Correspondence

Electron Beam Noisiness and Equivalent Thermal Temperature for High-Field Emission from a Low-Temperature Cathode*

Dyke[1] has demonstrated that substantial current can be drawn from an unheated cathode in the presence of high electric fields. This raises a question as to the utility of field emission at low cathode temperatures in connection with low-noise microwave amplifiers. Since cathode temperature is so important in limiting the minimum theoretical noise figure of such an amplifier when it uses thermal emission,[2-4] some workers have suggested the possibility of obtaining low-noise performance with field emission from low-temperature cathodes. The following is an evaluation of the "noisiness" of field-emitted beams from cold cathodes, and the derivation of an expression for the equivalent thermal-cathode temperature.

Pierce,[5] Bloom, and Peter[2] have shown that an electron beam carrying noise fluctuations which stem from two uncorrelated noise sources is characterized by a noise standing wave with maximum and minimum currents $I_{\max}$ and $I_{\min}$ such that

$$W|I_{\max}||I_{\min}| = \text{constant} \tag{1}$$

where W is the characteristic beam impedance. The constant is independent of any lossless region through which the beam might pass. The above expression has been termed the beam "noisiness"[6] and is important in determining minimum noise figure.[2-4] For thermal emission, in which the electrons leave a hot cathode with a Maxwellian velocity distribution, (1) becomes[2,5]

$$W|I_{\max}||I_{\min}| = 2\alpha kT_cB \tag{2}$$

where T_c is the cathode temperature. Assuming pure shot noise at the potential minimum, α is approximately unity.

Since the constant in (1) is independent of the region through which the beam passes, the constant can be evaluated readily for any process of emission by assuming that the anode is placed very close to the cathode and given a high potential. The emitted electrons then are immediately accelerated by the strong electrostatic fields which prevent the accumulation of space charge in the cathode-anode region. For the above temperature-limited operation, it is valid to consider the entire cathode-anode region as constituting a velocity jump. If one of the independent noise sources is due to current fluctuation at the cathode and the other is due to an uncorrelated velocity fluctuation there, (1) can be written

$$W|I_{\max}||I_{\min}| = \left[\overline{I^2}\frac{\overline{v_0^2}}{\eta^2}\frac{2eB}{I_0}\overline{\delta v^2}\right]^{1/2} \tag{3}$$

where

$\overline{I^2}$ is the mean-square current fluctuation at the anode,

$\overline{v_0^2}$ is the mean-square velocity of the electrons passing the anode plane, and

$\overline{\delta v^2}$ is the mean-square deviation in velocity, or the variance, of the electrons passing the anode plane.

When electrons emerge from a cathode in completely random fashion, the current fluctuation there is called shot noise and is given by

$$\overline{I^2} = 2eI_0B. \tag{4}$$

Complete randomness characterizes both thermal emission and high-field emission. Under the cathode-anode conditions assumed, (4) also gives the current fluctuation at the anode for both types of emission. Evaluation of $\overline{v_0^2}$ and $\overline{\delta v^2}$ for both cases requires a knowledge of the respective energy distribution functions.

The energy distribution for thermal emission is very nearly Maxwellian and is given by

$$P_t(w) = A_t \exp\left(-\frac{w}{kT_c}\right) \quad \text{for } w > eV_0$$
$$= 0 \qquad \text{for } w < eV_0 \tag{5}$$

where

$P_t(w)dw$ is the number of electrons passing the anode plane per unit time having kinetic energies between w and $w+dw$,

A_t is a constant,

w is the electron kinetic energy (*i.e.*, $mv^2/2$) normal to the cathode and anode planes, and

V_0 is the anode voltage.

Assuming $eV_0/kT_c \gg 1$ we obtain

$$\overline{v_0^2} = \frac{\int_0^\infty v^2P_t(w)dw}{\int_0^\infty P_t(w)dw} = 2\eta V_0 \tag{6}$$

and

$$\overline{\delta v^2} = \overline{v_0^2} - \left[\frac{\int_0^\infty vP_t(w)dw}{\int_0^\infty P_t(w)dw}\right]^2 = \left(\frac{kT_c}{2m}\right)\left(\frac{kT_c}{eV_0}\right). \tag{7}$$

Using (4), (6), and (7) in (3) gives

$$W|I_{\max}||I_{\min}| = 2kT_cB \tag{8}$$

which agrees with (2).

Conceptually and mathematically, field emission is more complicated than thermal emission. The wave-mechanical theory as enunciated by Sommerfeld, Bethe, Nordheim, and Fowler[7-9] assumes the electrons within the cathode metal to be incident upon a potential barrier at the metal surface. With no accelerating field the barrier is infinitely thick and the electrons cannot escape by penetration through the barrier. Unless energy is given the electrons by heating the cathode, the electrons cannot escape over the top of the barrier. However, the addition of an accelerating field reduces the barrier thickness and makes possible electron escape from a cold cathode by the so-called "tunnel effect." Field emission occurs when an appreciable number of electrons escape.

A Fermi-Dirac energy distribution for the number of electrons incident upon the barrier per unit time is assumed. The probability of transmission through the barrier, sometimes called the transmission coefficient, is a function of electron energy and barrier thickness, increasing with electron energy and decreasing with barrier thickness. The product of the Fermi-Dirac energy distribution and the transmission coefficient gives the distribution of the normal component of kinetic energy for the emitted electrons. The complete expression for the distribution function is complicated and difficult to use for the calculations of interest here. However, by making simplifying assumptions, Richter[10] has arrived at a tractable, approximate expression for the case of zero absolute temperature. Following Richter, we obtain for the energy distribution function of electrons passing the anode plane per unit time

$$P_f(w) = A_f(w_0 - w)\exp\left(-\frac{w_0 - w}{e\beta}\right) \quad \text{for } w_0 - e\mu < w < w_0$$
$$= 0 \qquad \text{otherwise} \tag{9}$$

* Received by the IRE, April 2, 1956. Prepared under AF Contract AF33(600)-27784.

[1] W. P. Dyke, "Progress in electron emission at high fields," Proc. IRE, vol. 43, pp. 162–167; February, 1955.

[2] S. Bloom and R. W. Peter, "A minimum noise figure for the traveling-wave tube," *RCA Rev.*, vol. 15, pp. 252–265; June, 1954.

[3] J. R. Pierce and W. E. Danielson, "Minimum noise figure of traveling-wave tubes with uniform helices," *J. Appl. Phys.*, vol. 25, pp. 1163–1165; September, 1954.

[4] H. A. Haus and F. N. H. Robinson, "The minimum noise figure of microwave beam amplifiers," Proc. IRE, vol. 43, pp. 981–991; August, 1955.

[5] J. R. Pierce, "A theorem concening noise in electron streams," *J. Appl. Phys.*, vol. 25, pp. 931–933; August, 1954.

[6] S. Bloom, "The effect of initial noise current and velocity correlation on the noise figure of traveling-wave tubes," *RCA Rev.*, vol. 16, pp. 179–196; June, 1955.

[7] A. Sommerfeld and H. Bethe, "Elketronentheorie der metalle," *Handbuch der Physik*, vol. 24/2, pp. 436–443; 1933.

[8] L. Nordheim, "Die theorie der elektronenemission der metalle," *Physik. Z.*, vol. 30, pp. 177–196; April, 1929.

[9] R. H. Fowler and L. Nordheim, "Electron emission in intense electric fields," *Proc. Roy. Soc. A*, vol. 119, pp. 173–181; June, 1928.

[10] G. Richter, "Zur geschwindigkeitsverteilung der feldelecktronen," *Z. für Physik*, vol. 119, pp. 406–414; September, 1942.

where

- A_f is independent of w,
- β is a voltage given numerically by $0.97\times 10^{-10}E/\sqrt{\phi}$,
- E is the accelerating field at the cathode in volts per meter,
- ϕ is the thermionic work function in volts,
- w_0 is an energy given by $(eV_0 - e\phi)$ in MKS units, and
- $e\mu$ is the energy range in the Fermi-Dirac distribution for zero absolute temperature.

