to the polynomial $R_{2n}'(x)$, very near to R_{2n} but whose coefficients c_i' are simpler functions of the coefficients a_i of the polynomial P_n.

In fact the coefficients c_i' are given by the formulas

$$\begin{aligned} c_{2n}' &= b_{2n} = a_n^2 \\ c_{2n-1}' &= -2nb_{2n}\delta = -2na_n^2\delta \\ &\vdots \\ c_{2i}' &= b_{2i} = \sum h a_{i-k} a_{i+k} \\ c_{2i-1}' &= -2ib_{2i}\delta = -2\delta i \sum h a_{i-k} a_{i-k} \\ &\vdots \\ c_1' &= -2b_2\delta = -2\delta(a_1^2 - a_0 a_2) \\ c_0' &= b_0 = a_0^2. \end{aligned}$$

If we will apply the Routh criterion, we may note that all the elements of the second row are multiplied by -2δ and that the elements of the third row are of the form

$$\frac{n-i+1}{n} b_{2(i-1)}.$$

Moreover the first element of the first row will always have a sign opposite to that of the first element of the second row.

Because of this, there is no need to consider the first row of Routh's set, and we may construct it starting from the elements of ths second row, divided by -2δ and from the elements of the third row multiplied by n. In such a way the criterion may be enunciated as follows.

The number of complex zeros of the polynomial with real coefficients $P_n(x) = a_i x^i$ is equal to the number of sign variations on the first column of the set:

$nA_n = \alpha_{n,1}$	$(n-1)A_{n-1} = \alpha_{n-1,1}$	$\cdots$	$(i+1)A_{i+1} = \alpha_{i+1,1}$	$iA_i = \alpha_{i,1}$	$\cdots$	$2A_2 = \alpha_{2,1}$	$A_1 = \alpha_{1,1}$
$A_{n-1} = \alpha_{n,2}$	$2A_{n-2} = \alpha_{n-1,2}$	$\cdots$	$(n-i)A_i = \alpha_{i+1,2}$	$(n-i+1)A_{i-1} = \alpha_{i,2}$	$\cdots$	$(n-1)A_1 = \alpha_{2,2}$	$nA_1 = \alpha_{1,2}$
$\alpha_{n,3}$	$\alpha_{n-1,3}$	$\cdots$	$\alpha_{i+1,3}$	$\alpha_{i,3}$	$\cdots$	$\alpha_{2,3}$	0
$\alpha_{n,4}$	$\alpha_{n-1,4}$	$\cdots$	$\alpha_{i+1,4}$	$\alpha_{i,4}$	$\cdots$	$\alpha_{2,4}$	0
$\vdots$	$\vdots$		$\vdots$	$\vdots$		$\vdots$	$\vdots$
$\alpha_{n,2n-1}$	0	$\cdots$	0	0	$\cdots$	0	0
$\alpha_{n,2n}$	0	$\cdots$	0	0	$\cdots$	0	0

where

$$A_i = \sum_0^{\min(i,n-i)}{}_k \; h a_{i-k} a_{i+k}$$

$$h = \begin{matrix} +1 & \text{if } k = 0 \\ 2(-1)^k & \text{if } k \neq 0 \end{matrix}$$

and

$$\alpha_{i,j} = \frac{-\begin{vmatrix} \alpha_{n,j-2} & \alpha_{i+1,j-2} \\ \alpha_{n,j-1} & \alpha_{i+1,j-2} \end{vmatrix}}{\alpha_{n,j-1}}.$$

To the above polynomial R_{2n}' we may also apply the Hurwitz criterion in the standard determinantal form; in this case too it is possible to introduce some simplifications by dividing the elements of the columns of the matrices to be considered by the same coefficient.

(More detailed information about the present criterion is given in a paper which is being published in the "Bollettino della Associazione Matematica Italiana.")

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Flow Graph Determination of the Over-All Scattering Matrix of Joined Multiports*

It is advantageous to be able to determine the over-all scattering matrix of a system of joined multiports from the known scattering matrixes of its components. Such a determination will allow us to synthesize a given multiport from simpler interconnected multiports. Conversely, we may extract the various components within a multiport.

The method will be illustrated with the simple case of a cascade of two, two ports as in Fig. 1(a). The results for this case have been solved[1] and presented elsewhere,[2] by other methods, allowing a comparison of results.

Draw S' and S'' in flow graph form as in Fig. 1(b), joining b_2' to a_1'' and b_1'' to a_2' as the output of one network will be the input of the other. The joining paths have a gain of unity indicating that the junction itself is matched. If this were not the case, we would have to add a third two port in between to account for the mismatch. Note that the inner ports are now within the composite network and the only external ports are those of a_1', b_1' and a_2'', b_2'' keeping the structure a two port. Calling a_1', b_1' the number one port, and a_2'', b_2'' the number two port, the S_{11} of the composite scattering matrix will be the total path gain from a_1' to b_1', and S_{21} will be the total path gain from a_1' to b_2'' etc. The structure in Fig. 1(b) has one loop which corresponds physically to the multiple reflection between the two networks due to their input mismatches (S_{22}' and S_{11}'') on each side of the junction. The path gains can be determined by inspection, using Mason's non-touching loop rule[3] with which we obtain

$$
\begin{aligned}
S_{11} &= S_{11}' + S_{21}'S_{11}''S_{12}'(1 - S_{22}'S_{11}'')^{-1}\\
S_{12} &= S_{12}''S_{12}'(1 - S_{22}'S_{11}'')^{-1}\\
S_{21} &= S_{21}'S_{21}''(1 - S_{22}'S_{11}'')^{-1}\\
S_{22} &= S_{22}'' + S_{12}''S_{22}'S_{21}''(1 - S_{22}'S_{11}'')^{-1}. \qquad (1)
\end{aligned}
$$

The equations in (1) are identical to those obtained more laboriously elsewhere.[1,2]

With this method we shall now show how the turnstile junction[4] is electrically equivalent to two π hybrids joined at their parallel arms. With the proper choice of reference planes a π hybrid will have the scattering matrix:

$$
S_\pi = \frac{1}{\sqrt{2}}\begin{vmatrix} 0 & 0 & 1 & 1\\ 0 & 0 & -1 & 1\\ 1 & -1 & 0 & 0\\ 1 & 1 & 0 & 0 \end{vmatrix}. \qquad (2)
$$

The π hybrid is a matched four port (with one symmetry plane) in which the emergent energy from two of the ports is either in phase or 180° out of phase.

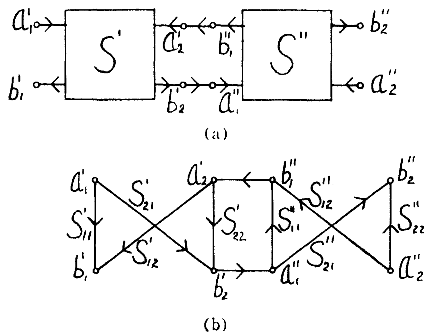

Fig. 1—(a) Cascade of two, two ports having scattering matrixes S' and S''. (b) Flow graph of the network of (a).

Fig 2(a) shows two four ports joined at one arm. The flow graph for this system with the four ports corresponding to (2) is shown in Fig. 2(b). Each vertical half of the flow graph is a representation of matrix (2). The isolation between ports is effected by the proper segregation of a's and b's. Ports 3 and 4 correspond to the series and parallel arms respectively. As all ports are matched, the junction will generate no loop and this flow graph may be evaluated very simply by path gain products.[5] Renaming ports 1', 2', 3', 1'', 2'', and 3'', to 1, 2, 3, 4, 5, and 6 respectively, and defining the scattering coefficients of the composite network in the usual manner we obtain:

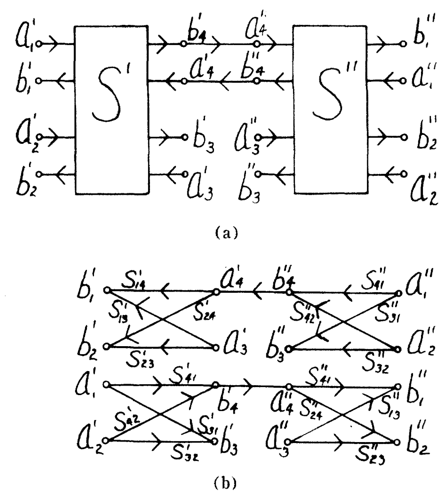

Fig. 2—(a) Two four ports joined at one arm. (b) Flow graph of two matched four ports joined at one arm.

$$
S = \begin{vmatrix}
0 & 0 & S_{13}' & S_{41}''S_{41}' & S_{42}''S_{41}' & 0\\
0 & 0 & S_{32}' & S_{41}''S_{42}' & S_{32}''S_{32}' & 0\\
S_{13}' & S_{32}' & 0 & 0 & 0 & 0\\
S_{41}''S_{41}' & S_{41}''S_{42}' & 0 & 0 & 0 & S_{13}'\\
S_{42}''S_{41}' & S_{32}''S_{32}' & 0 & 0 & 0 & S_{23}''\\
0 & 0 & 0 & S_{13} & S_{23}'' & 0
\end{vmatrix}. \qquad (3)
$$

Letting S' and $S'' = S_\pi$ of (2) we obtain:

$$
S = \begin{vmatrix}
0 & 0 & 1/\sqrt{2} & 1/2 & 1/2 & 0\\
0 & 0 & -1/\sqrt{2} & 1/2 & 1/2 & 0\\
1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 & 0 & 0\\
1/2 & 1/2 & 0 & 0 & 0 & 1/\sqrt{2}\\
1/2 & 1/2 & 0 & 0 & 0 & -1/\sqrt{2}\\
0 & 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0
\end{vmatrix}. \qquad (4)
$$

Renumbering ports 1, 2, 3, 4, 5, and 6 to 1, 4, 2, 5, 3, and 6, respectively, we obtain:

$$
S = \begin{vmatrix}
0 & 1/2 & 0 & 1/2 & 1/\sqrt{2} & 0\\
1/2 & 0 & 1/2 & 0 & 0 & 1/\sqrt{2}\\
0 & 1/2 & 0 & 1/2 & -1/\sqrt{2} & 0\\
1/2 & 0 & 1/2 & 0 & 0 & -1/\sqrt{2}\\
1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 & 0 & 0\\
0 & 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 & 0
\end{vmatrix}. \qquad (5)
$$

(To change the port numbers of a scattering matrix first interchange the corresponding rows and then the corresponding columns, or vice versa.) S of (5) is identical to that of the matched turnstile junction.[4] Thus we have derived the properties of one type of matched six port and done so with less effort than would have been possible by previous methods. In addition we have shown how this six port may be synthesized from two π hybrids joined at their parallel arms.

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* Received February 14, 1962.

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On the Significance of Instantaneous and Short-Term Correlation Functions for a Class of Stochastic Processes*

Introduction

With attempts to develop instantaneous adaptive-learning systems a real need has arisen to define clearly the concept of instantaneous correlation functions and their physical realization. Several authors[1] have considered this problem partially. The purpose of this note is to define the notion of instantaneous correlation as a limiting value of short-term correlation and to develop appropriate expressions for such correlation functions in terms of an orthogonal series.

Short-Term Correlation Functions

In the actual measurement process, the correlation function is measured over some interval of time, let us say $2T$. Since the correlation function thus obtained is a function of this short interval,[2] $2T$, the time t and correlation interval τ, the resulting function, $\phi_{2T}(t, \tau)$ is called the short-term correlation function and is defined by

$$\phi_{2T}(t,\tau) = \frac{1}{2T}\int_{t-T}^{t+T} x(\xi)x(\xi+\tau)d\xi. \quad (1)$$

For a stationary time process, as $T\to\infty$, (1) becomes the usual definition of a correlation function, namely

$$\phi(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x(\xi)x(\xi+\tau)d\xi. \quad (2)$$

This is evident by making use of the fact that for a stationary process

$$\lim_{T\to\infty}\phi_{2T}(t,\tau) = \lim_{T\to\infty}\phi_{2T}(0,\tau) \equiv \phi(\tau). \quad (3)$$

Instantaneous Correlation as a Limit of a Short-Term Correlation Function

It was shown above that as the interval T is lengthened, the correlation function of a stationary random process approaches the usual definition. What happens if T is shortened instead? In this case the correlation function is markedly dependent upon the interval length T as well as the origin in time t and the correlation variable τ. For very short intervals, T, the correlation is defined on an average only for that interval as is clearly evident. By analogy to the velocity of a particle of mass or the frequency of a wave motion, as the time of measurement is decreased the average velocity or frequency approaches what is defined as the instantaneous velocity or frequency. In the same way we define as the instantaneous correlation by allowing $T\to 0$.

Definition

The instantaneous correlation function $\phi(t, \tau)$ of a random process $x(t)$ is defined as the

$$\lim_{T\to 0}\phi_{2T}(t,\tau) = \lim_{T\to 0}\frac{1}{2T}\int_{t-T}^{t+T} x(\xi)x(\xi+\tau)d\xi = \phi(t,\tau). \quad (4)$$

To show that (4) is consistent with the usual notion of instantaneous quantities such as, say, velocity or frequency, the limit in (4) is now evaluated.

Let us define the function $\varphi(\xi, \tau)$:

$$\varphi(\xi\ \tau) \equiv \int x(\xi)x(\xi+\tau)d\xi \quad (5)$$

and form the difference function

$$\varphi(t+T,\tau) - \varphi(t-T,\tau) = \int_{t-T}^{t+T} x(\xi)x(\xi+\tau)d\xi. \quad (6)$$

Dividing both sides of (6) by $2T$ gives

$$\frac{\varphi(t+T,\tau)-\varphi(t-T,\tau)}{2T} = \frac{1}{2T}\int_{t-T}^{t+T} x(\xi)x(\xi+\tau)d\xi. \quad (7)$$

The difference quotient on the left side tends to a derivative as $T\to 0$ under certain conditions[3] of the function φ, *i.e.*,

$$\lim_{T\to 0}\frac{\varphi(t+T,\tau)-\varphi(t-T,\tau)}{2T} = \frac{\partial\varphi(t,\tau)}{\partial t}\cdot \quad (8)$$

Hence

$$\frac{\partial\phi(t,\tau)}{\partial t} = \lim_{T\to 0}\frac{1}{2T}\int_{t-T}^{t+T} x(\xi)x(\xi+\tau)d\xi. \quad (9)$$

It is clear then that the instantaneous correlation function of the random process $x(t)$ is the derivative of the function $\varphi(t, \tau)$. This is roughly analogous to the derivative idea arising in the notion of instantaneous velocity and frequency.

To evaluate (9), note that

$$\varphi(t,\tau) = \int_0^t x(\xi)x(\xi+\tau)d\xi + \varphi(0,\tau) \quad (10)$$

where $\varphi(0, \tau)$ is the initial function of integration. Taking partial derivatives of both sides of (10) with respect to the time variable t gives

$$\frac{\partial\varphi(t,\tau)}{\partial t} = \frac{\partial}{\partial t}\int_0^t x(\xi)x(\xi+\tau)d\xi + \frac{\partial\varphi(0,\tau)}{\partial t}\cdot \quad (11)$$

The last term in the right member of (11) is evidently zero. The first term of the right member of (11) is evaluated according to the rules[4] of differentiating an integral with respect to its upper limit t:

$$\frac{\partial}{\partial t}\int_0^t x(\xi)x(\xi+\tau)d\xi = x(t)x(t+\tau). \quad (12)$$

Thus it follows that

$$\frac{\partial\varphi(t,\tau)}{\partial t} = x(t)x(t+\tau) = \phi(t,\tau) \quad (13)$$

or

$$\lim_{T\to 0}\frac{1}{2T}\int_{t-T}^{t+T} x(\xi)x(\xi+\tau)d\xi = x(t)x(t+\tau). \quad (14)$$

From this it follows that the instantaneous correlation function is merely the product of the random function $x(t)$ and the function of its continuous translation $x(t+\tau)$.

Synthesis of Instantaneous Correlation Functions in Terms of a Complete Set of Orthogonal Functions

Wolf,[5] and Wolf and Dietz,[6,7] give a general theory for probing systems and correlation systems. It is shown that

$$A_n = \int_0^\infty h_n(\tau)\phi(\tau)d\tau, \quad (15)$$

where $h_n(\tau)$ are the impulse responses of a set of orthogonal filters, A_n = Wiener statistics obtained by averaging the product of the responses of the orthogonal filter to noise, and the noise. If $\{h_n(\tau)\}$ form a complete orthogonal set, a solution for $\phi(\tau)$ the correlation function is easily obtained,[5] *i.e.*,

$$\phi(\tau) = \sum_{n=1}^{\infty} A_n h_n(\tau). \quad (16)$$

From the theory given above, it follows that the instantaneous correlation function is obtained utilizing the mechanization given by Wolf[5] with $T\to 0$ where T is the averaging time and $g(t)$, the random variable, is assumed to have been on since $\tau = \to\infty$. Thus

$$A_n(T,t) = \int_{-\infty}^{\infty}\phi_T(t,\tau)h_n(\tau)d\tau \quad (17)$$

or

$$\phi_T(t,\tau) = \sum_{n=1}^{\infty} A_n(T,t)h_n(\tau). \quad (18)$$

Passing to the limit as $T\to 0$ gives

$$\phi(t,\tau) = \sum_{n=1}^{\infty} A_n(t)h_n(\tau) \quad (19)$$

where

$$A_n(t) = g(t)v_n(t) = \int_{-\infty}^{\infty} g(t)g(t+\tau)h_n(\tau)d\tau \quad (20)$$

is now a random variable.

The display of $\phi(t, \tau)$ is via a three-dimensional plot. The variable t measures the change in statistics of the correlation function as the origin of measurement changes. The variable τ measures the correlation spread. The third axis gives the correlation amplitude, $\phi(t, \tau)$.

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* Received February 19, 1962; revised manuscript received, March 5, 1962.

[1] D. R. Rothschild, "A note on instantaneous spectrum," Proc. IRE (*Correspondence*), vol. 49, p. 649; March, 1961.

[2] Over the interval T, the short-term correlation function $\phi_T(t, \tau)$ is defined as

$$\phi_T(t,\tau) = \frac{1}{T}\int_{t-T}^{t} x(\xi)x(\xi+\tau)d\xi. \quad (1a)$$

[3] F. Riesz and B. Sz. Nagy, "Functional Analysis," Ungar Press, ch. 1; 1957.

[4] I. S. Sokolnikoff, "Advanced Calculus," McGraw-Hill Book Co., Inc., New York, N. Y., p. 121; 1939.

[5] A. A. Wolf, "Some recent advances in the analysis and synthesis of nonlinear systems," *Trans. AIEE*, vol. 80, pp. 289–300; November, 1961.

[6] A. A. Wolf and J. H. Dietz, "A statistical theory for probing a class of linear and nonlinear systems for faults, parameter identification, and transmission characteristics," to be published.

[7] A. A. Wolf and J. H. Dietz, "Adaptive correlation techniques for signal classification, recognition, and filtering," to be published.

WWV And WWVH Standard Frequency and Time Transmissions*

The frequencies of the National Bureau of Standards radio stations WWV and WWVH are kept in agreement with respect to each other and have been maintained as constant as possible since December 1, 1957, with respect to an improved United States Frequency Standard (USFS).[1] The corrections reported here were arrived at by means of improved measurement methods based on transmissions from the NBS stations WWVB (60 kc) and WWVL (20 kc). The values given in the table are 5-day running averages of the daily 24-hour values for the period beginning at 1800 UT of each day listed.

WWV Frequency
With Respect to U. S. Frequency Standard

1962 June	Parts in 10^{10}†
1	−130.2
2	−130.2
3	−130.3
4	−130.3
5	−130.3
6	−130.3
7	−130.3
8	−130.2
9	−130.2
10	−130.2
11	−130.1
12	−130.1
13	−130.1
14	−130.0
15	−129.9
16	−129.8
17	−129.9
18	−129.9
19	−129.9
20	−129.9
21	−129.9
22	−129.8
23	−129.7
24	−129.8
25	−129.8
26	−129.8
27	−129.7
28	−129.6
29	−129.5
30	−129.3

† A minus sign indicates that the broadcast frequency was below nominal. The uncertainty associated with these values is $\pm 5\times 10^{-11}$.

The time signals of WWV and WWVH are also kept in agreement with each other. Since these signals are locked to the frequency of the transmissions, a continuous departure from UT2 may occur. Corrections are determined and published by the U. S. Naval Observatory. The time signals are maintained in close agreement with UT2 by properly offsetting the broadcast frequency from the USFS at the beginning of each year when necessary. This new system was commenced on January 1, 1960.

Subsequent changes were as follows:

Frequency Offset, with Reference to the USFS

January 1, 1960, −150 parts in 10^{10}
January 1, 1962, −130 parts in 10^{10}

Time Adjustments, with Reference to the Time Scale UT2

December 16, 1959, retardation, 20 milliseconds
January 1, 1961, retardation, 5 milliseconds
August 1, 1961, advancement, 50 milliseconds

Adjustments were made at 0000 UT on the foregoing dates; an advancement means that the signals were adjusted to occur at an earlier time than before.

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* Received July 19, 1962.

[1] See "National standards of time and frequency in the United States," Proc. IRE (*Correspondence*), vol. 48, pp. 105–106; January, 1960.

The Coherent Memory Spectrum Analyzer with Loop Gain K<1*

The coherent memory spectrum analyzer is a relatively simple and flexible system for real-time spectral analysis. The basic system (Fig. 1) has been treated by several writers.[1,2,3] These writers indicate that, in unity loop gain, the square of the envelope of the system output is equivalent to scanning the spectrum of the system input once every T seconds by a filter with power transfer given by

$$\left\{\frac{\text{Sin}\,[\pi(N+1)(fT - t/T)]}{\text{Sin}\,[\pi(fT - t/T)]}\right\}^2$$

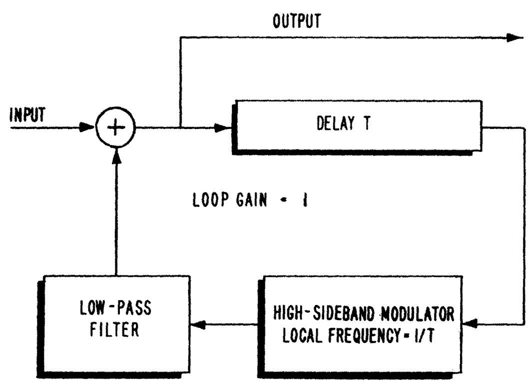

Fig. 1—Basic coherent memory spectrum analyzer.

where N is the number of times a given signal component is allowed to circulate in the loop before being removed by the low-pass filter. The shape or resolution of this filter depends on N, while the center frequency depends on the time (*i.e.*, the center frequency is t/T^2 plus any multiple of $1/T$). Because the response of the system is periodic in frequency, input spectrum must be limited to a bandwidth of less than $1/T$ for unambiguous analysis.

This writer has found that the general shape of the power transfer function can be improved by removing the low-pass filter and reducing the loop gain to $K<1$. To investigate this case, let the system input be

$$i(t) = Ae^{2\pi jft}. \tag{1}$$

Let the time be represented as

$$t = nT + \tau, \quad 0 \le \tau < T$$
$$n = 0, 1, 2, \cdots. \tag{2}$$

Then

$$i(t) = i_n(\tau) = Ae^{2\pi jf(nT+\tau)}. \tag{3}$$

After n loop circulations with loop gain K, the signal in the loop is

$$0_n(\tau) = \sum_{m=0}^{n} AK^m e^{2\pi j[f(n-m)T + f\tau + m\tau/T]}$$
$$= Ae^{2\pi jf(nT+\tau)}\frac{1-\{Ke^{2\pi j[\tau/T - fT]}\}^{n+1}}{1-Ke^{2\pi j[\tau/T-fT]}} \tag{4}$$

where the well-known relation

$$\sum_{n=0}^{N} x^n = \frac{1-x^{N+1}}{1-x} \tag{5}$$

has been used. The square of the instantaneous envelope of the signal in the loop is $|0_n(\tau)|^2$. Since the loop gain K is less than unity, the instantaneous envelope of the steady-state system output is easily obtained

$$\lim_{n\to\infty} |0_n(\tau)|^2$$
$$= \frac{A^2}{1+K^2-2K\cos 2\pi(\tau/T - fT)}. \tag{6}$$

Eq. (7) is the power response of the system to a single frequency input of frequency f and amplitude A. The output is periodic in $t=T$ and $f=1/T$, proportional to A^2, and peaks when

$$\tau/T - fT = 0, \quad \pm 1, \quad \pm 2, \cdots. \tag{7}$$

Therefore, the square of the envelope of the system output is equivalent to scanning the power spectrum of the input every T seconds with a filter with power transfer function given by

$$\frac{1}{1+K^2-2K\cos 2\pi(\tau/T - fT)}.$$

* Received May 17, 1962; revised manuscript received, June 5, 1962.

[1] S. Applebaum, U. S. Patent No. 2,997,650; August 22, 1961.

[2] H. J. Bickel, "Spectrum analysis with delay line filters," 1959 IRE WESCON Convention Record, pt. 8, pp. 59–67.

[3] J. Capon, "On the properties of an active time-variable network: the coherent memory filter," *Proc. Symp. on Active Networks and Feedback Systems*, Polytechnic Inst. of Brooklyn, Brooklyn, N. Y.; April, 1960.

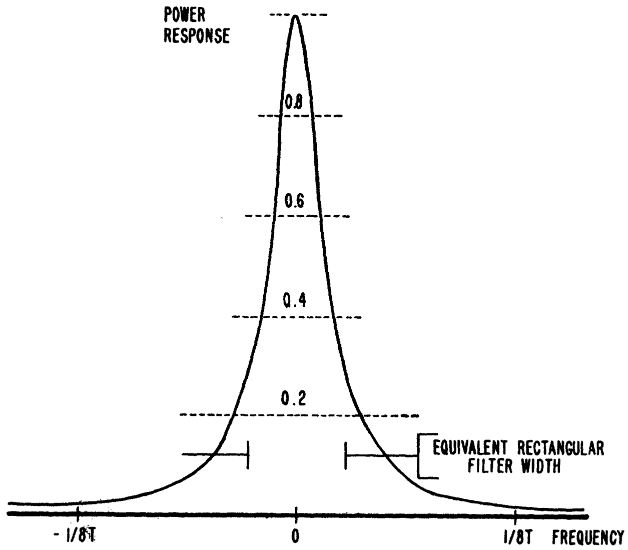

Fig. 2—Power transfer function for loop gain $K = 0.9$.

The filter shape is determined by the loop gain K, while the center frequency is time-dependent and equal to t/T^2 plus any multiple of $1/T$. The filter shape is shown in Fig. 2 for the case $K = 0.9$; notice the absence of sidelobes.

The equivalent rectangular bandwidth (*i.e.*, resolution) of a filter with power transfer $P(f)$ is defined as[4]

$$W_e = \frac{\left[\int_0^\infty P(f)df\right]^2}{\int_0^\infty P^2(f)df}. \tag{8}$$

Because the power transfer function of this system is periodic in frequency and because in practice the input spectrum is band-limited to the periodic interval (*i.e.*, $1/T$), (8) will be integrated over the periodic interval only. Making the substitution $2\pi(t/T - fT) = x$ yields

$$W_e = \frac{\left[\int_0^\pi [1 + K^2 - 2K\cos x]^{-1}dx\right]^2}{2\pi T\int_0^\pi [1 + K^2 - 2K\cos x]^{-2}dx} = \frac{1}{2T}\left[\frac{1-K^2}{1+K^2}\right] \tag{9}$$

which results in the useful relation

$$\frac{\text{Resolution}}{\text{System Bandwidth}} = \frac{1}{2}\left[\frac{1-K^2}{1+K^2}\right]. \tag{10}$$

Adjusting the loop gain, K, theoretically yields any desired resolution, a flexibility that is one of the advantages of the analyzer—although there are practical limitations. As $K \rightarrow 1$, the system becomes sensitive to small perturbations in the loop gain, but there are no problems with oscillations because the feedback at any frequency is zero as a result of the frequency shifting by the SSB modulator. The linear dynamic range and bandpass of the memory loop will also set practical limitations on the resolution.

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[4] R. B. Blackman and J. W. Tukey, "The Measurement of Power Spectra," Dover Publications, Inc., New York, N. Y., p. 19; 1958.

Shot Noise in Thin Film Transistors*

In this note the theory of shot noise in transistors is adapted to the thin film transistors consisting of metal-oxide-metal-oxide-metal or metal-oxide-metal-semiconductor structures.[1,2]

Let I_e be the emitter current, I_{cc} the leakage current between base and collector, and α_{dc} the dc current amplification factor; then the collector current $I_c = \alpha_{dc}I_e + I_{cc}$.

According to Zijlstra[3] the current in metal-oxide-metal diodes shows full shot noise at small currents and suppressed shot noise at larger currents. Consequently one would expect the same for the emitter current of the thin film transistor. The current distribution between base and collector is a partition problem and as a consequence the collector noise should consist of collected shot noise, partition noise and full shot noise of the current I_{cc}. Hence if the noise is represented by a current generator i_1 in parallel to the emitter, and a current generator i_2 in parallel to the collector (Fig. 1), and if F_c^2 is the noise suppression factor of the emitter noise,[4] one would expect

$$\overline{i_1^* i_1} = 2eF_c^2 I_e \Delta f \tag{1}$$

$$\overline{i_2^* i_2} = 2eF_c^2 I_e \Delta f \alpha_{dc}^2 + 2eI_e\alpha_{dc}(1 - \alpha_{dc})\Delta f + 2eI_{cc}\Delta f \tag{2}$$

$$\overline{i_1^* i_2} = 2eF_c^2 I_e \Delta f \alpha_{dc} \tag{3}$$

since the cross correlation between i_1 and i_2 is caused by the fact that the part $\alpha_{dc}I_e$ of the emitter current is collected.

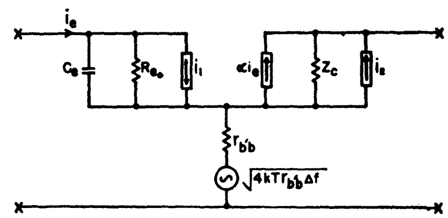

Fig. 1—Equivalent circuit of thin film transistor.

Since the emission and collection processes are *fast* processes, (1), (2) and (3) should hold for a wide range of frequencies. The high-frequency behavior of the diode is not caused by these processes, but by the emitter capacitance C_e.

The differences with the semiconductor transistor are as follows:

1) For the semiconductor transistor $F_c^2 = 1$.

2) I_e is not a simple exponential function of V_e, and hence the equations for $g_{e0} = \partial I_e/\partial V_e$ and $g_{ce0} = \partial I_c/\delta V_e$ no longer hold.

3) The frequency dependence of the thin film transistor must be attributed to the emitter capacitance C_e. Hence if $\alpha_0 = \partial I_c/\partial I_e$, the high-frequency current amplification factor α is

$$\alpha = \frac{\alpha_0}{1 + j\omega C_e R_{e0}} = \frac{\alpha_0}{1 + jf/f_0} \tag{4}$$

where $R_{e0} = 1/g_{e0}$, and $f_0 = (2\pi C_e R_{e0})^{-1}$ is the α-cutoff frequency of the thin film transistor.

Sometimes the noise is represented by an emitter emf e and a collector current generator i (Fig. 2). In that case,

$$e = i_1 Z_e; \quad i = i_2 - \alpha i_1; \quad Z_e = \frac{R_{e0}}{1 + jf/f_0}. \tag{5}$$

Consequently

$$\overline{e^* e} = \frac{2eF_c^2 I_e \Delta f R_{e0}^2}{(1 + f^2/f_0^2)} \tag{6}$$

$$\overline{i^* i} = 2eF_c^2 I_e \Delta f\left[\alpha_{dc}^2 - \frac{2\alpha_{dc}\alpha_0}{(1 + f^2/f_0^2)} + \frac{\alpha_0^2}{(1 + f^2/f_0^2)}\right] + 2eI_e\alpha_{dc}(1 - \alpha_{dc})\Delta f + 2eI_{cc}\Delta f \tag{7}$$

$$\overline{e^* i} = 2eF_c^2 I_e \Delta f R_{e0}\left[\frac{\alpha_{dc}}{1 - jf/f_0} - \frac{\alpha_0}{1 + f^2/f_0^2}\right]. \tag{8}$$

* Received June 25, 1962. Supported by U. S. Signal Corps Contract.

[1] C. A. Mead, "Operation of tunnel-emission devices," *J. Appl. Phys.*, vol. 32, pp. 646–652; April, 1961.

[2] J. P. Spratt, R. F. Schwarz, and W. M. Kane, "Hot electrons in metal films, injection and collection," *Phys. Rev. Lett.*, vol. 6, pp. 341–342; April 1, 1961.

[3] R. J. J. Zijlstra, "Noise in currents through thin insulating layers," *Physica* (to be published).

[4] A. van der Ziel, "Noise," Prentice-Hall, Inc., New York, N. Y.; 1954.