A plot of (9) for a typical set of parameters is shown by the solid-line curve of Fig. 1. The adjacent dotted-line curve is given to illustrate the effect of temperature. It was computed from an expression, more complete than (9), which takes into account temperature. Note that for the field-emission distribution the electron potentials are below the anode potential by approximately the work function. This is a consequence of the "tunneling." Shown for comparison are plots of the Maxwellian distribution (5) for two different values of cathode temperature.

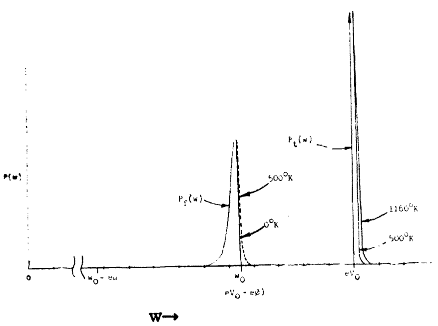

Fig. 1—A comparison of the energy distributions of electrons for high-field emission and for thermionic emission. The high-field emission curves assume a tungsten cathode and a field strength of 3.5×10^9 volts per meter. For tungsten $\phi = 4.5$ volts and $\mu = 5.7$ volts.

Using (9) and following the procedures indicated in (6) and (7) we obtain for the field emission case

$$\overline{v_0^2} = \frac{2w_0}{m} \tag{10}$$

and

$$\overline{\delta v^2} = \frac{(e\beta)^2}{w_0 m}. \tag{11}$$

Here we have assumed w_0 is much greater than the energy spread and that $\exp(-\mu/\beta) \ll 1$. Using (4), (10) and (11) in (3) gives for the beam noisiness

$$W\,|\,I_{\max}\,|\,|\,I_{\min}\,| = \sqrt{2}\,2e\beta B. \tag{12}$$

By equating (8) and (12) we obtain the equivalent thermal-cathode temperature for field emission from a zero temperature cathode. Thus

$$T_{\text{eq.}} = \sqrt{2}\,\frac{e\beta}{k} = 1.6\times 10^{-6}\,\frac{E}{\sqrt{\phi}}. \tag{13}$$

For the solid-line curve of Fig. 1, $T_{\text{eq.}}$ is 2,640°K.

As (13) indicates, the equivalent temperature can be reduced by decreasing E and by choosing a metal with a higher work function. Since the total current emitted is approximately proportional to[9]

$$\frac{E^2}{\phi}\exp\left(-6.8\times 10^9\,\frac{\phi^{3/2}}{E}\right),$$

a small change in either E or ϕ unfortunately has enormous effect upon the emission. For example, in order to reduce the equivalent temperature to approximately a typical thermal-cathode temperature, say 1,160°K, the E for the above example must be reduced by a factor of 2.27 which results in a current reduction by a factor of 7×10^{10}!

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VHF Diffraction by Mountains of the Alaska Range*

During the winter of 1953–1954 it was learned that television signals of usable quality were being received at Lake Minchumina, Alaska, from two transmitters at Anchorage, 200 miles away. As Mt. McKinley, over 20,000 feet high, lies almost directly on the line of sight between the two locations, it is evident that a diffracted wave is responsible for the result. Signals were received on two frequencies, 57 mc and 200 mc, with roughly equal intensities, the peak transmitted power of the latter signal being 400 watts and the antenna gain about 7.

A preliminary field investigation in the summer of 1954 suggested that this was an interesting opportunity for studying diffraction propagation over very long paths. On either side of the line between Anchorage and Minchumina are two very prominent peaks, Mt. McKinley (20,300 feet) and Mt. Foraker (17,395 feet), which are connected by a rugged ridge 15 miles long, studded with peaks between 10,000 and 14,000 feet high. Mt. McKinley is the highest mountain in North America, and probably the highest in the world above its base plateau. There are no extensive foothill systems. The valleys on either side are extremely flat, over a hundred miles across, and nearly uninhabited.

In order to study temporal variations of signal strength, a receiving station was installed at Lake Minchumina, consisting of a receiver, a graphic recorder, and a direction-finding antenna. The 200 mc (Channel-11) signal was chosen, to simplify antenna construction. Throughout the winter of 1954–1955 occasional severe fades were noted, accompanied by swings in the direction of arrival of up to 12°. On occasion the fading would be frequency selective so that the video signal would fade out while the audio signal remained strong, and vice versa. At all other times the signal varied in a less extreme manner, with periods of from one to several hours. In part, this fading has been correlated with the rise and fall of the tides in the estuary (Knick Arm) at Anchorage; however, it seems certain that the major influences are meteorological. This contrasts with previous experiences with diffracted signals over shorter paths, which show, in general, relatively small variations of signal strength.

The space-variation of signal strength was investigated, during the summer of 1955, by airborne measurements taken from a C-47 airplane equipped with a 200 mc receiver and a steerable, 4-element, Yagi antenna. During these tests, the Anchorage 200 mc station transmitted a "black" picture; that is, a signal unmodulated except for synch pulses. Flights were made along a 100 mile path, centered at Lake Minchumina and perpendicular to the direction of propagation, at altitudes of 5,000, 10,000, 15,000, and 20,000 feet. Navigation was accomplished with the aid of the Minchumina radio range, which has a magnetic bearing of 216°-36°, almost exactly perpendicular to the direction of propagation. Radar fixes were obtained at the beginning and end of each run, and dead reckoning was used to interpolate to positions between these fixes. The most noticeable feature of the results was the very rapid fluctuation of signal strength from point to point over distances of a few hundred feet. Signals of as much as 10 microvolts (antenna output) were obtained on local maxima, and at minima the signal often faded into the noise (about 2 microvolts). At all points in the radio shadow of the mountains the signal displayed this violent variation. During the 20,000 feet altitude run the plane was heading northeast along the radio range, and when it emerged from the radio shadow of the McKinley mountain complex the signal became steady and strong (19 microvolts). At this point the descent was started, and the signal declined steadily and smoothly until it disappeared into the noise at about 12,000 feet. It is evident, then, that the mountains are responsible for the presence of the signal in the vicinity of Minchumina, and that the existence of the pronounced fine structure of the diffraction pattern makes the siting of a receiving antenna extremely important. It was noted during analysis of the flight data that coarse-structure maxima were obtained when the plane was in line with the larger mountains. These maxima, however, were much less important than the fine-structure maxima.

In order to verify the fine structure some additional data were taken on the ground at Minchumina Airport. The transmitter was modulated with a video test pattern, and the audio channel was disabled. In a direction parallel with the direction of propagation the signal voltage (antenna output across 50 ohms from a 10-element Yagi, gain 7.5 db) varied between 2.5 and 10.5 microvolts with periods of from 600 to 700 feet. The maxima were not all of the same intensity, but the pattern repeated itself, approximately, throughout the entire 4,000 foot run. A similar result was experienced

* Received by the IRE, May 28, 1956. This work was performed as part of Air Force Contract No. AF19(604)-1089.

when the field intensity was explored in the direction perpendicular to the direction of propagation, except that in this case the minima were spaced from 50 to 100 feet apart.

Such a complicated interference pattern must be quite sensitive to small changes in phase of one or more of the interfering components; such changes might accompany meteorological changes, even though much of the transmission path is at high altitude. It is thought that the time-variations in signal strength and direction of arrival can be explained by changes in meteorological conditions over the paths of various signal components arriving via scattering and/or diffraction by widely-separated mountain peaks.

It was thought desirable to check the experimental signal strengths against those predicted by knife-edge diffraction theory. For the Anchorage-to-Minchumina path an obstacle height (above sea level) of 12,000 feet was assumed, this being approximately the average height of the profile between Mts. McKinley and Foraker. The other distances are as follows: transmitter to knife-edge, 132 miles; receiver to knife-edge, 68 miles; obstacle height above direct line between transmitter and receiver (using 4/3 earth radius): 15,400 feet. Using these data, the transmission loss between transmitter and receiver was calculated to be 150 db, assuming no image antennas. The maximum image gain from a 4-ray treatment would be 12 db. The signal actually received at Lake Minchumina with the receiving antenna at a theoretically optimum height corresponded with a transmission loss of 131 db. The signal was therefore some 7 db stronger than would be expected under the optimum 4-ray situation.