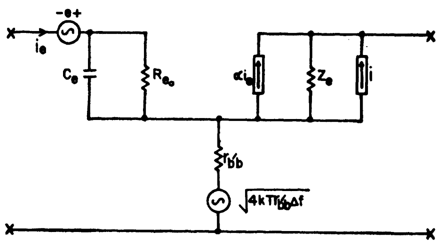

Fig. 2—Alternate equivalent circuit of thin film transistor.

When α_{dc} does not depend upon the emitter current I_e, $\alpha_0 = \alpha_{dc}$ and (7) and (8) can be simplified accordingly.

Figs. 1 and 2 incorporate also the thermal noise of the resistance $r_{b'b}$ of the base layer. The theory of the noise in thin film transistors is thus brought to the point where the standard procedures for calculating the noise resistance and the noise figure can be applied.[5]

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[5] A. van der Ziel, "Fluctuation Phenomena in Semiconductors," T. Butterworth, Ltd., London, Eng.; 1959.
E. R. Chenette and A. van der Ziel, "Accurate noise measurements on transistors," IRE TRANS. ON ELECTRON DEVICES, Vol. ED-9, pp. 123–128; March, 1962. See especially the Appendix.

A Variable-Parameter Direct-Current Switching Filter*

Using contacts to switch a direct current involves two difficulties. The first is the contact arc, which generates radio-frequency interference and oxidizes or erodes the contact material. The second is the rapid rise and decay of the switched current, which may generate undesirable circuit transients as well as interference. Switching filters for arc suppression or transient control often contain reactive elements. With a linear filter, the switch must dissipate previously stored energy during either the contact make or the contact break, and it seems impossible to relieve the switch of this burden. The filter discussed here, developed for envelope shaping in a radiotelegraph transmitter,[1] eliminates the contact dissipation by using a rectifier to vary the network structure.

Consider Fig. 1, where the circuit to be switched is represented by the battery E and load resistance R_0 to the left of the terminals. The network has two circuit equivalents. The first [Fig. 2(a)] is appropriate following the instant when the switch is closed. The second [Fig. 2(b)] is appropriate following the instant when the switch is opened. Because of the rectifier, neither equivalent is an unconditionally valid representation of Fig. 1 (for all time, $t \geq 0$) unless the rectifier current is either positive for all $t \geq 0$ or zero for all $t \geq 0$.

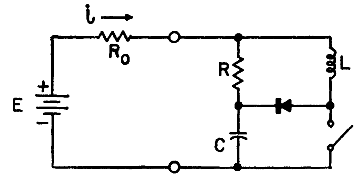
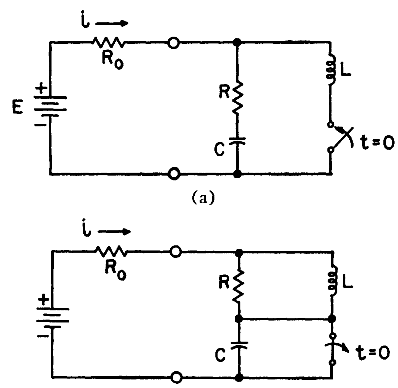


Fig. 1—Switching filter.



(a)

(b)

Fig. 2—Equivalent circuits.

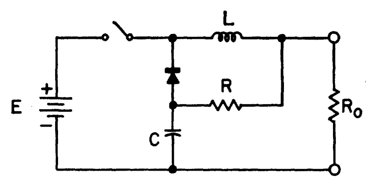


Fig. 3—Alternative filter circuit.

In Fig. 2(a), the contact current at closure is initially zero and immediately afterward rises slowly, being constrained by the inductance L. During this transient interval, the accumulated charge on capacitor C is drained through R and L in series. In Fig. 2(b), the contact voltage at break is initially zero and immediately afterward rises slowly, being constrained by the capacitance C. With the following definitions:

$$\begin{aligned} R_s &= R + R_0 \\ R_p &= RR_0/(R + R_0) \\ 2a &= (1/R_sC) + (R_p/L) \\ \omega^2 &= 1/LC, \end{aligned} \tag{1}$$

the load current following contact closure at $t=0$ is

$$\begin{aligned} i &= (E/R_0)(1 - e^{-at}\cos \mu t), \\ \mu^2 &= (R_0/R_s)\omega^2 - a^2. \end{aligned} \tag{2}$$

Following contact break at $t=0$, the load current is

$$i = (E/R_0)e^{-at}\cos \nu t, \quad \nu^2 = (R/R_s)\,\omega^2 - a^2. \tag{3}$$

The equivalent circuits are valid representations for all $t \geq 0$ if the currents (2) and (3) are either critically damped or over-damped. For $R = R_0$, $\mu^2 = \nu^2$; critical damping is obtained when $\mu^2 = \nu^2 = 0$. These conditions yield

$$\begin{aligned} a &= \omega/\sqrt{2} \\ Q &= \omega L/R_0 = 1/\omega CR_0 = \sqrt{2} \pm 1, \end{aligned} \tag{4}$$

so that there are two L/C ratios that provide critical damping. A design for the filter proceeds as follows. Let $R = R_0$, and choose a load-current time constant $1/a$. Then,

$$\begin{aligned} L &= (1 \pm \sqrt{1/2})(R_0/a) \\ C &= (1 \mp \sqrt{1/2})(1/aR_0), \end{aligned} \tag{5}$$

which result from eliminating ω in (4).

Fig. 3 is an alternate for the circuit of Fig. 1. These filters should be useful for reducing radio-frequency interference, for preserving relay and chopper contacts, and perhaps for reducing the dissipation in transistor and controlled-rectifier switches.

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* Received January 19, 1962.
[1] G. F. Montgomery, "Thoughts on keying filters," QST, vol. 45, pp. 64–65; November, 1961.

On the Approximation Problem for Band-Pass Delay Lines*

The design of band-pass networks having constant delay in the pass band has been considered a difficult problem because the approximation must be done in the frequency domain directly, rather than by transformation of results already worked out for the low-pass case. This is so because the usual low-pass-to-band-pass transformation does not preserve constant delay.

Consider, however, a pole cluster having constant delay in the low-pass interval, and apply the "half transformation":

$$p = s + j. \tag{1}$$

This has the effect of translating the entire cluster in a positive direction along the $j\omega$-axis. The cluster now bears the same relationship to unit frequency that it formerly bore to zero frequency, and therefore has constant delay in the band-interval case. While we must still add the complex-conjugate pole cluster for realizability, it can be said that the approximation problem is solved so long as this addition does not seriously impair the desired constancy of delay.

Neglect of the contribution from the negative cluster has been called the "narrow-band approximation," but an examination discloses that this contribution is negligible for astonishingly large bandwidths. The design procedure is therefore notably easy:

1) Scale the low-pass pole cluster so that its interval of constant delay is one half the interval which is desired in the band-pass network.
2) Add $\pm j$ to all poles.
3) The result can now be realized as a resistively terminated ladder of series

* Received March 8, 1962.

coils and shunt capacitors. By adding appropriate zeros, it is also realizable as an all-pass network, or in one of the usual band-pass configurations.

Design Example: We are given[1] a low-pass 3-pole cluster having a mean delay of 4.576 seconds from $\omega=0$ to $\omega=1$. The delay ripples are ± 0.1 sec, and are equal:

$$P_1 = -0.7263$$
$$P_2 = -0.6148 \pm 0.9493j.$$

For a desired bandwidth of 0.5, we multiply these numbers by 0.25, and add $\pm j$. The transformed poles are

$$-0.1816 \pm j$$
$$-0.1537 \pm 1.2373j$$
$$-0.1537 \pm 0.7627j$$

having a nearly constant delay of 18.3 sec from $\omega=0.75$ to $\omega=1.25$ with nearly equal ripples of ± 0.4 sec.

Since the negative pole cluster contributes a delay of 0.328 sec at $\omega=0.75$, and 0.196 sec at $\omega=1.25$, the error is 0.132 sec, *i.e.*, this is the error in the so-called "narrow-band approximation." Since the error due to the design value of ripple is over six times as great as this, it can reasonably be claimed that the approximation is tolerably accurate.

This transformation has received previous mention,[1] but its merits were not described and may have been overlooked. It was used as a first approximation to be followed by a root-improvement procedure. The result was, of course, marvelously accurate; but a very large family of applications exist for which the inherent accuracy of the transformation is more than adequate.

As a final comment, it is worthy of mention (and of further study) that this transformation also produces amplitude pass bands having arithmetic symmetry within a very close tolerance.

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[1] E. Ulbrich and H. Piloty, "Über den Entwurf von Allpässen, Tiefpässen and Bandpässen mit einer im Tschebyscheffschen Sinne approximierte konstanten Gruppenlaufzeit," *A.E.U.*, vol. 14, pp. 451–467; October, 1960.

A New Frequency Demultiplier with a Tunnel Diode*

A tunnel diode is well known as a negative resistance device. It can, however, be operated as a rectifier device, since its voltage-current characteristic is asymmetric. With this characteristic, a new frequency demultiplier has been developed. The circuit is very simple and contains no dc power source. Only one tunnel diode used in this circuit operates not only as a negative resistance but also as a rectifier device.

* Received February 5, 1962; revised manuscript received, February 27, 1962.

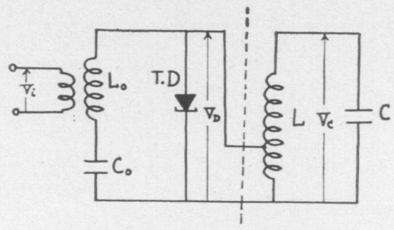

Fig. 1—The frequency demultiplier circuit.

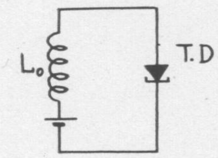

Fig. 2—The monostable circuit.

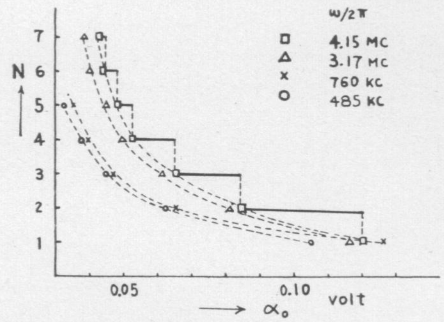

Fig. 3—The relations between the number N and the input signal amplitude.

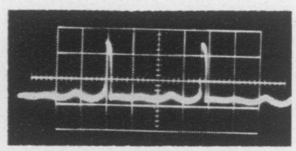

Fig. 4—The voltage waveform across the tunnel diode where $N=3$.

The tunnel diode is connected in series with inductance L_0 and capacitance C_0 as shown in Fig. 1. When external forcing oscillation with frequency $\omega_0 \approx 1/\sqrt{L_0C_0}$ is applied to this circuit, the resonant current is rectified by the tunnel diode with its rectifying characteristic mentioned above and charges the capacitance C_0. With this charge, the tunnel diode is biased forward and its operating point moves toward the peak on its characteristic curve. At the Nth cycle of the external forcing oscillation (input signal), the operating point passes over the peak and jumps to the higher voltage level. That is, this circuit is effectively similar to the monostable circuit as shown in Fig. 2 and in this case the capacitance C_0 may be considered as the dc power source supplying forward-bias voltage to the tunnel diode. Thus, the operating point passes down along the characteristic curve and jumps back from the valley to the lower voltage level (starting point).

In the same manner, the capacitance is recharged and the operating point moves toward the peak again.

Fig. 3 shows the relation between the number N and the input signal amplitude α_0 for various frequencies. It is easily seen that when the input signal is large, N may be small in number and when small, large in number.

The voltage waveform across the tunnel diode is the pulse form as shown in Fig. 4. We can also obtain the sine wave by connecting LC tank circuit ($LC=N^2L_0C_0$) to the tunnel diode.

In animal auditory mechanisms, sound entering the ear makes the tympanum vibrate and this vibration is transmitted to the cochlea, in which the auditory signal is changed to the pulse train. This pulse frequency is dependent on the auditory signal intensity and frequency. The description of the complete behavior of the auditory mechanism is beyond the scope of this correspondence. However, the pulse train observed in the above circuit is experimentally found to be much analogous to that of the output signal from the cochlea. This circuit may be considered, therefore, as an electronic model of the pulse-frequency-modulator of an auditory receptor in the cochlea.

The author wishes to express his thanks to Assistant Professor Jin-ichi Nagumo for suggesting this investigation as well as for constant guidance in the course of the work.

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Measurement of the Impurity Distribution in Diffused Layers in Germanium*

Two techniques have been developed to measure precisely the impurity distribution of diffused layers in thin, planar N-type germanium samples. A geometry is chosen so that the spatial relationships of the electric fields and voltages in the depletion layers can be mathematically related to the impurity density profile by means of Poisson's equation for one-dimensional geometry. The usual electrochemical transistor structure fulfills this requirement. Both the techniques described herein involve the measurement of punch-through voltages as a function of the position of the emitter in the diffused layer with respect to a fixed collector electrode located outside the diffused layer. The first (punch-through convergence) technique requires the measurement of both normal and inverse punch-through voltages as a function of the emitter position in the diffused layer. Such data is plotted in Fig. 1. The point at which the two curves converge is the location of the edge of the diffused layers. Fig. 1 also illustrates the precision of the method by show-

* Received February 6, 1962.

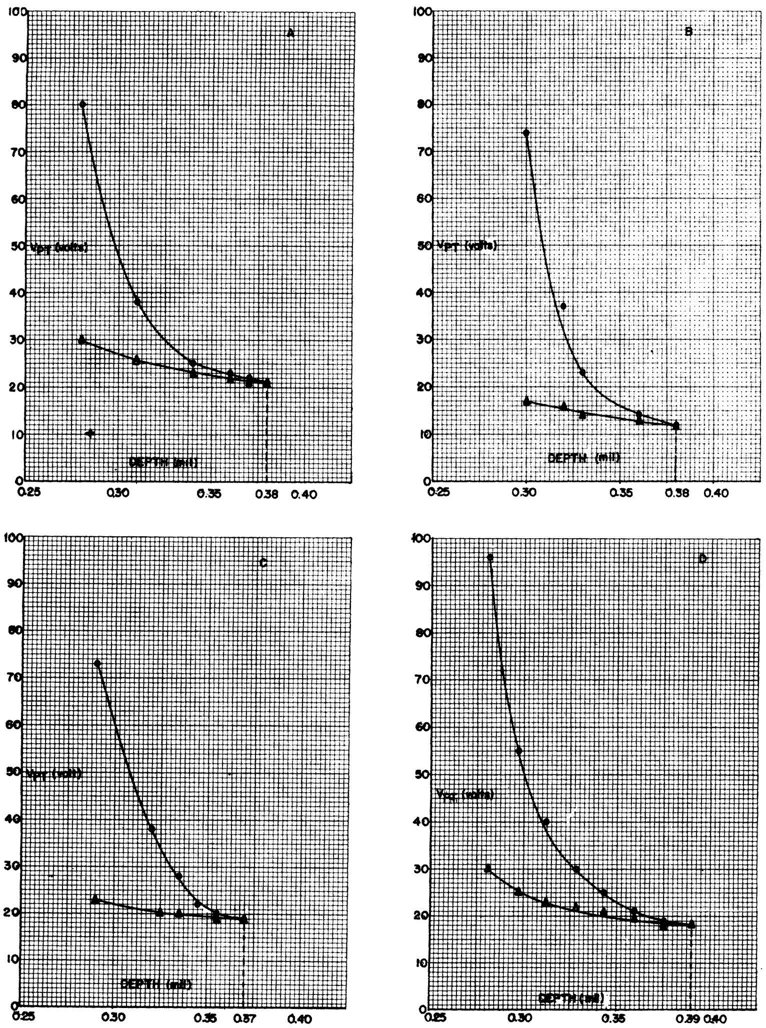

Fig. 1.

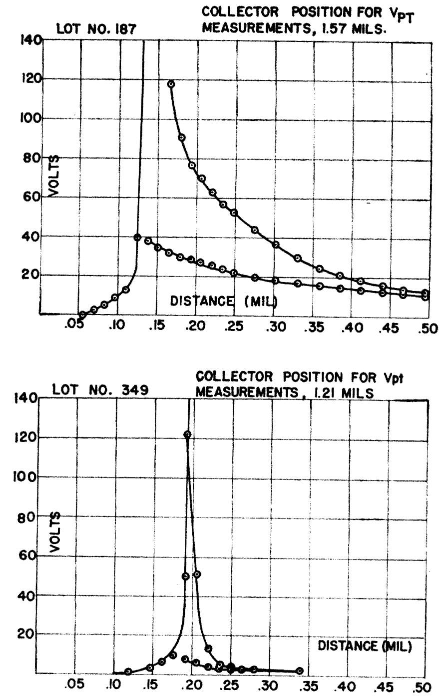

Fig. 2.

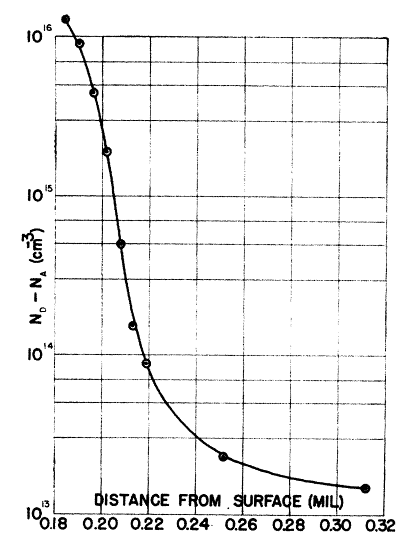

Fig. 3.

ing the data collected by two persons, on two completely separate sets of equipment, on four samples from the same diffused lot.

While it is not necessary to place the collector electrode at any particular location, the value of the data is increased if the collector is placed where it would be in the transistor. The upper punch-through voltage curve then relates the position of the edge of the collector-depletion layer to the applied collector voltage. By this means one can determine quantitatively the variation in base width and collector-depletion-layer width with the applied collector voltage.

Fig. 2 shows clearly the advantage this technique has over the diode-breakdown voltage technique. Diode-breakdown voltage measurements have been the most useful method for evaluation and control of diffused layers. For a number of reasons diode-breakdown voltage measurements are not very useful near the edge of the diffused layer. The punch-through voltage curves in Fig. 2 show the difference in depth of two diffused layers to be about 0.25 mil, while the breakdown voltage data would seem to show a difference of about 0.06 mil. Furthermore, the diode-breakdown voltage measurements would indicate that the bottom sample has a deeper diffused layer while the punch-through convergence technique shows that the top sample actually has the deeper diffused layer. The time required to collect data for curves such as shown in Fig. 1 is about 45 minutes.

The second technique to be discussed involves the calculation of the impurity distribution from measurements of the punch-through voltage from the fixed collector to the nonfixed emitter electrode as a function of the position of the latter electrode. Poisson's equation relates the impurity density to the gradient of the field:

$$N = \frac{\mathcal{E}}{q} \frac{dE}{dx}.$$

The value of dE/dx can be determined from successive measurements of punch-through voltage V and base width W, by the approximation:

$$\frac{dE}{dx} \approx \frac{\Delta V}{W \Delta W}.$$

Fig. 3 shows the result of such work on an N-type germanium blank into which a layer of arsenic has been diffused.

In the second technique described the depletion layer at the junction at which punch-through is detected has been neglected.

The etching, plating, and measuring equipments, and the techniques developed for electrochemical transistor technology made the approach described in this letter practical. The results of these approaches, in turn, have improved the control of the semiconductor material involved in electrochemical technology.

The authors wish to acknowledge the fact that Dr. G. L. Lang, presently at Carnegie Institute of Technology, made a major contribution in the early development of the theory of this method in 1958, and that L. Pomante, formerly of their laboratory, made some of the measurements.

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Phase Shift in Ferrites at High Power*

Although the high power absorption effects observed in ferrites are twofold in nature (thermal and nonlinear), the phase shift is expected to exhibit only thermal effects. That the phase shift characteristics are temperature sensitive is known from the strong dependence of magnetization on temperature and the measurements of Martin,[1] and Geiszler and Henschke.[2] The absence of nonlinear behavior can be demonstrated once the thermal characteristic effects are eliminated.

This communication describes a technique devised at this laboratory (as part of an experimental investigation of ferrite materials) for eliminating or minimizing heating effects in the measurement of the phase shift.

Measurements were carried out in regular and reduced S-band waveguides for longitudinally and transversally magnetized ferrite slabs. The results indicate that the phase change is practically independent of power level (up to the power measured).

A block diagram of the equipment used is given in Fig. 1. The high power source was a 4J39 magnetron with an operating frequency of 3525 Mc and an output peak power of 750 kw at a 0.5-μsec pulse width and a repetition rate of 1000 pps. The power was fed into a power splitter leading to two arms, one containing the ferrite test sample, the other a variable calibrated Riblet-type phase shifter and power divider. The phase shift of the standard phase shifter, consisting of a short-slot hybrid and variable short, is a linear function of the position of the short, which was measured with an Ames gage. The power divider was constructed of three short-slot hybrids and an adjustable short.

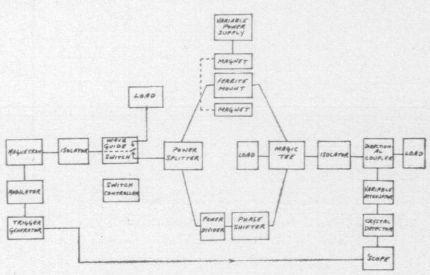

Fig. 1—Block diagram of equipment used for high-power shift measurements of ferrite materials.

The introduction of the power divider in one of the arms provided a means of equalizing the amplitudes of the power in the two arms of the comparison circuit. In the process of varying the power by the power divider, an additional phase shift is also introduced. Thus, the measured ferrite phase shift is the sum of the phase shifts of the Riblet phase shifter and power divider. The signals from both arms were then recombined in a magic-T and detected by a crystal. The detected signal was applied to an oscilloscope. When the detected signal is a minimum the two arms of the bridge are in phase balance.

The procedure for the phase shift measurement was to adjust the Riblet phase shifter and power divider for a minimum deflection on the oscilloscope, when the sample was present, without and with an applied magnetic field. The phase difference was then determined from a change in setting of the phase shifter and power divider. Due to the high sensitivity of the setup, the phase measurements were accurate to within two degrees.

Precautions against heating effects were taken through the introduction of a high-speed waveguide switch past the magnetron and by means of a stepwise determination of the minimum (see below). A control circuit was designed to adjust the switching time interval (the time in which the shutter opens completely and returns to its original position). During the switching cycle the magnetron power applied to the ferrite is swept in amplitude from 0- to 375-kw peak power and then back to 0 power. By changing the voltage and/or the resistance in the control circuit, the time interval is adjustable from 20 to 100 msec. Once a minimum was found, the power was turned off to allow the ferrite to cool. The power was then reapplied and the short adjusted slightly for a minimum. This process was repeated till no further adjustment was necessary.

It will be shown that, since only one switching cycle was necessary for the final determination of the phase shift, the rise in temperature was negligible. The energy incident on the ferrite during one cycle is

$$W = (2/\pi)NP_p u,$$

W = energy in joules
N = number of pulses = 20
P_p = peak power = 375×10^3 w
u = pulse width = 0.5×10^{-6} sec

The factor $2/\pi$ arises from the fact that the sweep in power from 0 to maximum and back to zero has a cosine dependence. Assuming that, under the worst condition, all the energy is absorbed, the rise in temperature is

$T = W/(cm)$
T = temperature in °C
c = approximate specific heat of ferrites = 708 joules/kg/°C
m = mass in kg.

Thus, Ferrite Motorola Y-188 with $m=320$ gm and Trans-Tech 469 with $m=80$ gm had, at most, an increase of 0.042°C and 0.011°C, respectively.

Low-power measurements were also performed for comparison. The phase changes of one representative sample (out of twelve) as a function of magnetic field is given in Fig. 2. The curves clearly show that hardly any phase distortion is introduced with the increase of peak power.

Martin[1] and Brown[3] obtained similar results. Martin noticed in ferrite slabs at X band that at a repetition rate of 1 pps (no heating effect), throughout the range of high power used (up to 120 kw), the phase changed at most a few degrees over that observed at low power. In an X-band limiter, Brown found that the phase shift was constant over a 30-db range of input power.

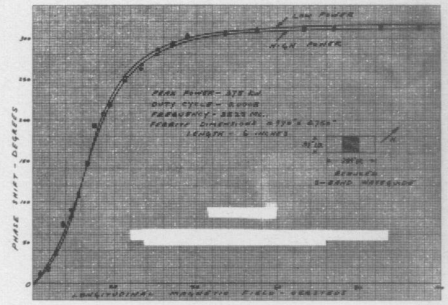

Fig. 2—Phase shift of Ferrite Motorola Y-188 at high and low power vs longitudinal magnetic field.

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* Received February 6, 1962; revised manuscript received, February 13, 1962.

[1] R. L. Martin, "High power effects in ferrite slabs at X-band," *J. Appl. Phys.*, vol. 30, pp. 159–160; April, 1959.

[2] T. D. Geiszler and R. A. Henschke, "Broad band reciprocal phase shifters," *J. Appl. Phys.*, vol. 31, pp. 174–175; May, 1960.

[3] J. Brown, "On phase distortion of ferro-magnetic limiter," Proc. IRE, vol. 49, pp. 362; January, 1961.

A New Class of Distributed RC Ladder Networks*

A class of solutions to the linear second order differential equation

$$v'' - (r'/r)v' - srcv = 0, \qquad (1)$$

in which s is a constant, r and c are functions of the independent variable x, and the prime denotes differentiation with respect to x, has been discussed by Kazansky[1] and Jacobs.[2] Sugai[3] also derives this class from a method of Hildebrand.[4] It is defined by making constant the logarithmic derivative of r/c expressed as a function of

$$z = \int_0^x \sqrt{rc}\, dx.$$

That is, let

$$G(z) = d(\ln \sqrt{r/c})/dz = 2m \qquad (2)$$

where m is a real constant.

Described here is a broader class of networks which encompasses that discussed above and which provides network behavior that cannot be achieved with the $G(z)=2m$ class.

* Received February 2, 1962. This material constitutes a portion of the work in progress on a Doctoral Dissertation at the Polytechnic Institute of Brooklyn.

[1] B. G. Kazansky, "Outline of a theory of non-uniform transmission lines," *Proc. IEE*, pt. C, vol. 105, pp. 126–138; March, 1958.

[2] I. Jacobs, "A generalization of the exponential transmission line," Proc. IRE, vol. 47, pp. 97–98; January, 1959.

[3] I. Sugai, "A generalized Hildebrand's method for nonuniform transmission lines," Proc. IRE (*Correspondence*), vol. 49, p. 1944; December, 1961.

[4] F. B. Hildebrand, "Advanced Calculus for Engineers," Prentice-Hall, Inc., New York, N. Y., p. 50; 1948.

The basis for this class is the Liouville Transformation[5] in which the independent and dependent variables are simultaneously transformed according to

$$z = \int_0^x \sqrt{rc}\, dx \qquad (3)$$

and

$$u = (c/r)^{1/4} v. \qquad (4)$$

The transformed differential equation is now in the Liouville Normal Form,

$$\ddot{u} - [s - F(z)]u = 0, \qquad (5)$$

where

$$\begin{aligned} F(z) &= (1/rc)\{(1/4)[(c/r)'/(c/r)](1/2)[(rc)'/(rc)] \\ &\quad - (1/4)[(c/r)''/(c/r)] \\ &\quad + (3/16)[(c/r)'/(c/r)]^2\}. \qquad (6) \end{aligned}$$

This relation can be written in many forms but it is particularly useful to express it in terms of the function $G(x)$. Note $G(x)$ is simply related to $G(z)$ by virtue of (3) and its derivative,

$$dz/dx = \sqrt{rc}.$$

Of course $F(z)$ can also be expressed as a function of x, thus,

$$F(x) = (1/4)\{2[(r/c)/(r/c)'](G^2)' - (G^2)\}. \quad (7)$$

If, now, the function $F(x)$ is equated to a constant then the Liouville Normal Form of the differential equation becomes readily solvable since it would have constant coefficients. This operation specifies a relation between r and c which defines this new class of networks. That is,

$$F(x) = -m^2$$

or

$$2[(r/c)/(r/c)'](G^2)' - (G^2) = -4m^2. \quad (8)$$

This last equation is a first order linear differential equation in (G^2) with variable coefficients. Its solution may be obtained by the method of variation of parameters as

$$G^2 = 4m^2 + K\sqrt{r/c} \qquad (9)$$

where K is a constant of integration. This equation is the criterion for this new class of solutions.

It is seen from (9) that when K is chosen as zero (9) yields the criterion (2) for the earlier class which has been referred to as a "Generalized Exponential Class."[2]

The class introduced here and defined by (9) might be called a "Generalized Hyperbolic Class" (G.H.C.) since the solutions of (5) with $F(z) = -m^2$ are hyperbolic in nature.

The G.H.C. criterion (9) has four cases according to

a) $m^2 = 0$, $K = 0$
b) $m^2 \neq 0$, $K = 0$
c) $m^2 = 0$, $K \neq 0$
d) $m^2 \neq 0$, $K \neq 0$.

Case a) is the simplest and corresponds to the "Proportional" Network[6] in which r/c is independent of x. The second case b), as was mentioned above, is the "Generalized Exponential Class." Cases c) and d) are the two new solutions introduced by the G.H.C. and the performance of networks of these types differs from that of a) and b). This performance in terms of network parameters and graphical determination of their zeros has been studied in detail. The relation between r and c for the two new G.H.C. cases has also been worked out and specific practical examples formulated. It is hoped that these details will be published at some later date.

Other possibilities besides letting $F(z) = -m^2$ exist which, by taking advantage of differential equations whose solutions are known, lead to still other classes of networks. For example the Liouville Normal Form (5) can be made into a Bessel equation by the choice

$$F(z) = -m^2 - [a^2 - (1/4)]/z^2 \quad (10)$$

where a = constant.

This leads to the following as a criterion for this "Generalized Bessel Class" of networks:

$$G^2 = 4m^2 + K\sqrt{r/c} - (4a^2 - 1)\sqrt{r/c} \cdot \int_0^x \frac{(\sqrt{r/c})'/(r/c)}{\left[\int_0^x \sqrt{rc}\, dx\right]^2} dx. \qquad (11)$$

It is seen from this criterion that the G.H.C. criterion (9) is a special case of (11) wherein $a = \frac{1}{2}$.

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[5] E. Kamke, "Differentialgleichungen, Lösungsmethoden und Lösungen," Edwards Bros., Inc., Ann Arbor, Mich., p. 261; 1945.

[6] M. J. Hellstrom, "Symmetrical RC distributed networks," PROC. IRE, vol. 50, pp. 97–98; January, 1962.

The Magnetic Monopole and the Principle of Parity*

In a recent letter, Leen asked for a logical argument for the nonexistence of an isolated magnetic pole.[1] How about the following reasoning using the well-known principle of parity: Let us assume a current i. Around this current we have a circular magnetic field H. We test this magnetic field by our (hypothetical) isolated magnetic pole $+p$ fixing this pole by, say, a string on the conductor carrying the current i (see Fig. 1). The magnetic pole rotates around i under the action of the force F induced by the magnetic field H—let us assume *counterclockwise*. Now we quote the principle of parity stating that a possible experiment seen in a mirror is a possible experiment too. Looking in a mirror, however, we see our pole rotating *clockwise* around i.

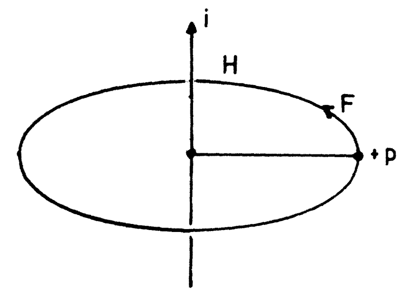

Fig. 1.

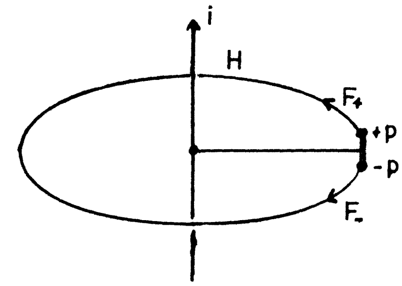

Fig. 2.

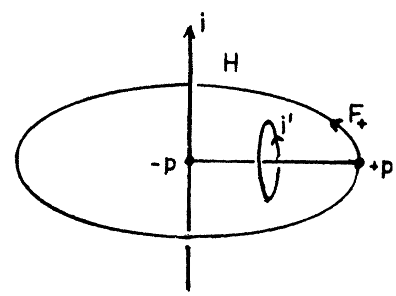

Fig. 3.

This contradiction solves only if we assume that an isolated magnetic pole $+p$ must not exist, but that a magnetic dipole $(+p, -p)$ does. Fig. 2 shows that for a dipole, the forces F_+ on $+p$ and F_- on $-p$ cancel; the dipole does not move.