An additional experiment was conducted over a different path, to obtain another check of the validity of the knife-edge approximation. This path extended from College, Alaska (near Fairbanks) to the crossing of the Paxson-Cantwell Road and the Maclaren River, a distance of 135 miles, and crossed the Alaska Range in the vicinity of Mt. Hayes. The height of the knife-edge above sea level was assumed to be 8,000 feet, and the distance from transmitter (at College) to knife-edge was 97 miles. The transmitter was a modified SCR-270 radar set with a frequency of 107 mc. Using a 4/3 earth radius the one-ray transmission loss was computed to be 132 db. Adding 12 db for maximum image gain would give 120 db transmission loss. These figures are in agreement with the measured transmission loss of 126 db. No measurements were made to determine how this signal varied with position or time.

Despite the assumptions made necessary by lack of detailed topographical information, the above results appear to give substantial support to the validity of the knife-edge approximation.

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Measurement Considerations in High-Frequency Power Gain of Junction Transistors*

Introduction

A recent paper[1] concerning high-frequency power gain of junction transistors included experimental results of measurements of power gain for a number of grown-junction transistors. Since the appearance of the paper, the writer has been criticized for the method used to measure power gain, *viz.*, by employing *no* neutralization and by *not* conjugate matching in the input circuit. For example, one question that has been raised is whether a value of high-frequency gain for an unneutralized transistor has any meaning.

This letter explains in more detail why the measurements were made as they were. This was not done in the original paper because much of the necessary background material more properly belonged in a then-unpublished paper by J. G. Linvill.[2] Also given here is an extension of the theoretical results presented earlier to two types of neutralization commonly used in common-emitter transistor amplifiers.

Transistor Stability

For a particular transistor in a particular configuration, there are ranges of frequencies for which the transistor is *unconditionally stable; i.e.*, if no external feedback is employed, oscillations cannot be obtained with any possible passive terminations.[3] In these frequency ranges the transistor *per se* has an unambiguous maximum available power gain. The value of this gain can be calculated in terms of the four-pole parameters as shown by Linvill, and it can be measured by employing conjugate matching at both input and output with no risk of converting the transistor amplifier to an oscillator. On the other hand, in the other frequency ranges where the transistor is potentially unstable, oscillations may occur for particular terminations, and it is necessary to apply some sort of constraint in defining a "maximum" gain, since in reality the maximum gain is infinity. The constraint may take the form of neutralization,[4] of unilateralization,[5] or of requiring a given degree of "stability," with a suitable analytical definition for the latter.[6]

Since the type of constraint employed necessarily influences the value of "maximum" gain obtained, the writer preferred to avoid this situation if possible and to consider high-frequency power gain under conditions for which the transistor was unconditionally stable.

Investigation of the conditions for unconditional stability[3] for the usual ideal model of the junction transistor including ohmic base resistance r_b' and collector-base capacitance C_c, leads to the following results for *high* frequencies.[7] For the common-emitter configuration, unconditional stability is obtained for radian frequencies ω *greater* than a critical frequency[8]

$$\omega_{\text{crit}} \equiv 0.4(r_\epsilon'/r_b')\omega_a, \tag{1}$$

where r_ϵ' is the Shockley *et al* emitter resistance $(kT/q_e I_\epsilon)$, and ω_a is the inherent radian alpha-cutoff frequency of the transistor. This equation is valid for either the fused-junction or the grown-junction transistor, since ω_{crit} is always less (by a factor of 2.5) than the frequency for which the distributed nature of the base resistance of a grown-junction transistor becomes significant, *i.e.*, for $\omega < 2.5\omega_{\text{crit}}$, the complex base impedance z_b' of a grown-junction transistor is resistive and is equal to r_b'.[9] On the other hand, for either the common-base or the common-collector configuration, unconditional stability is obtained only at frequencies *greater* than the inherent alpha-cutoff frequency ω_a.

* Received by the IRE, April 23, 1956.

[1] R. L. Pritchard, "High-frequency power gain of junction transistors," Proc. IRE, vol. 43, pp. 1075–1085; September, 1955.

[2] J. G. Linvill, "The relationship of transistor parameters to amplifier performance," presented at IRE-AIEE-Univ. of Penn. Conf. on Transistor Circuits, Philadelphia, Pa., February 17, 1955, submitted for publication, *Bell Sys. Tech. J.*

[3] A stability criterion for determining whether or not a particular four-pole is unconditionally stable has been derived in terms of its four-pole parameters by Linvill (see footnote reference 2), and independently by A. P. Stern, "Considerations on the stability of active elements and applications," 1956 IRE Conv. Rec., part 4. The criterion was quoted by A. P. Stern, C. A. Aldridge, and W. F. Chow, "Internal feedback and neutralization of transistor amplifiers," Proc. IRE, vol. 43, p. 839, (5); July, 1955.

[4] Using the definition quoted by C. C. Cheng, "Neutralization and unilateralization," Trans. IRE, vol. CT-2, p. 138; June, 1955, neutralization refers to "the process of balancing out an undesirable effect."

[5] Using the definition of Cheng, see footnote reference 4, and Stern *et al.*, see footnote reference 3, unilateralization refers to a method of rendering a bilateral network unilateral. Note that unilateralization is a special case of neutralization, but the converse is not necessarily true. Both of these authors also have pointed out the large number of different ways by which neutralization can be effected. See also G. Y. Chu, "Unilateralization of junction-transistor amplifiers at high frequencies," Proc. IRE, vol. 43, pp. 1001–1006; August, 1955.

[6] For example, see J. F. Gibbons, "Transistor amplifier performance," paper presented at IRE-AIEE-Univ. of Penn. Transistor Circuits Conf., Philadelphia, Pa., February 17, 1956. Also, D. D. Holmes and T. O. Stanley, "Stability considerations in transistor IF amplifiers," 1956 IRE Conv. Rec., part 4, and A. P. Stern, footnote reference 3.

[7] High frequencies are defined here as (radian) frequencies ω much greater than the common-emitter current-amplification-factor cutoff frequency $(1-\alpha_0)\omega_a$, where ω_a is the (radian) inherent alpha-cutoff frequency of the transistor; more exactly, as $\omega^2 \gg (1-\alpha_0)^2\omega_a^2$.

[8] Pritchard, *op. cit.*, footnote 13. Also derived independently by Stern, *op. cit.*

[9] Pritchard, *ibid.*, (9a).

The critical frequency of (1) is sufficiently low in general relative to ω_a that the common-emitter configuration provides a fairly wide range of "high" frequencies over which power gain can be measured *without* having to employ neutralization for stability. Accordingly, for the writer's measurements the common-emitter configuration was employed, and a measuring frequency of 5 mc was chosen for convenience as being above the critical frequency in general for the transistors used.

The reason for employing a purely resistive generator impedance rather than using conjugate matching in the input circuit was simply convenience. In general, such a measurement would *not* yield the maximum gain. However, as noted in the original paper, calculations show that for the junction-transistor model at *high* frequencies, greater than ω_{crit}, the maximum gain obtainable with a purely resistive generator is at most a db less than the *true* maximum gain, which *would* be measured with conjugate matching in the input circuit.

Furthermore, an additional calculation for the theoretical model, taking account of low-frequency parameters, has shown that potential instability for this method of measuring power gain with a purely resistive generator impedance occurs only at frequencies *less* than ω_{crit} of (1). In fact, the critical frequency in *this* case is the geometric mean of ω_{crit} and the common-emitter current-amplification-factor cutoff frequency $(1-\alpha_0)\omega_a$. For frequencies greater than *this* critical frequency, the value of gain measured is the available power gain quoted earlier, multiplied by a function of frequency that is equal to two at this critical frequency but then decreases rapidly to one with increasing frequency.