But let us fix the dipole $(+p, -p)$ so that the pole $-p$ falls within the path of the current i (see Fig. 3). We again observe the counterclockwise rotation of $+p$ around i. To avoid the former contradiction against the principle of parity we must now consider the dipole (like Ampère) as produced by a circular current i'. If we look in the mirror now, this current reverses its direction, the magnetic dipole reverses too, and we see a likewise possible experiment.

Consequently the principle of parity implies the nonexistence of the magnetic monopole and the "axial" character of the magnetic dipole.

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* Received February 6, 1962.

[1] M. W. Leen, "Physical basis for electromagnetic theory," PROC. IRE (*Correspondence*), vol. 50, p. 90; January, 1962.

Author's Comment[2]

The comments of Schnupp are certainly pertinent to an examination of the minimum number of postulates required for the derivation of Maxwell's equations. However, the author does not agree that the principle of parity is useful for that purpose.

Consider a current flowing through a line element $\boldsymbol{dl}$, in a positive direction along the Z axis of a right-handed system of coordinates. Then the magnetic field at a point P_2, a distance r from the Z axis, has the direction and magnitude given by

$$\boldsymbol{B} = \left(\frac{\mu_0 i}{4\pi}\right)\frac{\boldsymbol{dl} \times \boldsymbol{r}}{r^3}.$$

The direction of $\boldsymbol{B}$ is determined by the conventional direction of a vector product in a right-handed coordinate system as shown in the solid lines in Fig. 4. In addition, if we place the magnetic dipole N-S at P_2 with its axis inclined to the direction of $\boldsymbol{B}$, it will be subject to a clockwise torque (looking from P_2 to P_1) tending to align it with $\boldsymbol{B}$.

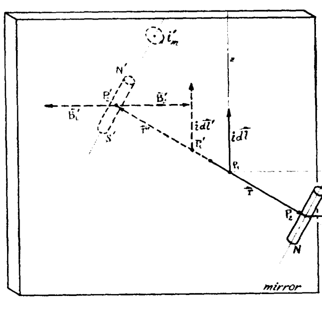

Fig. 4.

Now if we apply the principle of parity as quoted by Schnupp, and construct the mirror image as shown in dotted lines, it is seen that the current element $\boldsymbol{idl}'$ and the distance $\boldsymbol{r}'$ form, with $\boldsymbol{B}_s'$ (the mirror image of $\boldsymbol{B}$) a left handed system, whereas the true field due to $\boldsymbol{idl}'$ is

$$\boldsymbol{B}_L' = \left(\frac{\mu_0 i}{4\pi}\right)\frac{\boldsymbol{dl}' \times \boldsymbol{r}'}{r^3}.$$

Now if we postulate a (south) magnetic monopole at P_2, the force it experiences is in the direction of $\boldsymbol{B}$ while its image at P_2' would experience a force in the direction of $\boldsymbol{B}_L'$. This is essentially in agreement with Schnupp's argument up to this point. However, the apparent contradiction between the results of the real experiment at P_2' and the image at P_2' of the real experiment at P_2 can be resolved without concluding that monopoles do not exist.

Note that the direction of rotation of a small current loop, i_m, is reversed by imaging in the mirror and hence the dipole N-S is reversed by imaging to take the position N'-S'. Thus we can resolve the dilemma by assuming that reflection in a mirror reverses the polarity of a monopole (if one were to exist) as well as a dipole.

To summarize, the derivation of Maxwell's equations in the previously quoted articles[3,4] requires in addition to the assumption of the linearity of electromagnetic phenomena, the assumption of the nonexistence of magnetic monopoles. The principle of parity cannot be used as a substitute.

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[2] Received February 20, 1962.

[3] P. Clavier, "Electromagnetic theory from a mathematical viewpoint," Proc. IRE (*Correspondence*), vol. 48, pp. 1494–1495; August, 1960.

[4] I. Sugai, "Vectors for waves and electrons," Proc. IRE (*Correspondence*), vol. 49, pp. 628–629; March, 1961.

The Effect of Mutual Inductance Upon Tunnel Diode Locked Pair Switching*

In previous discussions of the design and operation of the tunnel diode locked pair logic circuit, it has been assumed that mutual inductance between the loops of the two diodes could be neglected.[1–4] This note presents some results of a computer analysis which indicate that significant improvement in high-speed switching may be obtained by adding mutual coupling.

The circuit to be considered is shown in Fig. 1. For digital computer analysis the tunnel diodes have been represented by an analytic volt-ampere relation which closely approximates a germanium unit with 10 ma of peak current. A piecewise linear diode approximation was used on an analog computer. The coupled inductances of the two loops are represented by their "tee" equivalent.

If the load resistance R is large enough so the rise time into the load

$$\tau = M/R \tag{1}$$

may be neglected, the switching transient depends only upon the difference between total and mutual inductance or L-M. This effect is shown in Fig. 2. If L-M is made too large and highest speed operation is desired, the circuit does not switch but oscillates with both diodes in phase so no power is coupled to the output. The condition for this oscillation is

$$L > \frac{R_S R_N C}{1 - k} \tag{2}$$

where $k = M/L$ is the coefficient of coupling. In Fig. 3 the maximum inductance for which switching occurs is plotted vs $1/1-k$. The supply waveforms are step functions which correspond to operation at the maximum frequency. The relation is linear and the slope implies $R_N = 35$ ohms which is not unreasonable for the "average" negative resistance of a 10 ma diode. Examination of the switching waveforms for several combinations of circuit parameters indicates that switching time is essentially unimpaired if

$$L - M < \tfrac{1}{2} R_S R_N C. \tag{3}$$

Circuits have been constructed with interlocked supply resistors as shown in Fig. 4. Both diode loops surround approximately the same area so the majority of the magnetic flux couples both loops. The inductance and coupling coefficient of this fixture have been measured as about 1.5 nanohenry

* Received June 18, 1962.

[1] E. Goto, *et al.*, "Esaki diode high speed logical circuits," IRE Transactions on Electronic Computers, vol. EC-9, pp. 25–29; March, 1960.

[2] W. F. Chow, "Tunnel diode digital circuitry," IRE Transactions on Electronic Computers, vol. EC-9, pp. 295–301; September, 1960.

[3] J. J. Gibson, *et al.*, "Tunnel diode balanced pair switching characteristics," *Digest of 1962 Internat'l. Solid-State Circuits Conf.*, Philadelphia, Pa., February, 1962, Lewis Winner, New York, N. Y., pp. 54–55; 1962.

[4] L. Esaki, "Characterization of tunnel diode performance in terms of device figure of merit and circuit time constant," *IBM J. Res. & Dev.*, vol. 6, pp. 170–178; April, 1962.

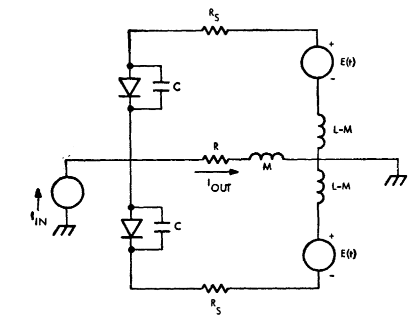

Fig. 1—Tunnel diode locked pair including mutual inductance.

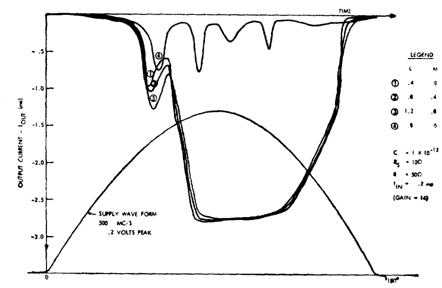

Fig. 2—Switching transient for various values of inductance L and mutual inductance M with a sinusoidal supply waveform. (Analog computer solution.)

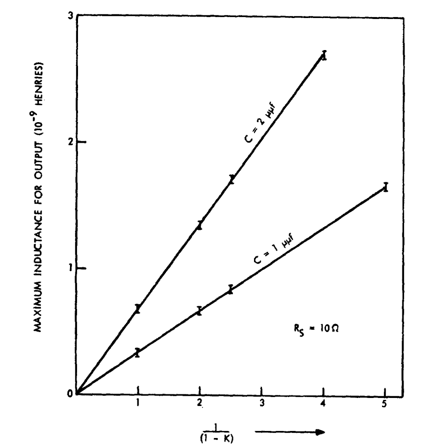

Fig. 3—Maximum inductance for output vs $1/1-k$. Circuit parameters are as in Fig. 2. Supply waveforms are 0.2 volt steps. (Digital computer solution.)

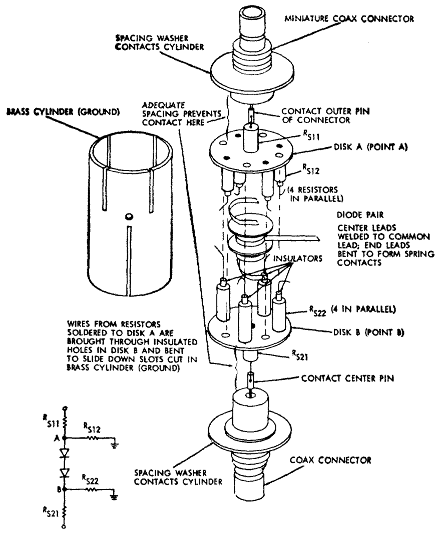

Fig. 4—Exploded view of diode mount. (Not to scale.)

and 0.7, respectively. Using 10 ma diodes with about 3 $\mu\mu$f of capacitance, a current gain of 50 has been measured at 200 mc where current gain is the ratio of maximum output current into the load resistor divided by one half of the input current necessary to switch from one output polarity to the other.

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Designing a Phase-Locked Loop as a Doppler Tracker*

The transfer function and low-pass filter of a phase-locked loop may be precisely determined to give adequate transient response with minimum noise bandwidth when the loop is used as a tracking filter. This note gives a straightforward design procedure.

The well-known linear equivalent of a phase-locked loop is shown in Fig. 1, and the root locus is shown in Fig. 2.[1] The open-loop transfer function is composed of the two poles and the zero on the real axis, and we desire to place the closed-loop poles so that the absolute magnitude of the phase error $e(t)$ is less than approximately 0.6 rad when tracking a given Doppler signal. (This will result in a near optimum choice between noise bandwidth and threshold.)

* Received February 7, 1962. This research was supported by AF Space Systems Division Contract No. AF 04(647)-829.

[1] C. S. Weaver, "A new approach to the linear design and analysis of phase-locked loops," IRE Trans. on Space Electronics and Telemetry, vol. SET-5, pp. 166–178; December, 1959.

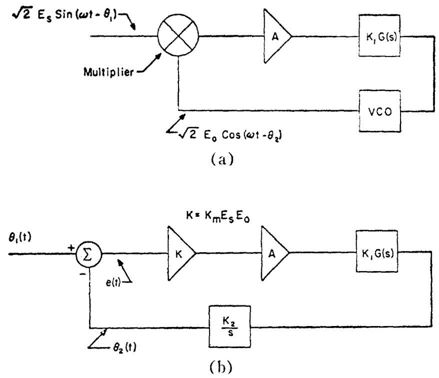

Fig. 1—(a) A phase-locked loop. (b) Linear equivalent.

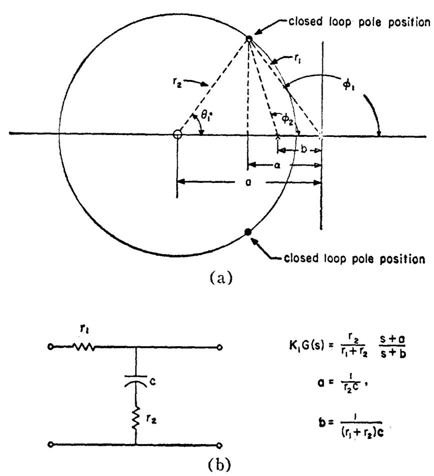

Fig. 2—(a) The root locus when $KG(s)$ is as shown in (b). (b) The loop filter.

From Fig. 2(a) we see that the transfer function from $\theta_1(t)$ to $e(t)$ is

$$\frac{E(S)}{\theta_1(S)} = \frac{1}{1 + KAK_1K_2\dfrac{S+a}{S+b}\dfrac{1}{S}} = \frac{S(S+b)}{S(S+b) + K_{OL}(S+a)} \tag{1}$$

where K_{OL}, the open-loop gain, is

$$K_{OL} = KAK_1K_2.$$

Over most of the range of a typical Doppler curve the slope is very nearly a straight line, or a ramp in frequency. Let this slope be D cps. Because phase is the integral of frequency,

$$\theta_1(S) = \frac{2\pi D}{S^2} \times \frac{1}{S} = \frac{2\pi D}{S^3}\cdot \tag{2}$$

and

$$E(S) = \frac{2\pi D}{S^3}\,\frac{S(S+b)}{S(S+b)+K_{OL}(S+a)} = \frac{2\pi D}{S^2}\,\frac{S+b}{S(S+b)+K_{OL}(S+a)}\cdot \tag{3}$$

From a table of Laplace transforms we see that $e(t)$ eventually settles down into a ramp plus a constant given by

$$e(t) = \frac{2\pi D}{r_1^2}\left[bt + 1 - \frac{2b\alpha}{r_1^2}\right]. \tag{4}$$

For a large tracking range (compared to the loop bandwidth) b should be small compared to r_1. Then $e(t)$ is approximately

$$e(t) \cong \frac{2\pi D}{r_1^2}\,[bt + 1] < 0.6. \tag{5}$$

If ϕ_1 is 135°, there will be an overshoot of less than 10 per cent. Then

$$r_1 > \sqrt{\frac{2\pi D}{(0.6)}\,(bt + 1)} \tag{6}$$

will guarantee that we are in the low-phase error region.

The root locus condition requires that

$$180° = \theta_1 - \phi_1 - \phi_2. \tag{7}$$

Then the design procedure is as follows: r_1 is found from inequality (6), and a is adjusted to satisfy (7). The open-loop gain is

$$K_{OL} = \frac{r_1 r_3}{r_2}\cdot$$

If possible the VCO frequency with no phase error should be set at the center of the Doppler curve (zero frequency shift). Then t in inequality (6) would be no larger than the time, t_0, it takes the frequency ramp to go from the higher frequency limit of the Doppler curve to the center of the curve. If the VCO must be set at some other point, the largest time of inequality (6) will be $2t_0$.

Since the noise bandwidth is proportional to r_1, it is obvious that the narrowest bandwidths may be obtained by making b equal zero (an active integrator). This ideal integrator also eliminated the VCO center-frequency setting problem.

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An Experimental Technique for Parametric Devices*

In experimental work at radio frequencies an alteration to a circuit may be accomplished by unsoldering one component and soldering in another. Unfortunately, in the microwave region of the spectrum, an equivalent change usually demands an elaborate machining operation. We present here a description of a method which we have used to facilitate rapid alterations to an experimental parametric amplifier, together with a brief note on the performance of the amplifier.

The amplifier is completely coaxial, and in its experimental form consists basically of a center conductor and an array of brass washers. These washers all have the same outer diameter (2.25 in), but vary in inner diameter and thickness. They are clamped in a rack by a slight longitudinal pressure,

* Received February 12, 1962.

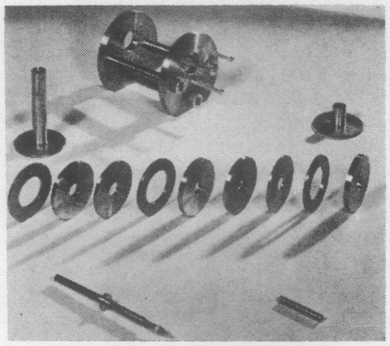

Fig. 1—Experimental L-band parametric amplifier, exploded view.

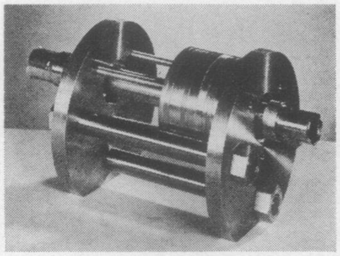

Fig. 2—The amplifier assembled.

and together comprise the outer conductor of a coaxial line. Some of the washers have a relatively large inner diameter, and, to form a radial rejection filter, one of these special washers is sandwiched between two of normal inner diameter. The rejection frequency, is, of course, determined by the inner diameter of the special washer. The washers and the clamping rack, together with the other parts of the amplifier, are shown spread out in Fig. 1.

The essential feature of the clamping rack is a pair of heavy drill rods which support the assembly of washers. These rigidly connected, parallel drill rods have a space between them which is considerably less than the diameter of the washers. Thus, they accurately position the washers, but they do not prevent their removal and rapid replacement once the center conductor has been shifted lengthwise. A stock of washers of various dimensions is available to permit modification of the amplifier's configuration.