Neutralization or Unilateralization

Although it is *not* necessary to employ neutralization to maintain stability of transistors in the common-emitter configuration for frequencies greater than ω_{crit}, some sort of neutralization often *is* employed to minimize interaction between cascaded amplifier stages. Accordingly, it is of interest to present results of calculations of the maximum available power gain at high frequencies for the transistor model with the y type of neutralization or unilateralization commonly employed,[10] as shown in Fig. 1. In this circuit a feedback admittance y_f is connected between output and input terminals of the transistor through an ideal, lossless, phase-reversing transformer of turns ratio $n{:}1$. For neutralization, the feedback admittance generally is a neutralizing capacitor C_N such that

$$y_f = j\omega C_N = -nIm(y_{12}), \qquad (2)$$

whereas for unilateralization, by definition,

$$y_f = -ny_{12}, \qquad (3)$$

where y_{12} is the short-circuit feedback admittance of the transistor (numerically negative owing to the current-polarity convention used).

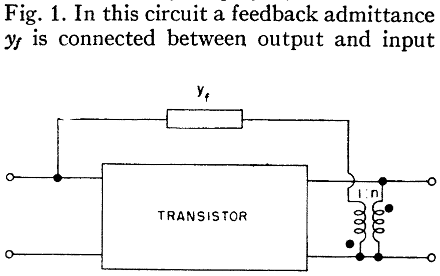

Fig. 1—Transistor with feedback (neutralizing) admittance and phase-reversing transformer.

At frequencies $\omega > \omega_{crit}$, the maximum available power gain of the transistor *per se* $(y_f=0)$ is given in terms of the equation presented earlier (for the constant $-r_b'$ model) as:

$$G_{max} = G_{av} \cdot K_G(c), \qquad \omega > \omega_{crit}, \qquad (4)$$

where

$$G_{av} \equiv \frac{0.2}{\omega^2}\left(\frac{\omega_a}{r_b' C_c}\right), \qquad (5)$$

and $K_G(c)$ is Linvill's gain factor[2] as a function of his criticalness factor c, which for this case is simply $c=(\omega_{crit}/\omega)$. Normally K_G is essentially equal to one, but as c approaches one, indicating impending instability, K_G rapidly approaches the value 2.

If the capacitive-type neutralization is employed (2), the corresponding expression for maximum gain is

$$G_{max} = G_{av}\left[\frac{(\omega/\omega_{crit})^2+4}{(\omega/\omega_{crit})^2+2}\right]K_G(c'), \qquad (6)$$

where c' is the criticalness factor for the new amplifier and is a function of (ω/ω_{crit}). Note that this expression is independent of the turns ratio n (for the lossless transformer).

For the case of y unilateralization (3), the maximum available power gain *is* a function of the turns ratio n. However, if n is made sufficiently large relative to (r_e'/r_b'), the dependence can be essentially eliminated, in which case the maximum power gain is

$$G_{max} = G_{av}[1 + (2\omega_{crit}/\omega)^2]. \qquad (7)$$

The gain factor K_G for this case is equal to one, since the criticalness factor vanishes for the case of zero feedback.

A comparison between the power gain for these three cases is presented in Fig. 2, which shows the ratio G_{max}/G_{av} as a function of (ω/ω_{crit}). Note that as frequency is decreased below ω_{crit}, the maximum y unilateralized gain increases quite rapidly with respect to G_{av} (which already is increasing at the rate of 6 db per octave of decreasing frequency). Ultimately, however, the power gain is limited by low-frequency parameters which have not been included in these calculations.

It should be emphasized that these results correspond to only *two* of *many* possible methods of neutralizing a transistor amplifier.

Conclusion

A few additional remarks concerning the critical frequency of (1) may be of some interest. In particular, note that the value of C_c does not appear explicitly. This may be surprising at first inasmuch as C_c is the source of collector-base feedback, which gives rise to instability. However, at high frequencies, where the phase of alpha is significant, collector-base capacitance also contributes the dominant part of collector-emitter (output) conductance, which tends to stabilize the transistor. Hence, if two transistors at the same dc emitter current have identical values of ω_a, and of the *product* $r_b'C_c$, so that their high-frequency gains are identical, the transistor with the lower C_c will be stable down to a lower frequency, not because it has lower capacitance, but because it has higher base resistance!

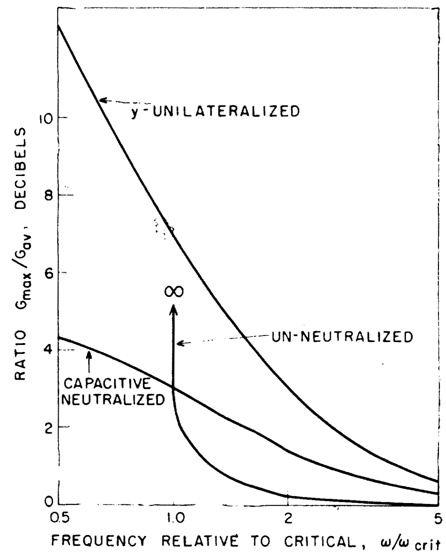

Fig. 2—Ratio of maximum available power gain to figure-of-merit power gain as a function of relative frequency for transistor amplifier; unneutralized, y neutralized, and y unilateralized.

Since a grown-junction type of transistor generally has a low collector capacitance and high ohmic base resistance relative to the fused-junction transistor, this may explain why the former type of transistor is more likely to be stable at the low end of the high-frequency range, *e.g.*, at 455 kc.

In conclusion, it should be emphasized again that all of the above discussion concerns frequencies that are high relative to the common-emitter current-amplification-factor cutoff frequency $(1-a_0)f_a$. However, a given frequency, *e.g.*, 455 kc, which may be high for one transistor, may be low for another in which $(1-\alpha_0)\omega_a$ and ω_a are both very high. For example, a transistor with f_a of the order of 500 mc, and $\alpha_0=0.98$[11] *may* be unconditionally stable at 455 kc simply because this is essentially dc for such a transistor![12]

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[10] See, for example, D. D. Holmes, T. O. Stanley, and L. A. Freedman, "A developmental pocket-size broadcast receiver employing transistors," Proc. IRE, vol. 43, pp. 663–664; June, 1954. Also, see L. J. Giacoletto, "Performance of a radio-frequency alloy junction transistor in different circuits," *Transistors I*, RCA Labs., Princeton, N. J., pp. 431–457; March, 1956.

[11] For example, the diffused-base transistor; C. A. Lee, "A high-frequency diffused base germanium transistor," *Bell Syst. Tech. J.*, vol. 35, p. 23; January, 1956.

[12] At low frequencies a junction transistor is unconditionally stable from dc *up to* a second critical frequency, which generally is somewhat less than $(1-\alpha_0)\omega_a$ and which is inversely proportional to C_c and to collector-base resistance. In this frequency range, C_c gives rise to potential instability in the same manner as does grid-plate capacitance in a conventional triode electron tube (Miller effect).

On the Waveform of a Radio Atmospheric at Short Ranges*

In view of the very great number of past experimental investigations[1] of waveforms of atmospherics, it seems worthwhile to give further consideration to this problem from a theoretical standpoint. Probably the first approach in this direction was carried out by Ollendorf.[2] The lightning flashes were represented by dipole moments which vary with time, so that the fields produced can be obtained with the aid of a Hertz vector. In a previous theoretical analysis, the author[3] considered a vertical electric dipole on a curved imperfectly conducting earth for the case of a current excitation which varied exponentially with time. It is the purpose of this note to present some calculations in graphical form showing the nature of the transient response of idealized lightning discharge at short ranges where the ionospherically reflected wave can be neglected or separately accounted for.

The instantaneous product of the dipole current and vertical height is denoted by $P(t)$ and is represented as follows

$$P(t) = \sum_{n=0}^{\infty} p_n e^{-\Gamma_n t} u(t) \qquad (1)$$

with $u(t)=0$ for $t<0$ and $=1$ for $t>0$, where p_n and Γ_n are real coefficients. It is known from the work of Bruce and Golde[4] and others[1] that only about two or three terms in the above expansion are adequate to represent the electric moment of a typical discharge. Employing the results of a previous theoretical analysis,[3] the electric field $E(t)$ at a distance D meters on a homogeneous earth of conductivity σ is

$$E(t) = \frac{2\times 10^{-7}}{D} u(t') \sum_{n=0}^{\infty} p_n B_n(t') \qquad (2)$$

where

$$B_n(t) = \left(\frac{t}{2\alpha^2} + \Gamma_n - \frac{C}{D}\right) e^{-t^2/4\alpha^2} + \frac{C^2}{\Gamma_n D^2} + \left(\frac{C}{D} - \frac{C^2}{\Gamma_n D^2} - \Gamma_n\right) e^{-\Gamma_n t}\left[1 + \pi^{1/2}\alpha\Gamma_n e^{\Gamma_n^2\alpha^2} \cdot \left[\operatorname{erf}\left(\frac{t}{2\alpha} - \alpha\Gamma_n\right) + \operatorname{erf}(\alpha\Gamma_n)\right]\right] \qquad (3)$$

where $t'=t-D/C$, $C=3\times 10^8$ meter/sec, $\operatorname{erf}(z) = 2\pi^{-1/2}\int_0^z e^{-x^2}dx$, and

$$\alpha = \left(\frac{D}{21.6\pi\sigma}\right)^{1/2} \times 10^{-9}.$$

In the preceding formula, the effect of earth curvature has not been included. It can be shown[3] that the flat earth approximation is valid for transient fields if t' is greater than about 0.2 microseconds and the range D is not much greater than 50 km. If the earth conductivity is allowed to approach infinity, the preceding equation reduces to

$$B_n(t) = \delta(t) + \frac{C^2}{\Gamma_n D^2}(1 - e^{-\Gamma_n t}) + \left(\frac{C}{D} - \Gamma_n\right) e^{-\Gamma_n t} \qquad (4)$$

where $\delta(t)$ is the unit impulse or "Dirac" function.

As an example, the response $B(t)$ is calculated for a source function, given by

$$P(t) = P_m\left[e^{-\Gamma_1 t} - e^{-\Gamma_2 t}\right] \qquad (5)$$

which is clearly a special case of (1) where only two terms are considered. P_m is a constant which is independent of time and would be proportional to the peak magnitude of the product of the current times the height of stroke column. Choosing $\Gamma_1 = 0.02\times 10^6$ sec^{-1} and $\Gamma_2 = 0.50\times 10^6$ the function $P(t)/P_m$, which can be called the stroke dipole moment, is plotted in Fig. 1. It is believed that this pulse shape is representative of the main return stroke of the lightning discharge having a build up time of about 10 μsec and pulse width of about 50 μsec.

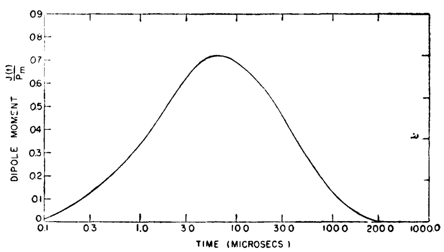

Fig. 1—Idealized stroke dipole moment: $P(t)/P_m = e^{-\Gamma_1 t} - e^{-\Gamma_2 t}$.

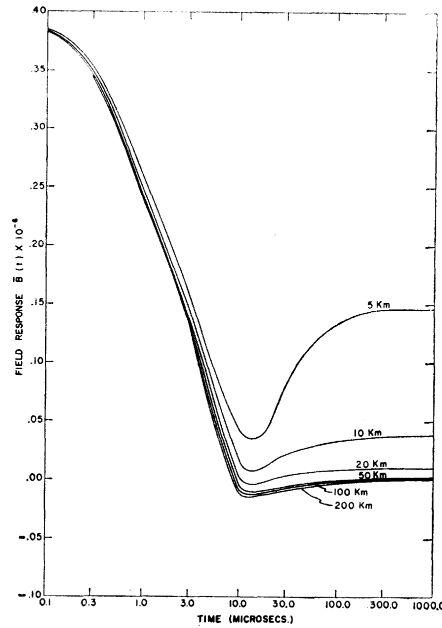

Fig. 2—Field response for idealized stroke dipole moment; $\sigma=\infty$.

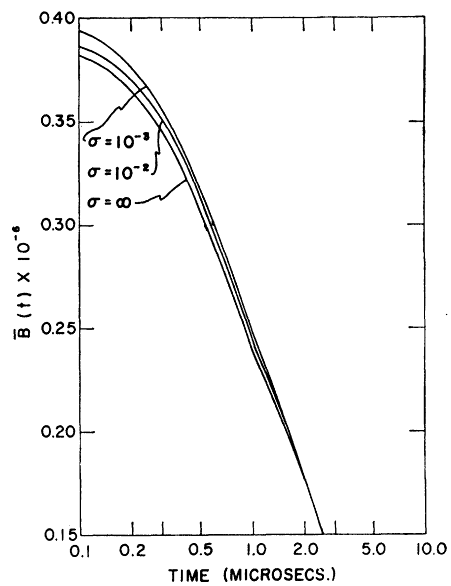

Fig. 3—Field response for idealized stroke dipole moment; $D=100$ km.

The field response $B(t)$ for this particular source is shown plotted in Fig. 2 for various ranges over a perfectly conducting ground. It is interesting to note that at larger ranges the shape of the response curves approach an asymptotic value which is closely proportional to the first time derivative of the source dipole moment. At shorter ranges, the shape of the response curves is very different and approaches the time integral of the dipole moment. The curves illustrated in Fig. 2 would be modified at small times to some extent if the finite conductivity of the ground were considered. As an example, the function $B(t)$ for a range of 100 km is shown plotted in Fig. 3 for ground conductivities of ∞, 10^{-2}, and 10^{-3} mhos per meter. The earth curvature would also have some effect on the response curves at very small times, say less than 0.2 μsec. In the event that one should become interested in this region of the time domain, reference can be made to an earlier paper by the author.[3]

It is hoped that more detailed experimental results on atmospheric waveforms will be forthcoming. For the purpose of comparing the experimental results with theory, it is highly desirable that simultaneous wave forms be made at two or more stations whose distances to the source is in the range from 5 to 200 km. The recent simultaneous recordings taken at the University of Florida[5] seem very promising. Despite the fact that the distances to strokes were not known accurately, the qualitative transformation of the wave shape with distance is in agreement with the theoretical curves presented here.

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* Received by the IRE, April 9, 1956.

[1] An excellent summary of past work in this field is given in "Bibliography relating to lightning atmospheric location," Lightning and Transients Res. Inst. Rep. No. 181, Minneapolis, Minn., 1955.

[2] F. Ollendorf, "Radiation field of lightning," *Elekt. Nachr.-Tech.*, vol. 7, pp. 108–119; March, 1930.

[3] J. R. Wait, "Transient fields of a vertical dipole over a homogeneous curved ground," *Can. J. Phys.* vol. 34, pp. 27–35; January, 1946.

[4] C. E. R. Bruce and R. Golde, "The mechanism of the lightning stroke," *J. IEE*, vol. 88, pp. 487–497; December, 1941.

[5] A. W. Sullivan, S. P. Hersperger, R. F. Brown, and J. D. Wells, "Investigation of atmospheric radio noise," Scientific Report No. 9, Dept. of Elect. Eng., University of Florida, October, 1955.

A Balanced, Unregulated, Dual Power Supply*

When using certain types of dc amplifiers requiring two voltages of opposite polarity and equal amplitude or servo potentiometers that require a physically fixed zero voltage reference point or any instrument requiring two voltages of opposite polarity and equal amplitude, it is desirable to have a power supply that will vary both ± voltages simultaneously and by like amount regardless of changes occurring in the ac line.

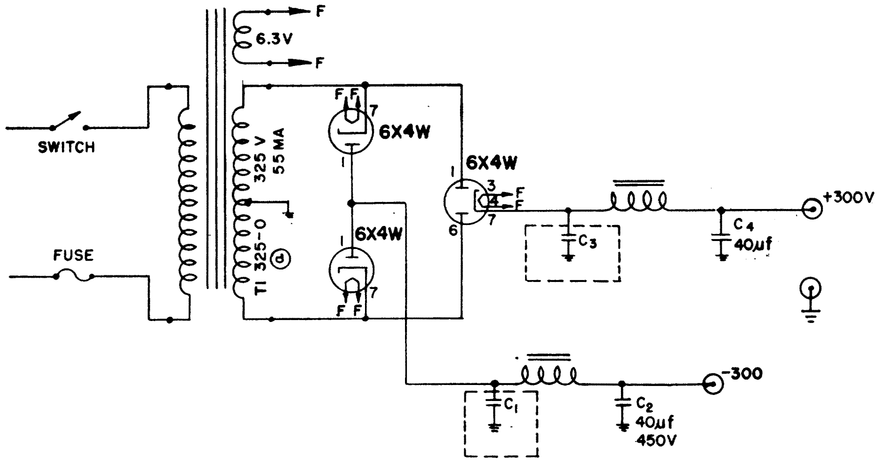

Fig. 1—A balanced dual power supply. (Note: C_1C_3, small value capacitors, used if necessary to adjust the level of $B\pm$ voltages.)