The actual amplifier, which is seen assembled in Fig. 2, is a one-port, nondegenerate difference frequency reactance amplifier. The normal inner and outer diameters have been chosen to match General Radio standard connectors. The diode is mounted at the end of a coaxial line; dc connection between the diode and one of the washers is made by three radial wires spaced 120 degrees. These wires have appreciable inductance at the pump frequency. This washer is insulated for dc with 0.003-in teflon dielectric, thus permitting the application of dc bias. This insulated washer, through which the bias voltage is applied, is the fourth washer from the right in Fig. 1.

The diode is a Microwave Associates Type MA450E. The impedance presented to the diode by the source is controlled by a matching device in the signal line, while the idler resonant cavity is made up of a short section of the coaxial line which is bounded by two idler rejection filters.

The pump frequency is 9800 Mc. The pump energy is fed in (from the right in Fig. 2) and is capacitively coupled to the diode. A radial rejection filter in the signal line isolates the pump and signal circuits.

The signal frequency is 1315 Mc and the 3-db bandwidth at 20-db gain is 30 Mc. The over-all noise figure is 2.0 db, which includes a contribution of 0.3 db from the circulator and 0.1 db from the second stage. At a sacrifice of a further 0.25 db in noise figure, the gain-bandwidth product may be increased from 300 to 400 Mc.

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On the Use of Pulse Compression for the Enhancement of Radar Echoes from Diffuse Targets*

A diffuse radar target is one which is made up of a large number of discrete targets each of which is quite small. The specific example of this kind of target considered here is a weather cloud which is composed of myriads of water droplets. These droplets may range in diameter from on the the order of a micron to on the order of a centimeter. In this case the phasor $\check{E}_i$ representing the (radar) voltage return from the ith droplet of this ensemble is given by[1]

$$\check{E}_i = B_i a_i^3 e^{j(\omega t - 4\pi r_i/\lambda + \phi_i)}, \tag{1}$$

where

- B_i is a factor which is a function of transmitted power, frequency antenna gain, range, propagation attenuation, and the complex dielectric constant of the drop;
- a_i is the radius of the ith drop;
- ω is the radar angular frequency;
- r_i is the range of the ith drop;
- λ is the free space wavelength of the radar energy; and
- ϕ_i is the phase shift incurred upon scattering by the drop.

The time waveform of a radar echo from a diffuse target will be composed of a summation of echoes from each drop, each of which will be of magnitude $B_i a_i^3$ and of duration T where T is the duration of the transmitted pulse. Therefore, at a time t_0 the composite phasor $\check{E}$ of the return is given by the summation[2]

$$\check{E} = \sum_{i=1}^{N_0} B a_i^3 e^{j(\omega t_0 - 4\pi r_i/\lambda + \phi_i)}, \tag{2}$$

where N_0 is the total number of drops in the effectively illuminated volume V_0 associated with t_0. Since N_0 is in actuality quite large, it is sometimes desirable to describe the drop size distribution by a continuous function $\rho(a)$ which is assumed to be homogeneous over V_0. This function is defined by letting $\rho(a)da$ be equal to the number of drops per unit volume with radius falling between a and $a+da$.

The summation in (2) may be thought of as a sum of N_0 two-dimensional vectors in phasor space. Further if $N_0 \gg 1$ and $Tc \gg \lambda$, where c is the speed of light, then the phase angle of each term may be considered uncorrelated or random. The problem, therefore, is one of a random walk in two dimensions in which the size of each step is subject to a probability density given by

$$p(E_i) = \frac{\rho[a_i = f(E_i)]}{3B^{1/3}E_i^{2/3}\int_0^\infty \rho(a)da}, \tag{3}$$

where E_i is the magnitude of $\check{E}_i$ and $f(E_i)$ is the appropriate function [from (1)] relating a_i to E_i. The solution of the two-dimensional random walk problem for large N in which the step size varies according to a probability distribution is given in the literature[3] and is

$$W(E)dE = \frac{2E}{N\langle E_i^2\rangle}\exp\left[\frac{-E^2}{N\langle E_i^2\rangle}\right]dE, \tag{4}$$

where E is the magnitude of $\check{E}$, $W(E)dE$ is the probability that the magnitude of the (phasor) summation of (2) will fall in the range between E and $E+dE$, and $\langle E_i^2\rangle$ is the expected value of the square of the step size. From (1) and (3) $\langle E_i^2\rangle$ is found to be

$$\langle E_i^2\rangle = \int_0^\infty E_i^2 p(E_i)dE_i$$
$$= \frac{B^2}{\int_0^\infty \rho(a)da}\int_0^\infty a^6\rho(a)da. \tag{5}$$

It is evident from (4) that the statistics of the magnitude of the total voltage return are in the form of a Rayleigh distribution which depends only upon the product $N\langle E_i^2\rangle$. In order to determine the effect of a pulse compression process upon these parameters and the resultant effect upon the probability distribution consider an idealized rectangular representation of pulse compression. If the transmitted pulse is modulated in such a way that a pulse compression may be effected, then upon compression each individual pulse return in the time waveform will become shorter by some factor $\beta = \tau/T$ where τ is the duration of the compressed pulse. Conservation of energy dictates that $E_{ia} = E_{ib}/\beta^{1/2}$ which means

$$B_a = B_b/\beta^{1/2}, \tag{6}$$

where the subscripts a and b stand for after and before compression. Eq. (6) reiterates the fact that for the case of a discrete target pulse compression (ideally) increases the effective signal magnitude by $\beta^{-1/2}$. At the same time there will also be an improvement in range resolution by the factor β. This

* Received February 16, 1962.

[1] This equation assumes that the Rayleigh approximation to the back-scattering cross section of the drop is valid, *i.e.*, $a \ll \lambda$. In this case the back-scattering cross section is proportional to a^6.

[2] Note that the subscript has been dropped from the factor B since it may be assumed that B_i will not vary significantly over V_0.

[3] J. L. Lawson and G. E. Uhlenbeck, "Threshold Signals," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., New York, N. Y., vol. 24, pp. 52–53; 1950.

range resolution improvement is due to a reduction, by the factor β, in the effectively illuminated volume. It follows from this fact that

$$N_a = \beta N_b \tag{7}$$

for regions greater than a distance $Tc/2$ (in the direction of propagation) from the edges of the target region. It should be observed that N_b is in general sufficiently large so that the inequality $N_a \gg 1$ will remain valid for any reasonable value of β.

Using (5)–(7), it can be seen that

$$N_a \langle E_i^2 \rangle_a = N_b \langle E_i^2 \rangle_b, \tag{8}$$

and as a consequence the statistics of E_r remain invariant upon pulse compression.

The physical interpretation of this conclusion is that although the compression process increases the magnitude of the return from each individual target element, the number of individual returns that come in simultaneously at any given time is proportionately reduced. And as has been shown above, these two effects exactly cancel in the summation process leaving the total time waveform (statistics) essentially unchanged. Thus, when pulse compression is considered it may be seen that there is a certain amount of parallelism as well as a notable difference between the cases of diffuse and discrete targets. When system performance has reached a peak power limitation and average power is increased by increasing the pulse duration, range resolu- is lost proportionately in both cases. However, the (peak) SNR is proportionately improved in the case of a diffuse target (due to an increase in N) while the (peak) SNR associated with the discrete target remains unchanged.[4] If this longer pulse is now modulated and compressed, range resolution is regained in both cases, but the (peak) SNR in the diffuse case remains unchanged while that associated with the discrete target is proportionately increased. Thus, it is seen that the end result is the same in both cases; the compression process itself only regains lost resolution for the diffuse target while it improves both resolution and (peak) SNR for discrete target case.

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[4] The system noise bandwidth is assumed constant here since both cases are subject to the same constraints.

Industrial Patentry*

The occasion of issue of the 3,000,000th Patent calls to mind some troublesome aspects of industrial patentry.

Basically, the patent evolution was conceived to provide incentive to an individual, the inventor, and to provide a period of time during which he could reap the fruits of his invention while protected against imitation. These fruits are profit and prestige.

In our industrial process, the individual must sign a patent assignment agreement, usually as a condition of employment. Such agreements often cite company patent plans which vary in the liberalness of their terms according to the company. Since an agreement usually connotes a half-way meeting place where the parties give toward an equitable exchange, the inventor is here cautioned that if he looks for a great deal more than *the employment itself*, he generally looks in vain. There is no easy way, or obvious plan, to circumvent this loss of some of the original point to patentry in our present industrial system. But this does not mean that we should not be looking for better ways. Perhaps we should at least review the premises in the following numbered series of paragraphs.

1) The patent system is intended to provide an incentive toward original thought. The thought process is an attribute of *homo sapiens*. While confessing to little research on the subject, I feel that patentry, in its essentials, relates the concept of the individual, and the concept of incentive, in an intimate union. A patent can issue to more than one party but it is my opinion that the total constructive good of a patent is inversely proportional to the number of people involved.
2) The patent system is intended to provide protection against theft or depredation. It provides the inventor with an avenue of redress. It is not automatic. The inventor must initiate the action to restate his primacy and thus the protection is primarily a defensive advantage.
3) Industrial patentry adds a new dimension to the patent system and I am not sure that this is clearly recognized. In most of the present industrial agreements known to the author, the inventor exchanges his patent rights for employment. The agreement favors the employer. This is probably a justifiable starting place because the individual is usually more articulate in his objectives than the company. The individual can discern easier *what is best for him* than the company can discern *what is best for it*. It is the nature of all such agreements therefore to obligate the individual in certain specific matters without obligating the company very much, if at all.

In an assignment to an industrial organization, it is intended, I am sure, that the foregoing elements 1) and 2) be preserved. We shall of course assume good faith by all parties. Without it there is little point to this discussion. Even with good faith, however, it is apparent that the elements of incentive and protection are not entirely preserved.

Let us look at some delicate balances. If an incentive plan is designed around the yield from a patent (royalty or similar reckoning) the inventor may become diligent on a single matter which has little real future for the company, because it is the one extra benefit *he* could receive.

If the incentive is designed around the man's progress in the company, he might become interested in patentry which disclosed little of a new nature just in order to accumulate a number of patents.

If the plan is conceived primarily to protect the company, with little reduction-to-practice, the individual may "dry up." He may in all good faith become devoid of ideas simply because there seems to be little future in them. He might even go further and start to think that somehow the original germ of his thoughts arrived outside the framework of his obligation to his company to divulge them.

Regardless of the original good faith, if industrial implementation of the patent process results in extreme pursuits and serious imbalance in either direction, the whole process becomes a drain on our resources rather than a boon to them.

The fundamental question remains to be answered. By what method can we keep the individual's innovation motivation high and keep his entire energy devoted to his company, while providing his company with a good reason to have a patent plan and a good reason to implement it? While based on incentive, the question above should also relate to protection, in which both the individual and the company have a stake. The individual has a technical prestige (a forgotten part of which certainly involves an investment in education and experience) to safeguard and enhance, and the company has its primary business investment to protect.

The following personal observations may be helpful. If a price tag can be set on each patent prosecution, then I feel that much of the "junk" will be abandoned early by the inventor and his company. The common sense of the inventor, and the common sense of the company's "guiding light" in its patent effort should prevail.

I feel that the company "guiding light" in the patent area should not have any line technical interest which could alter his objectivity. He should, however, be thoroughly sympathetic to the patent concept. It should go without saying, too, that no element in the industrial patent effort should be rewarded on the basis of size or volume. This includes the head of the patent department.

I feel that upon proper presentation to a Board of Directors, annually, and after their deliberation, the individuals who have achieved patents or filings of note should be conspicuously honored in accordance with a definitive result of the Board's deliberation. This honor should be both professional and monetary. Admittedly, this idea could not survive without a continuous executive "push."

Perhaps, the basic nature of successful patentry is compounded of a judicious interplay between those two autocrats, prestige and profit. The corporate nature of industry may have to be reborn so that it can be expressed in a unified, and therefore unitary, approach to profit. Somehow we must get back to a single autocratic being dedicated to profit. This being can then find common cause with that other autocrat, prestige.

* Received February 23, 1962.

Prestige is more naturally an individualistic attribute. A dual relationship between these two autocrats, wherein prestige and profit can be interchanged, will restore the original virtue of the patent concept. It appears that when the relationship strays beyond this duality, to a three-poled problem and on to companies, committees, really to pluralities of any kind, the whole thing breaks down.

I am seeking the observations of others on this subject. Perhaps the viewpoint deserves further clarification or even outright refutation. Certainly there are more answers to choose from than the ones offered here.

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On Network Realizability Conditions*

Recently Hazony and Nain[1] stated a set of realizability constraints on the Z matrix of a passive n-port. As pointed out by Slepian[2] these conditions are not sufficient for realization. Since the revised conditions stated by Hazony and Nain[3] are also not sufficient, it seems that further clarification is necessary. This is especially true since other somewhat different, but admittedly incomplete, conditions also have been given.[4]

Here we give the necessary and sufficient conditions for an impedance matrix Z to correspond to a finite, passive n-port. Because of space limitations we refer elsewhere for the proof and don't even attempt a definition of terms such as linear, finite, passive and n-port.

Consider a linear, time-invariant, passive n-port N described by an $n \times n Z$ matrix. Then by exciting with increasing exponentials we can see that Z is necessarily positive real,[5] *i.e.*,

1) $Z(p)$ is analytic in Re $p>0$
2) $Z^*(p)=Z(p^*)$ in Re $p>0$
3) $Z_H(p)$ is positive semi-definite in Re $p>0$.

Here $p=\sigma+j\omega$, a superscript asterisk denotes complex conjugation, and Z_H is the Hermitian part of Z.

If N is finite then Z must be rational and condition 2) implies real coefficients; we then call Z real-rational. In the real-rational case, that considered by Hazony and Nain,[3] 3) implies 1). In a form closer to that given given by Hazony and Nain[1] the positive real conditions in the real-rational case can be stated as follows:

Theorem: The necessary and sufficient conditions that an $n \times n$ matrix $Z(p)$ be the impedance matrix of a finite, passive n-port N are

1) Z is real-rational and
2) Z is analytic in Re $p>0$ and
3) Poles of Z on Re $p=0$ are simple (including infinity) and
4) The residue matrix of Z for each pole on Re $p=0$ (including infinity) is Hermitian with every principal minor non-negative and
5) All principal minors of Z_H are non-negative for each p on Re $p=0$ for which they are defined.

Conditions 4) and 5) are equivalent to the respective statements that the residue matrices (on Re $p=0$) and $Z_H(j\omega)$ are positive semi-definite. Each of these conditions can be given a physical interpretation. Thus 1) states that N can be built with a finite number of real valued elements (including gyrators and transformers). 2) and 3) state that N is stable but perhaps not asymptotically stable. 4) indicates that poles on $p=j\omega$ are due to lossless subnetworks. Finally 5) shows that the average power input in the sinusoidal steady state is nonnegative (recall that the steady state can't be defined for open circuit natural frequencies). Several synthesis methods prove the sufficiency; such are those of Oono and Yasuura,[6] Belevitch[7] and Newcomb.[8] The necessity proof relates the conditions of the theorem to the positive-real definition.[8] This follows the standard pr test[9] and is available in notes for lectures given at Stanford.

To some extent we can compare this with the statements in the previous notes. We can write

$$Z = R_{SY} + R_{SS} + jX_{SY} + jX_{SS}$$

where the R's and X's are real matrices and the subscripts SY and SS stand for symmetric and skew-symmetric, respectively. In Hazony and Nain[3] the condition det (Re Z) = det $(R_{SY}+R_{SS}) \geq 0$ for $p=j\omega$ is stated. Note that this doesn't agree with condition 5) of the above theorem which requires det $(R_{SY}+jX_{SS}) \geq 0$. Of course the requirement of det (Re $Z(j\omega)) \geq 0$ is a wrong condition, as seen by the example

$$Z(p) = \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix}$$

which has det $Z(p)=3>0$ but can't be realized by a passive N. In Hazony[4] the condition det $R_{SY} \geq 0$ for $p=j\omega$ is given. This is seen to be a necessary condition, but much more is required, since det $Z_H(j\omega) \geq 0$ must hold. The necessity of det $R_{SY} \geq 0$, $p=j\omega$, can be seen by connecting transformers to N in the manner attributed to Brune.[10] However, this interpretation fails when looking at $Z_H(j\omega)$ where the nonphysical complex transformer would have to be used.

ACKNOWLEDGMENT

The author would like to thank R. Espinosa who brought the relations of Hazony[4] to his attention, Professor C. A. Desoer who guided the work of which the above theorem is a portion, and Professor E. S. Kuh who suggested the existence of the theorem.

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* Received February 26, 1962.