Fig. 1 displays a schematic diagram of a balanced, unregulated, dual power supply that is inexpensive to construct, but one that will hold both ± voltages to an equal amplitude without the use of regulator tubes.

The power supply is of the bridge type except for the grounded center tap of the power transformer secondary which provides two distinct voltages of equal amplitude and opposite polarity. These two voltages are easily adjusted to equal amplitude by adjustment of the value of the capacitors, C_1 and C_3 (Fig. 1).

Due to the type of circuitry used, a transformer with a full wave rectifier rating of 325-0-325 volts at 55 milliamperes of current can be used to provide +300 V at (55 ma) (.707) = 38.9 ma and −300 V at (55 ma) (.707) = 38.9 ma without overrating the transformer.

Hence, the rated current should be specified in rms current within the winding. In practice, however, the current rating of most $B\pm$ supply transformers is given in dc output current to make it easier for the design engineer. This current rating assumes a full wave rectifier with a condenser-input filter. If the type of power supply circuit used is any other type, the current rating of the transformer must be modified accordingly.

In the balanced-bridge type power supply used in this application, the current through the transformer winding is sinusoidal in the absence of filters. When working into a LC filter, the sine wave will become somewhat distorted. In this application, a grounded center tap was used rather than the conventional bridge-type circuit, thus giving two balanced supplies of 300 volts each rather than one supply of 600 volts as would be obtained from a conventional bridge circuit with the transformer secondary center tap ungrounded.

Fig. 2 displays a graphed result of tests made of the balanced, unregulated, dual power supply, used in conjunction with a dc amplifier, offering an essentially balanced load, both at zero volts output and maximum signal output. The only necessary condition was to assure that the two power supply voltages would remain balanced although changes in the ac line voltage occurred. In the test, two dc amplifiers were connected in cascade, and sufficient additional resistance to make up a total load of plus 40 ma and minus 40 ma of current were used. An input signal was fed into the first cascaded amplifier; the output of the second amplifier was then used to buck out the input signal, the difference then being the error voltage of the amplifier's output.

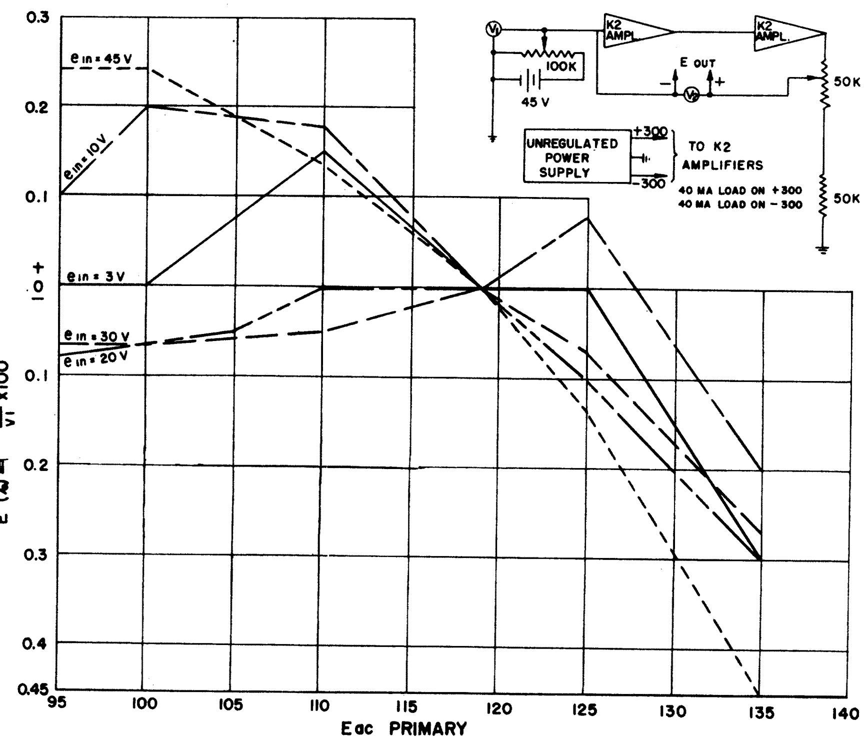

Fig. 2—Varying of ac input voltage into unregulated power supply vs output voltage of Philbrick K^2 amplifier.

A graph (Fig. 2) discloses the percentage of output voltage error of the amplifiers when using a known input voltage to the amplifiers and various transformer input voltages.

The dc amplifiers were zeroed with the ac line voltage set at 118 volts. Voltages ranging from 95 volts to 135 volts ac were fed into the primary of the power transformer. The largest error voltage graphed proved to be less than ½ of 1 per cent, with the ac primary input voltage to the power transformer set at 135 volts and with the dc input voltage to the amplifier set at 45 volts. All other error readings graphed fell within 3/10 of 1 per cent.

The physical size of the dual power supply may be kept very small as compared to regulated power supplies serving the same purpose. Only three tubes and sockets, one power transformer, two chokes, and two filter condensers are required.[1] Identical chokes and filter components are used in both positive and negative legs of the supply to maintain like conditions in both sections.

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* Received by the IRE, March 7, 1956.

[1] Filter condensers C_{1-3} (Fig. 1) are small value capacitors and used only if necessary to adjust $B\pm$ voltage requirements.

Systemic Learning*

W. P. Tanner, Jr.[1] has recently advanced a suggestion which promises to add considerably to our understanding of that basic intelligent process which we call learning. The idea as advanced is embryonic in development, but it can be seen to embody concepts of system design such as those much used by communication engineers, together with dependence on mathematical probability theory and its outgrowth—statistical decision theory. If we accept certain philosophical limitations due to dependence on probability theory as being relatively nonrestrictive, then we may use the first characteristic to suggest the name systemic learning in order to differentiate between Tanner's suggestion and most preconceived notions of learning.

Briefly, Tanner's suggestion begins with the system indicated in Fig. 1. It consists of an input, an output, and a relation between the two. We usually say that learning reflects a change in the *relation*. Tanner continues with an adjunct as indicated in Fig. 2, where the adjunct may be regarded as a (possibly optimum) rule for changing the relation as based on experience (estimates of probabilities) when the system of Fig. 1 operates in an environment.

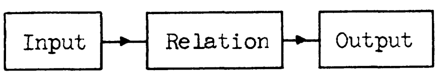

Fig. 1—A general system.

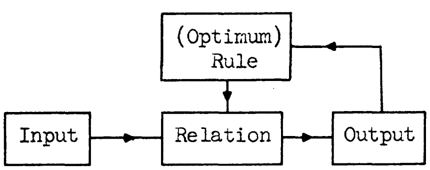

Fig. 2—A self-modifying system.

If now the adjunct is included with the original system to form a new system, we see that the new system is essentially fixed in character, since it is provided, by its design, with the only (fixed) rule which it needs to adapt itself to environmental influences in a manner which may well be optimum. Being fixed, the new system cannot change in over-all design by any experience; hence it need not be regarded as exhibiting learning. The actual "behavior" of the original system we can call systemic learning; and then we say, with Tanner, that there is no such thing as systemic learning, meaning thereby that what we call systemic learning is not unique but depends on our choice of system, and that with proper choice of system there is no systemic learning. If learning as commonly understood should be equivalent to systemic learning as here discussed, then we are free to say that there is no such thing as learning.

It seems to the writer than Tanner was led to the above point of view from his experience with human observers in strictly defined detection experiments modeled after statistical decision theory (hence mathematical probability theory) and in which he found his observers performing in a manner not far from optimum, however defined within the framework of the theory.

Being familiar with this previous work the writer has added his own engineering background and philosophical interests to frame the above discussion for the benefit of others with engineering background. It does not appear proper to go into questions of philosophical and religious implication of the above ideas, but if the writer's feeling is borne out, there will be many of them. They depend in part on the first mentioned philosophical implications of mathematical probability theory.

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* Received by the IRE, May 7, 1956.
[1] In private communication.

On the Use of a Special Word for the Quantity "Angular Velocity"*

In electric-circuit analysis, the use of the quantity "angular velocity," ω, in place of the related quantity "frequency," f, has great mathematical utility and simplicity. However, there is no simple designation for the term when speaking of it, leading to corruptions such as "angular frequency," "radian frequency," or even the erroneous term "frequency" alone. The repeated occurrence of the quantity "angular velocity" in written or spoken technical language makes the two-word designation unwieldy. It is accordingly suggested that a word be coined to designate this characteristic quantity of a circuit or of a sinusoidal time function. The words "pulsatance" and "pulsation" have been so used, but carry little general acceptance, are ambiguous, and may be confused with pulses or pulsation in the ordinary sense, to which they are related only as frequency is related. The lack of acceptance of such terms implies difficulty, so it is proposed that another term be selected. The main purpose of selecting a new term is to obtain one which is not ambiguous.

It is the purpose of this article to invite discussion and suggestions for possible words.

Three possible words are here suggested. The first two end in "cy" to show the close relation to "frequency," and both have stems that imply rotation or angular velocity. They are "rotency" and "angulancy." The third is derived from the reasonably meaningful corruption, "radian frequency": "frecrad." (The simpler spelling is preferred to "frequerad".) A 60-cycle (per second) signal would have a "frecrad" of 377 radians per second, for example (or an angulancy or rotency of 377 radians per second).

Let us further proceed to the generalized complex-quantity angular velocity that is becoming so common in complex-plane plots of poles and zeros of networks. Let us consider the complex S plane where $S=\sigma+j\omega$. In the ω-direction, the units are radians per second; in the σ-direction, the units are nepers per second; in either case the natural time function is a complex exponential $f(t)=K\epsilon^{st}$, regardless of the relative magnitudes of the "real" and "imaginary" components. Here are two distinct units—radians per second and nepers per second—that are intimately related. It is proposed that a common term—the "nerad"—be supplied for the general case. A complex network will thus have for one of its natural "frecrads" a value of so many "nerads." Since the term is coined, the "per-second" is preferably specifically included in its meaning: one "nerad" is equivalent to one *radian per second*, or one *neper per second*, or any combination of the two having unit magnitude. If the behavior is wholly undamped, the "frecrad" should be expressed in radians per second; if completely nonoscillatory, the "frecrad" should be expressed in nepers per second; if the behavior is a damped oscillation, the "frecrad" is complex, and expressed in nerads.

What do you think?

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Frequency Doubling and Mixing in Ferrites*

Ayres, Vartanian, and Melchor[1] have shown recently that the equation of motion for the magnetization vector in a ferrite predicts a frequency doubling, and they have experimentally observed this doubling. They showed that with the usual arrangement, in which the biasing field H_{dc} is applied along the z axis, the rf z magnetization m_z varies at twice the applied frequency; this can be used to induce a double-frequency voltage in a loop with the proper experimental arrangement. It was also pointed out by Ayres, Vartanian, and Melchor that frequency mixing would occur if two frequencies were applied to the ferrite; however, this mixing was not discussed. We have carried out a simple analysis of this effect in terms of applied magnetic fields and demagnetizing factors.[2] The results are valid for sample shapes and sizes for which the concept of demagnetizing factors is valid. Damping is neglected throughout the analysis. It is the purpose here to summarize some of the more interesting results. The equations of motion for the magnetizing vector are

$$\dot{m}_x = \gamma(m_y H_z - M_z h_y), \tag{1}$$

$$\dot{m}_y = \gamma(M_z h_x - m_x H_z), \tag{2}$$

$$\dot{m}_z = \gamma(m_x h_y - m_y h_x). \tag{3}$$

* Received by the IRE, May 14, 1956. This work was carried out under Contract AF19(604)-1084, while the author was an RCA Fellow in Electronics.
[1] W. P. Ayres, P. H. Vartanian, and J. L. Melchor, "Frequency doubling in ferrites," *J. Appl. Phys.*, vol. 27, p. 188; February, 1956.
[2] J. E. Pippin, "Frequency Doubling and Mixing in Ferrites," Sci. Rep. No. 2, AFCRC-TN-56-369, Gordon McKay Lab., Harvard Univ., Cambridge, Mass.; May 5, 1956.

As usual, we let small letters denote time varying quantities and capital letters denote dc quantities. In the first two equations it is assumed that $m_z \ll M_z$ and $h_z \ll H_z$. These equations, which are in terms of internal fields, can be written in terms of applied fields and demagnetizing factors in the usual way. If the applied field consists of two sinusoids with frequencies ω_1 and ω_2, then the first two equations can be solved under the above assumptions of small m_z and h_z to give

$$\boldsymbol{m} = \boldsymbol{\chi}_1 \cdot \boldsymbol{h}_{a_1} + \boldsymbol{\chi}_2 \cdot \boldsymbol{h}_{a_2}, \tag{4}$$

where here the vectors $\boldsymbol{m}$ and $\boldsymbol{h}_a$ refer only to the x and y fields; the subscript a means *applied*. The components χ_{xx}, χ_{xy}, and χ_{yy} of the tensor $\boldsymbol{\chi}$ are just the Polder susceptibility tensor components written in terms of applied fields. (This is the "external susceptibility" tensor.) The subscripts 1 and 2 refer to frequencies ω_1 and ω_2. There will be one such tensor for each applied frequency, the only variation being the value of ω appearing in the components of the tensor. If (4) is substituted in (3), after (3) has been rewritten in terms of applied fields, and the very considerable algebra carried through, it is found that $\dot{m}_z$ contains terms of frequency $2\omega_1$, $2\omega_2$, $\omega_1+\omega_2$, and $\omega_1-\omega_2$. These terms are given in detail elsewhere;[2] they are not written here because of their complexity. However, the special case of a *cylindrically symmetric* ferrite sample will be considered here. Specifically, if

$$h_{x_a} = h_{x_1} \cos \omega_1 t + h_{x_2} \cos (\omega_2 t + \phi)$$

and

$$h_{y_a} = h_{y_1} \cos (\omega_1 t - \alpha) + h_{y_2} \cos (\omega_2 t + \phi - \delta),$$

then for a cylindrically symmetric sample the double frequency terms D in $\dot{m}_z$ are given by

$$D = \frac{\gamma}{2} \chi_{xy_1}[h_{x_1}^2 \sin 2\omega_1 t + h_{y_1}^2 \sin (2\omega_1 t - 2\alpha)] + \frac{\gamma}{2} \chi_{xy_2}[h_{x_2}^2 \sin (2\omega_2 t + 2\phi) + h_{y_2}^2 \sin (2\omega_2 t + 2\phi - 2\delta)].$$

The sum and difference frequency terms s and d in $\dot{m}_z$ are given by

$$\left.\begin{matrix} s \\ d \end{matrix}\right\} = \frac{\gamma}{2} (\chi_{xy_1} \pm \chi_{xy_2}) \{ h_{x_1}h_{x_2} \sin [(\omega_1 \pm \omega_2)t \pm \phi] + h_{y_1}h_{y_2} \sin [(\omega_1 \pm \omega_2)t \pm \phi - \alpha \mp \delta] \} + \frac{\gamma}{2} (\chi_{xx_1} - \chi_{xx_2}) \cdot \{ h_{x_1}h_{y_2} \cos [(\omega_1 \pm \omega_2)t \pm \phi \mp \delta] - h_{x_2}h_{y_1} \cos [(\omega_1 \pm \omega_2)t \pm \phi - \alpha] \},$$

where the upper of the double signs applies for s and the lower for d. Upon inspection of these equations the following conclusions are evident.