[1] D. Hazony and H. J. Nain, "A synthesis procedure for an n-port network," PROC. IRE, vol. 49, pp. 1431–1432; September, 1961.

[2] P. Slepian, "Comments on a synthesis procedure for an n-port network," PROC. IRE (*Correspondence*), vol. 50, p. 81; January, 1962.

[3] D. Hazony and H. J. Nain, "Author's comments," PROC. IRE (*Correspondence*), vol. 50, p. 81; January, 1962.

[4] D. Hazony, "Two extensions of the Darlington synthesis procedure," IRE TRANS. ON CIRCUIT THEORY, vol. CT-9, pp. 284–288; September, 1961, See p. 287.

[5] D. C. Youla, L. J. Castriota, and H. J. Carlin, "Bounded real scattering matrices and the foundations of linear passive network theory," IRE TRANS. ON CIRCUIT THEORY, vol. CT-6, pp. 102–124; March, 1959. See p. 122 (Def. 21).

[6] Y. Oono and K. Yasuura, "Synthesis of finite passive $2n$-terminal networks with prescribed scattering matrices," *Memoirs of the Faculty of Engineering*, Kyushu University, vol. 14, pp. 125–177; May 1954. See pp. 153–158 and 163–167.

[7] V. Belevitch, "On the Brune process for n-ports," IRE TRANS. ON CIRCUIT THEORY, vol. CT-7, pp. 280–296; September, 1960.

[8] R. Newcomb, "Synthesis of Non-Reciprocal and Reciprocal Finite Passive $2N$-Poles," Ph.D. Thesis, University of California, Berkeley; 1960.

[9] D. F. Tuttle, Jr., "Network Synthesis," vol. 1, John Wiley and Sons, Inc., New York, N. Y., p. 182; 1958.

[10] E. A. Guillemin, "Synthesis of Passive Networks," John Wiley and Sons, Inc., New York, N. Y., pp. 7–9; 1957.

On the Origin of the Word "Radio"*

In regard to the Editorial in the September, 1961 issue of the PROCEEDINGS, I think that the reference mentioned to the earliest use of the prefix "radio" in the magazine *Tit-Bits* in May, 1898 has been taken from the Oxford English Dictionary, vol. 8, page 101. There is, however, a somewhat earlier use of that prefix—and in a somewhat more technical journal than *Tit-Bits*. The coining of the word "radioconductor," or rather its French equivalent "radioconducteur," no doubt goes back to Edouard Branly. The earliest use of the word "radioconducteur"—as far as I can see—appears in a footnote in a paper by Branly.[1] An English translation of this paper can be found in *Electrician*.[2] The translation of the footnote runs thus:

> My tubes of filings received the name "coherers" from Lodge and this name has been generally accepted. The expression is based on an incomplete examination of the phenomenon and on an inaccurate interpretation. I proposed the name "radioconductors" which recalls the essential property of discontinuous conductors of being excited by electric radiation. M. Ducretet uses my various radioconductors in the apparatus he has constructed to realise "Hertzian Telegraphy" without wires.

It seems to be most likely that Branly termed his device "radioconducteur" only

* Received February 9, 1962.

[1] E. Branly, "Sur la Conductibilité Électrique des Substances Conductrices Discontinues, à propos de la Télégraphie San Fils," *Compt. rend. Acad. Sci., Paris*, vol. 125, pp. 939–942; December 6, 1897. See p. 941.

[2] E. Branly, "On the electrical conductivity of discontinuous conducting substances," *Electrician*, vol. 40, p. 333; December 31, 1897.

after G. Marconi had used it in his first experiments on wireless telegraphic transmission in 1896. Branly, a physics professor at the Institut Catholique in Paris, had discovered the phenomenon in 1890.[3] He published a somewhat more detailed description in 1891[4] and a shortened version in English can be found in *Electrician*.[5]

Branly certainly did not try a transmission of intelligence over great distances. In fact, he only found that the phenomenon of a column of metal filings becoming conductive due to a spark discharge can be observed over a distance of about 20 meters (about 70 ft). It should, however, be borne in mind that Marconi used the Branly "radioconducteur" in his transatlantic transmission experiment.

There is a very interesting paper by Sir Oliver Lodge[6] in which he investigates the coherer principle under a wider aspect, though by no means belittling the achievements of Branly. There is an interesting passage in Lodge (p. 91):

> Mr. Marconi is to be congratulated on the results of his enterprise, the newspaper press of this and other countries have taken the matter up, popular magazine articles have been written about it, and so now the British Public has heard, apparently for the first time, that there are such things as electric waves, which can travel across space and through apparent obstacles to a considerable distance, and be detected in a startling fashion. Thus the public has been educated by a secret box more than it could have been by many volumes of Philosophical Transactions and Physical Society Proceedings; our old friends the Hertz waves and coherers have entered upon their stage of notoriety, and have become affairs of national and almost international importance.

In regard to the term "radiophone" there is an interesting paper in *Engineering*.[7] On the development of the term "radiophony" Preece writes (page 29):

> Many engineers have been investigating the subject, and it is rather amusing to notice the various titles adopted by them. Graham Bell, in his paper on the 27th August, 1880, adopted the title "Upon Production and Reproduction of Sound by Light." As recently as 21st April, 1881, in a paper read before the American Society of Science, he discusses the same question under the title, "Upon the Production of Sound by Radiant Energy." M. Mercadier, the head of the technical school of the telegraph administration in Paris, an extremely able experimenter, as well as a very clever physicist, has written several papers from week to week, published in *La Lumière Electrique* (and which have been read before the French Academy), which invariably have been entitled "Notes on Radiophony."

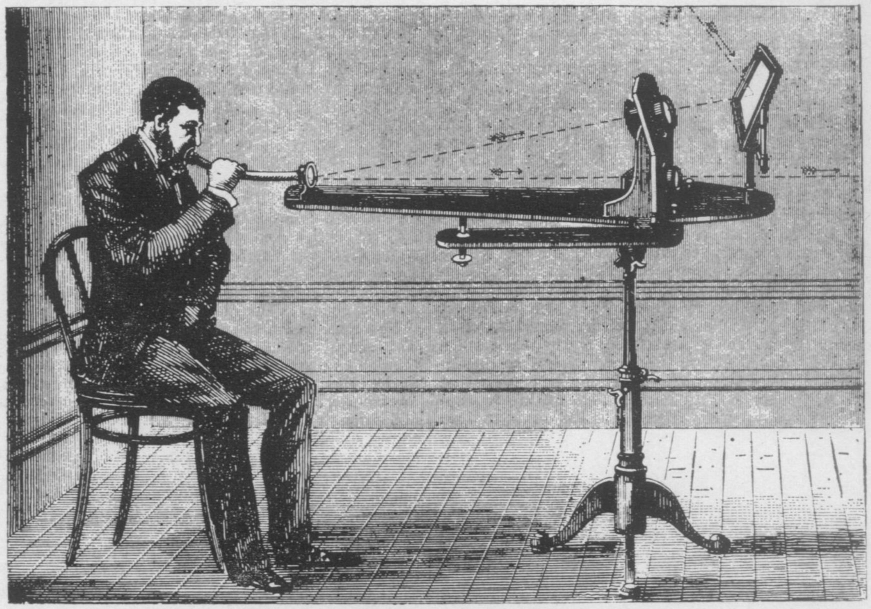

Fig. 1—Bell's articulating photophone. The transmitter. (From *Nature*, November 4, 1880.)

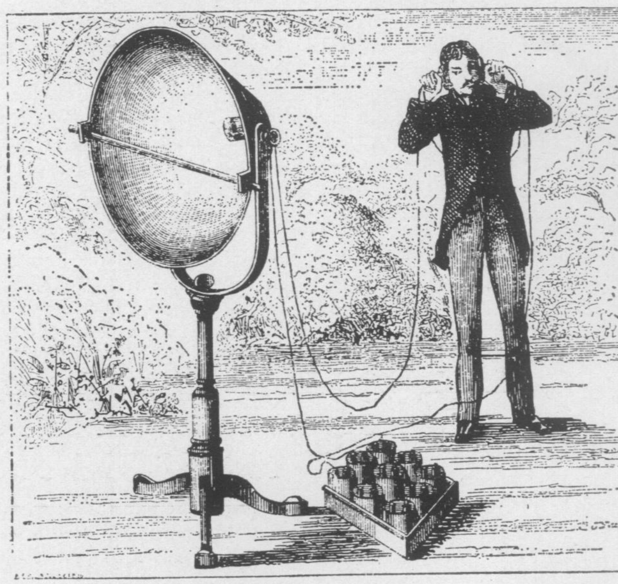

Fig. 2—Bell's articulating photophone. The selenium receiver. (From *Nature*, November 4, 1880.)

Preece continues:

> By radiophony, the term adopted by Mercadier, Bell and Tainter, and myself, I simply mean the production of sounds by radiant energy. . . . By "radidiant energy" physicists now speak of the motion of the ether, that highly elastic medium, which fills all space. The heat of the sun, the light of the stars, the effects of which we call actinism, and all the physical effects that pass between astronomical bodies are transmitted by this medium, ether, and its movements or vibrations are called radiant energy. Sometimes this term is called "radiation," and it is a very frequent thing to see in papers at the present day the word "radiation" so employed.

During my investigations on the word "radio," I came across a short note in *Nature*[8] in which there is an astonishing picture (p. 18) (shown here as Figs. 1 and 2). The similarity of Fig. 2 to the receiving end of a modern microwave link is very surprising. More can be found on Bell's photophone in chapter 16 of the Bell biography by C. Mackenzie.[9]

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On the Characteristic Function of a Stationary Random Process with Gaussian Envelope*

In two recent letters to the editor[1,2] Bunkin and Gudzenko have pointed out the following properties of a stationary random process $x(t)$:

1) If such a process is written in the form

$$x(t) = A(t)\cos\left[\omega_0 t - \theta(t)\right], \quad (1)$$

the introduction of two random functions instead of one puts a restriction on the distribution (density[3]) of $\theta(t)$ which must necessarily be uniform. Furthermore the characteristic function $f(\xi)$ corresponding to $p(x)$, the distribution of the original process $x(t)$, is related to $W(A)$, the distribution of the envelope, by the simple integral transformation

$$f(\xi) = \int_0^\infty J_0(\xi A)W(A)dA. \quad (2)$$

2) As a consequence of (1) among all the possible stationary processes only the ones with symmetrical distributions, *i.e.*, $p(x)=p(-x)$, can be written in the form 1 and an immediate relation may be established from (2) between the even moments of the distributions $p(x)$ and $W(A)$. Both properties can be derived assuming only independence of time of the distribution $W(A)$ and the conditional distributiln $W_\theta(\theta|A)$.

The purpose of this note is to use (2) to find the characteristic function of a stationary random process with Gaussian envelope.

Stochastic processes with approximately Gaussian envelopes have been reported lately in the literature, namely in the case of noisy oscillators[4,5] and in the so-called "shallow" fading.[6] Strictly speaking these processes are not stationary; Golay[7] has even shown intuitively the fundamental impossibility of defining a probability distribution for a noisy oscillator in view of the "random walk" type of behavior of the phase perturbations.

Still an engineering approach such as the one used by Brennan[5] in the problem of mild fading is particularly useful and can be extended even to non-monochromatic oscillators provided that only AM noise is considered. Substantially one thinks of intervals of time long enough to define a meaningful distribution but short enough to make negligible the dependence of time of the distribution itself.

The variance of the Gaussian envelope is assumed much smaller than the square of the mean value: this corresponds to most of the physical situations and makes the distribution vanishingly small at zero.

The required characteristic function is then

$$f(\xi) = \frac{1}{\sqrt{2\pi\sigma^2}}\int_0^\infty J_0(\xi A) \cdot\exp\left[-\frac{(A-A_0)^2}{2\sigma^2}\right]dA. \quad (3)$$

Changing the variable of integration and extending the lower limit of the integral in view of the assumed small variance

$$f(\xi) \cong \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty} J_0[(A_0+x)\xi] \cdot\exp\left(-\frac{x^2}{2\sigma^2}\right)dx \quad (4)$$

and using the addition formula for the Bessel function of zero order

$$f(\xi) = \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty}\left[J_0(A_0\xi)J_0(x\xi) + \sum_{n=1}^{\infty}2\cdot(-1)^n J_n(A_0\xi)J_n(x\xi)\right] \cdot\exp\left(-\frac{x^2}{2\sigma^2}\right)dx. \quad (5)$$

The series can be integrated termwise and the resulting series is rapidly convergent in the range of interesting values, furthermore all the terms of odd order vanish. Retaining then only the leading term, the value of the integral is

$$f(\xi) = J_0(A_0\xi)\,{}_1F_1\left[\frac{1}{2};1;-\frac{\sigma^2\xi^2}{2}\right]. \quad (6)$$

Taking advantage of the identity

$${}_1F_1\left[\tfrac{1}{2};1;-x\right] = \exp\left(-\frac{x}{2}\right)I_0\left(\frac{x}{2}\right), \quad (7)$$

the final form of the characteristic function is

$$f(\xi) = J_0(A_0\xi)\exp\left(-\frac{\sigma^2\xi^2}{4}\right)I_0\left(\frac{\sigma^2\xi^2}{4}\right) \quad (8)$$

As it was to be expected in view of the smallness of the ratio σ^2/A_0^2, the leading term of the characteristic function, as obtained by a series expansion of the modified Bessel function, is similar to the characteristic function of a process with a generalized Rayleigh distribution of envelope (familiar case of a sine wave and additive normal noise[8]).

The probability distribution $p(x)$ of the original process $x(t)$, is by definition

$$p(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(\xi)e^{-jx\xi}d\xi. \quad (9)$$

The leading term (aside from a scaled variance) is then the distribution found for instance by Rice[8] (case of high SNR) while correction terms can be evaluated by termwise integration.

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Comment on "A New Precision Low-Level Bolometer Bridge"*

In response to the IRE evaluation[1] of the bolometer bridge described by Reisener and Birx[2] I would like to suggest that bridge sensitivity is not the only criterion to be considered in making an evaluation as to what comprises the "best bridge detection and calibration method available today." In the existing microwave measurements art there are a variety of adaptations of the bolometer bridge concept, and the choice of what is best for a given application usually involves additional considerations such as operating convenience, cost, and the required accuracy of the measurement.

For example, in an application where operating convenience is the prime requirement, the "best" available method would probably include the use of one of the commercially available power meters which use audio frequency power for bridge balancing. In an application where accuracy is the most important consideration, one may forego the convenience of automatic bridge balancing and choose among the manually balanced dc bridges, some of which are also commercially available. Finally the "Self-Balancing D.C. Bolometer Bridge"[3] developed in this laboratory provides the accuracy expectancy of a high quality manual bridge and while retaining much of the operating convenience of the self-balancing audio bridges. This bridge has been commercially available for the past two years.

Admittedly, if one is interested in power measurements below the microwatt level, the greater sensitivity claimed for the method described by Reisener and Birx will presumably take precedence over all other considerations. At the milliwatt level, however, the cited techniques provide substantial improvements over the latter method in the areas of operating convenience and/or accuracy.

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A Method for Obtaining Compatible Single-Sideband Modulation*

Recently Kahn published an article about a new type of amplitude modulation called compatible single-sideband modulation.[1] His methods to obtain CSSB modulation are rather complicated and therefore it seems to be useful to refer to our article,[2] in which a new, simpler method is described. The principle of this method consists of multiplication of a full-carrier SSB signal by itself and suppression of all the frequency components except those around the double-carrier frequency.

This very simple operation provides in the case of single tone modulation a full compatible SSB signal, consisting of only three frequency components, as is shown by the following calculations:

The full-carrier SSB signal

$$\left(\text{carrier frequency} = \frac{1}{2\pi}\,\frac{\omega}{2}\right)$$

$$\cos\frac{\omega}{2}t + a\cos\left(\frac{\omega}{2} + p\right)t$$

gives after multiplication by itself and filtering around the double carrier frequency a signal

$$\cos\omega t + 2a\cos(\omega + p)t + a^2\cos(\omega + 2p)t$$

with an undistorted envelope

$$1 + a^2 + 2a\cos pt.$$

The same calculation for two cosine waves gives for the envelope

$$1 + a^2 + b^2 + 2a\cos pt + 2b\cos qt + 2ab\cos(p - q)t.$$

The last formula shows that the multiplication provides an ideal solution for CSSB modulation if it is possible to eliminate the intermodulation distortion in the envelope. It is remarkable that the increasing of the modulation depth by multiplication also reduces the intermodulation by a factor two. There are different means to eliminate the intermodulation: One is to separate the intermodulation terms by comparing the envelope, produced by a peak detector, with the original audio signal. Then the carrier ω is balanced modulated in amplitude with these terms and the output of the modulator is added in counter phase to the multiplied SSB signal. In the envelope, the intermodulation now almost disappears. Another method is to limit the CSSB signal and to modulate the so-obtained phase-modulated carrier with the undistorted audio signal. In this case it is very simple to determine the spectrum of the multiplied SSB signal after limiting by working the other way round: What spectrum gives

$$\cos\omega t + 2a\cos(\omega + p)t + a^2\cos(\omega + 2p)t$$

after modulation with $a^2+2a\cos pt$. One finally finds for the spectrum

$$A_1 = a \quad A_0 = (1 - a^2) \quad A_{-1} = -a(1 - a^2)$$

$$A_{-2} = +a^2(1 - a^2) \quad A_{-3} = -a^3(1 - a^2), \text{ etc.}$$

(A_1 = upper sideband, A_0 = carrier, $A_{-1\ldots n}$ = lower sideband.) If the phase-modulated carrier is modulated not with a signal $a^2+2a\cos pt$, but with one of the shape $2b\cos pt$, the sideband components (B_{-1}, B_{-2}, etc.) do not totally disappear. The suppression depends on the choice of $K=b/a$ (see Figs. 1 and 2).

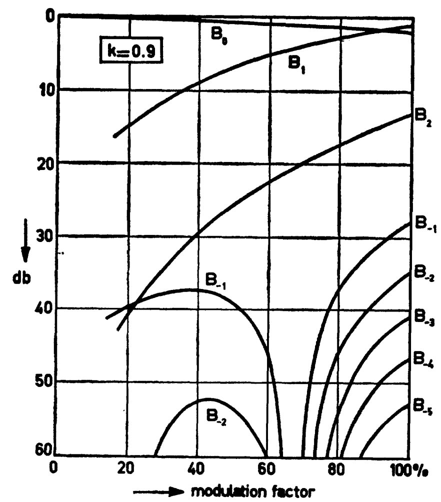

Fig. 1—Amplitudes of various sideband components as a function of the depth of amplitude modulation for adjustment parameter $K=0.9$.

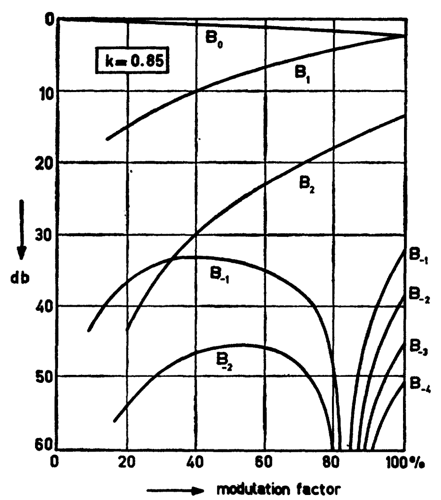

Fig. 2—Amplitudes of various sideband components as a function of the depth of amplitude modulation for adjustment parameter $K=0.85$.

It is very easy to extend the calculations in the case of an audio signal, built up of two cosine waves ($a\cos pt+b\cos qt$), by using the same method as has been shown for a single tone.

Now in the unwanted sideband a spectrum term $-ab\cos(\omega-p+q)t$ appears. Bearing in mind that $a+b$ does not exceed 0.5, it is easily seen the amplitude of this term is always below the level of -24 db.

In the laboratory an experimental transmitter, of which the block diagram is given in Fig. 3 below, has been in use for over a year and is shown to people interested in broadcasting.

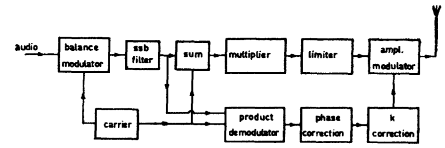

Fig. 3—Block diagram of the CSSB modulator.

The measurement results are in accordance with the theory.

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* Received February 26, 1962.

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Author's Reply[3]

I would like to take this opportunity to comment on the above communication and on the paper[2] authored by van Kessel, Stumpers and Uyen.

Actually, the system they discussed was developed at the Kahn Research Laboratories as a first step in a program to determine the optimum phase stretch technique and is covered by our patent. Theoretically, it was obvious that this technique would not approach the optimum CSSB arrangement, but it was so simple to set up experimentally that we conducted tests and the results of van Kessel, Stumpers and Uyen are very similar to these early experiments. The reason that it was felt that this squared technique was not as satisfactory as the later developed 1.4 phase stretch method was that it was incapable of 100 per cent envelope modulation without serious degradation of the undesired sideband characteristic. (See Fig. 5 of their paper.[2])

The authors point out that a K or correction factor is required in the amplitude modulation branch if the undesired sideband radiation is to be kept at a fairly low value. They indicate that a factor of 0.85 would be about optimum. This means that the system's peak amplitude modulation, *i.e.*, the envelope modulation, is limited to 85 per cent and would produce a loss in signal-to-noise of approximately 1.5 db. While many may feel that this is not of great moment, AM broadcasters are continuously striving to produce full modulation and in many cases it is unfortunately true that even overmodulation is common. In any case, 100 per cent modulation capability is necessary and this requirement is fully met by modern CSSB equipment. (Actually, measurements of 120 per cent modulated CSSB waves show relatively good spectrum characteristics.) The reason for the modulation limita-

[3] Received April 16, 1962.

tion in the squared technique is that the desired phase modulation curve is not closely approximated by the squared phase modulation function (see Fig. 9 of Reference 1).

As to the method of elimination of intermodulation distortion, this technique is identical to the technique used and discussed in my early papers. That is, one either derives the envelope function from the input signal or from a product modulator. In this manner, both the techniques described by van Kessel, Stumpers and Uyen and our modern CSSB systems are theoretically free of envelope distortion. Measurements of equipment indicate that an over-all harmonic distortion figure of less than 1 per cent is easily obtained.

I am, of course, pleased to see that such distinguished engineers have independently derived similar positive conclusions about the CSSB advantages and I appreciate the very kind comments made by these authors in their full paper and also by the Editor of the *European Broadcasting Union Review.* May I suggest that the reader secure the complete paper by van Kessel, Stumpers, and Uyen,[2] as it is most interesting and important.

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A Discussion of Recently Proposed Aether Drift Experiments*

It has been suggested in recent correspondence that the hypothesis of an aether, in addition to providing a satisfactory interpretation of relativistic phenomena, predicts a number of first-order effects which are not predicted by special relativity. On this basis a number of experiments have been proposed to measure the aether drift; these include Rapier's atomic clock experiment [1], Ruderfer's proposed observations on planets and satellites [2], and Carnahan's aberration experiment [3].

It is the purpose of this note to point out that the theory given for some, and probably all, of these proposals is fallacious, and that experiments of a considerably more recondite nature may be necessary to demonstrate the existence (or otherwise) of an aether.

The simplest of the proposals was that of Rapier [1], who suggested making a direct measurement of the one-way passage of a light signal between two synchronized atomic clocks. It is insufficient to refute this with the traditional argument that it is impossible to show that the clocks remain synchronized on separation [4], for if a positive effect were observed its magnitude would depend on the orientation of the light path relative to the aether and would presumably show periodic variations correlated to the earth's rotational and orbital motion; any desynchronization resulting from the separation process could be detected by varying the speed and path of separation.

Unfortunately, however, the aether hypothesis predicts a precisely null result. The fallacy arises from neglect of the second-order frequency change experienced by a moving clock; during the separation of the clocks their difference in frequency builds up into a first-order phase shift which exactly compensates for the effect of aether drift on the transit time of the light signal. In the simplest case, if the velocity relative to the aether is v and one clock is taken to a distance L with velocity u, the absolute time lost by the transported clock is

$$\frac{L}{u}\left\{\left[1-\frac{v^2}{c^2}\right]^{1/2}-\left[1-\left(\frac{v+u}{c}\right)^2\right]^{1/2}\right\}$$

$$\simeq \frac{Lv}{c^2} \text{ (to first order in } v/c).$$

Since this absolute time taken by the light signal is

$$\frac{L}{c}\left[1-\frac{v}{c}\right],$$

the measured time is L/c regardless of the value of v. An exact calculation (assuming the frequency to be a function of velocity but not of acceleration) shows that the result is correct to all powers of v/c and independent of the paths taken by the clocks. The only observable effect is the well-known "clock paradox" which depends only on the changes in speed of the clocks measured in any inertial system and is independent of the value of a superimposed aether drift [5], [6], [7].

The same argument applies to the second suggestion of Ruderfer [2], requiring observations of a stable oscillator in an earth satellite. The anticipated phase-modulation will be exactly compensated by the periodic phase shift of the clock itself resulting from its motion through the aether. This prediction is, in fact, consistent with the results of phase comparisons between highly stable frequency transmissions made daily over distances of thousands of miles. No periodic phase modulations (other than those caused by spurious propagation variations) appear to have been observed [8].

Ruderfer's first proposal, however, involving detailed observations of planetary motion, has to be examined with more caution, since an exact solution of the problem is impossible without knowing the mechanism of gravitational interaction or the likely effect on it of motion through the aether. Nevertheless it is difficult to see how Ruderfer's first-order "timing error" could manifest itself; simple position-time measurements of a planet would yield no information at all since, for example, a straightforward calculation shows that a circular orbit traveling through the aether at speed v would have the same appearance as an elliptical orbit with eccentricity $v_0v/2c^2$, where v_0 is the planet's orbital velocity. This eccentricity cannot be independently checked by distance measurements since it is of the same order (probably 10^{-5} or less) as the intractable second-order relativistic effects. Attempts to detect the "timing error" by observations of a natural satellite of the planet should also give a null result, for the same reason as the earth satellite experiment discussed above, assuming that gravitational clocks undergo the same frequency shifts as atomic and electromagnetic clocks.

Finally we can consider the possibility of aberration experiments. As Carnahan points out [3] these depend for their success on finding a source of radiation whose propagation direction, as measured by an observer at rest in the aether, is unaffected by the motion of the source through the aether. Since the fundamental mechanism of the emission of radiation (or photons) is unknown, it would be premature to assert that such a source cannot exist. But in so far as radiating systems can be analyzed in terms of simple dipoles, or in terms of simple interactions between fundamental particles, elementary considerations of energy and momentum balance indicate that the direction of radiation as seen by the source is fixed, and no aberration effects are therefore observable.

In view of the advantages of an aether hypothesis in interpreting relativistic phenomena [7], it is of interest to inquire whether there might be any other ways of detecting the aether. It is conceivable, for example, that experiments might reveal a slight variation in the velocity of photons with frequncy [9]; the principles of special relativity would then become merely good approximations, and comparisons between high-energy γ rays and light rays would probably enable an aether drift to be detected. Other possibilities could arise from departures from conventional electrodynamics in strong electric or magnetic fields [10], or from detailed studies of the properties of fundamental particles in motion.

In the absence of experimental proof of the absence of such effects it is not permissible to assert on philosophical grounds that attempts to measure the aether drift are meaningless. Nevertheless it seems unlikely that this will be achieved by experiments of the type suggested in recent correspondence.

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Some Remarks on "Penetration of the Ionosphere by Very-Low-Frequency Radio Signals"*

In a recent article,[1] Leiphart, Zeek, Bearce, and Toth have published some interesting data furnished by the Navy satellite "LOFTI I" on the propagation characteristics of VLF radio signals through the ionosphere. Their conclusions, based on the experimental results, seem to confirm early theoretical investigations done by the writer.[2] Although it is likely that the authors of the article have carried on similar investigations, it may be of interest to communicate the writer's results.

1) The authors have recorded a propagation loss of 38 db by day. They have calculated the absorption loss to be 27 db; inter-reflection losses were not calculated. The writer used the "LOFTI I" data to calculate inter-reflection as follows: Fig. 2 of the article in question shows that the lower boundary of the ionosphere can be approximated by a sharp density interface of 2×10^5 electrons per cubic centimeter. The wave is assumed to be linearly polarized; the interface reflection is then calculated on the basis of a gyrotropic semi-infinite uniform plasma.[2] A curve giving reflection losses at a sharp interface between free space and such a plasma is shown in Fig. 1 of this communication. In accord with the "LOFTI I" experiment, a gyrofrequency of 1 Mc per second and a signal frequency of 18 kc per second were used in the calculations. The plasma frequency for the assumed electron density at the interface is 4×10^6 cps; the corresponding reflection loss read off from Fig. 1 is 12 db.

The total propagation loss, reflection loss (12 db), plus absorption loss (27 db) amounts to 39 db which is in close agreement with the 38 db figure obtained in the "LOFTI I" experiment.[1] The additional inter-reflections above 100 km created by the electron density gradient of the medium have been neglected because the gradient is small over several wavelengths except for a narrow region about 200 km. Calculations show that this small region introduces an additional reflection loss less than 4 db.

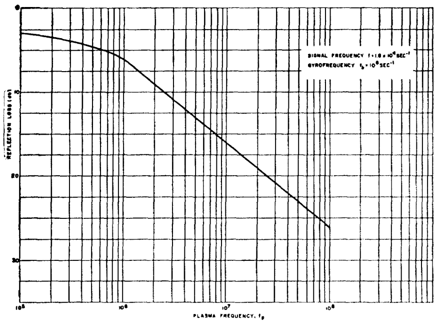

Fig. 1—Reflection loss vs plasma frequency at the interface between free space and a semi-infinite lossless uniform gyrotropic plasma.

2) The assumption of a sharp interface does not hold well at night as seen from Fig. 2 of Leiphart, *et al.* However, if the electron density for an equivalent uniform ionosphere is chosen at 200 km (the electron density gradient above this point begins to get small over a length comparable to a wavelength), one obtains an electron density of 10^5 electrons per cubic centimeter and a corresponding plasma frequency of 2.8×10^6 cps. The reflection loss read off Fig. 1 of this communication is about 10.5 db. The absorption loss calculated by the authors is 2 db.[1] The total loss amounts to 12.5 db, which again is in close agreement with the 13 db figure quoted in the "LOFTI I" experiment.

In neither calculation were spreading losses taken into account. The writer is intending to extend a previous analysis[3] of trapped modes in gyrotropic plasma slabs bounded by sharp interfaces in electron density to the case of varying density at the interface in order to account for the spreading loss.

3) A transmission delay as great as 30 times free space delay was recorded by the authors. This order of magnitude is also in agreement with some of the writer's earlier investigations.[4] The results of this latter investigation shown in Fig. 2 of this communication give group and phase velocities vs altitude; the plot is based on a gyrofrequency of 1 Mc per second, a signal frequency of 18 kc and on the data of Fig. 1 in the article discussed. It is seen that the phase and group velocities of the VLF signals are between 1/10 and 1/100 of the speed of light in vaccum over the major part of their path which accounts for a transmission delay of the order of 10 to 100 times the free space delay.

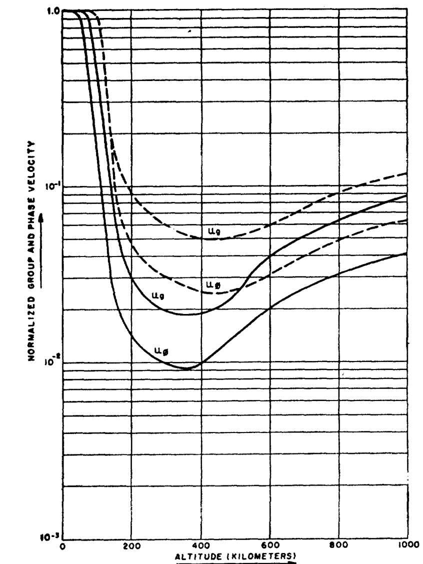

Fig. 2—Group and phase velocities of a VLF electromagnetic wave guided by the earth's magnetic field.

— — — — RH wave velocity-night
———— RH wave velocity-day
V_g = group velocity
V_ϕ = phase velocity

$$\mu_g = \frac{\sqrt{1+\dfrac{\nu_p^2}{\nu_b-1}}}{1+\left[\dfrac{\nu_b/2}{\nu_b-1}\right]\dfrac{\nu_p^2}{\nu_b-1}} = \frac{V_g}{C_v} = \text{normalized group velocity}$$

$$\mu_\phi = \frac{1}{\sqrt{1+\dfrac{\nu_p^2}{\nu_b-1}}} = \frac{V_\phi}{C_v} = \text{normalized phase velocity}$$

$$\nu_p = \frac{f_p}{f}$$

$$\nu_b = \frac{f_b}{f}$$

C_v = speed of light in vacuum
f_p = plasma resonant frequency
f_b = gyrofrequency = 10^6 sec^{-1}
f = signal frequency = 1.8×10^4 sec^{-1}.

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