1) If for a particular frequency ω_1 (or ω_2), the excitation is circularly polarized in either sense ($h_{x_1} = h_{y_1}$; $\alpha = \pm\pi/2$), *there is no doubling.* The amplitude of the double frequency term is zero and the magnetization vector precesses in a circle. Furthermore, the amplitude of the double frequency term is a maximum when the excitation is linearly polarized, for a given amplitude of excitation.
2) If the two excitations are circularly polarized in the same sense ($\alpha = \delta = \pm\pi/2$), then the amplitude of the sum term is zero, and the amplitude of the difference term is, in general, not zero.
3) If the two excitations are circularly polarized in opposite senses ($\alpha = -\delta = \pm\pi/2$), then the amplitude of the difference term is zero, and the amplitude of the sum term is, in general, not zero.
4) If $\dot{m}_z$ is used to induce a voltage in some manner, then statements 1), 2), and 3) show that it is possible to obtain *single sideband* mixing by applying two circularly polarized waves to the ferrite.

The writer wishes to acknowledge the valuable advice of Prof. C. L. Hogan.

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The Optimum Tapered Transmission Line Matching Section*

A recent paper by R. E. Collin[1] treats the problem of the design of an impedance matching transmission line taper of optimum characteristic impedance contour, *i.e.*, optimum in the sense that for a given maximum reflection coefficient tolerance in the pass band the taper has minimum length, or, conversely, for a given length the taper has minimum reflection coefficient tolerance in the pass band. Since the problem treated by Collin is identical with the one I treated in a paper which appeared several months earlier,[2] some comment on the relationship between the two papers seems to be called for.

Since the two papers treat the identical problem, one would think that they should arrive at identical results even though possibly by different routes. That this is, in fact, true is nowhere indicated in the body of Collin's paper, and, instead, it is implied throughout that one must settle for a rather involved compromise design in which the sidelobes of reflection coefficient ultimately decay inversely with frequency. This results from the unnecessary assumption that the characteristic impedance is a continuous function in the closed interval including the end points of the taper. When this assumption is removed, a simple design formula for the exact optimum taper can be obtained as indicated in my paper.[2] Although Collin indicates at the very end of his paper in an appendix that the exact optimum taper can be obtained (which violates his assumption of continuous Z_0) by considering a limiting form of his solution, an erroneous impression has already been given, and unless the reader perseveres to the very end of the paper he will never find out that a much simpler design than that indicated is available and applicable.

The above seems to indicate to me the danger of carrying over too closely the work from one field (in this case, antenna aperture theory) to another field in which the mathematics is analogous. While the mathematics may be exactly analogous, there may be important characteristics of the systems which differ, and, hence, strongly affect the nature of the results. The assumption of a discontinuous change of characteristic impedance in the transmission line taper is completely consistent with the physics of the problem. The corresponding assumption in antenna aperture theory, however, implies the presence of an impulse function in the excitation function, and, hence, implies infinite radiated power—an obviously nonphysical condition. This is exactly the reason that Taylor[3] found it necessary to go to a rather deep analysis in treating the problem of optimum distributions on continuously excited antenna apertures.

While Collin's paper furnishes a very instructive example of mathematical analysis, it could have been considerably simplified by removal at the outset of the unnecessary assumption of continuous Z_0. The results so obtained (as indicated in his appendix) are then equivalent to the results of my earlier paper. The Fourier series representation for the ideal characteristic impedance contour which was ultimately obtained by Collin is quite interesting and converges fairly rapidly (requires twelve terms for three figure accuracy in at least one practical case). There is a lot to be said computationally for the definite integral representation of my paper, however. Not the least of these is that it has been computed and tabulated,[2] thus rendering the design of an optimum transmission line taper particularly direct and convenient.

Finally, I would like to thank Dr. Collin for pointing out the omission in (12) of my paper. The equation will read correctly with the addition indicated by Dr. Collin except at the end point $x = -l/2$, or, more precisely, it may be corrected to read:

$$\begin{aligned} \ln (Z_0) &= \frac{1}{2} \ln (Z_1 Z_2) + \frac{\rho_0}{\cosh (A)} \cdot \left\{ A^2 \phi(2x/l, A) + U\left(x - \frac{l}{2}\right) - U\left(-x - \frac{l}{2}\right) \right\}, & |x| \leq l/2, \\ &= \ln (Z_2), & x > l/2, \\ &= \ln (Z_1), & x < -l/2. \end{aligned} \tag{12}$$

This omission had also been called to my attention in a private communication by G. J. Wheeler of the Raytheon Wayland Laboratory. He also noted that on the last page of the paper one should read $l/\lambda = 0.587$ rather than $\beta l = 0.587$. While I must apologize for my faulty proof reading, I would like to point out that these in no way affect the results of the paper nor do they affect

* Received by the IRE, April 27, 1956.
[1] R. E. Collin, "The optimum tapered transmission line matching section," PROC. IRE, vol. 44, pp. 539–548; April, 1956.
[2] R. W. Klopfenstein, "A transmission line taper of improved design," PROC. IRE, vol. 44, pp. 31–35; January, 1956.
[3] T. T. Taylor, "Design of line-source antennas for narrow beamwidth and low sidelobes," TRANS. IRE vol. AP-3, pp. 16–28; January, 1955.

the tabulated values of $\phi(z, A)$ which were given.

I hope that these remarks may have clarified the situation in regard to the design of optimum impedance matching transmission line tapers.

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In connection with R. E. Collin's paper,[1] I should like to make the following comments.[4]

In his paper, Collin refers to my letter to the Editor of PROCEEDINGS[5] in which I showed how a nonlinear Riccati equation for the reflection coefficient ρ could be linearized by neglecting the squared term ρ^2 that is assumed to be small compared to unity. The solution of the resultant linear differential equation was interpreted as being the Fourier transform of $P(x) = (1/2) \cdot (d\ln Z_0(x)/dx)$, where $P(x)$ is the reflection from a length dx of the tapered line at a distance x, and $Z_0(x)$ is the characteristic impedance varying along the line. Collin refers to my letter in connection with tapered transmission lines that have triangular $P(x)$ distributions. He calls these tapers Gaussian. However, in a more extensive research work,[6] indicated in the last part of the letter,[5] I have already introduced the designation Gaussian for tapered lines that have Gaussian $P(x)$ distributions. A set of graphs showing $Z_0(x)$, $P(x)$, and $\rho(l/\lambda)$ (l=length of the tapered line, λ=wavelength) for different Gaussian tapers is shown in reference 6. The latter designation seems to me to be the more consistent one because the Gaussian distribution is an important self-reciprocal Fourier transform that is well known from probability theory. This notation is also analogous to notations used in other engineering fields, for example, in antenna theory.[7] In engineering we usually deal with incomplete Gaussian distributions. The Fourier transforms of these distributions have been studied by Millington[8] by means of the saddlepoint method.

A quick literature survey reveals that the extensive Fourier transform work mentioned above, in addition to being referred to in numerous papers and books, is reviewed in:

1) *Science Abstracts*, Section B, Electrical Engineering, Vol. 54, p. 458, 1951; (Abstract 3439).
2) *Wireless Engineer*, Vol. 28, p. A226, December, 1951; (Abstract 2909).
3) PROCEEDINGS OF THE IRE, Vol. 40, p. 116, January, 1952; (Abstract 2909).
4) *Mathematical Review*, Vol. 13, p. 803, 1952.
5) *Annales des Télécommunications*, Vol. 8, p. A6, 1953; (Abstract 51173).

It appears that these reviews have not been brought to the attention of Dr. Collin.

Lately a brief summary of this work has been given in *Proceedings*.[9]

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[4] Received by the IRE, May 2, 1956. This work was supported in part by the Army (Signal Corps), the Air Force (Office of Sci. Res., Air Res. and Dev. Com.), and the Navy (Office of Naval Res.).

[5] E. F. Bolinder, "Fourier transforms in the theory of inhomogeneous transmission lines," PROC. IRE, vol. 38, p. 1354; November, 1950.

[6] E. F. Bolinder, "Fourier transforms in the theory of inhomogeneous transmission lines," *Trans. Roy. Inst. Tech.*, Stockholm, Sweden, No. 48, pp. 84; 1951.

[7] J. F. Ramsay, "Fourier transforms in aerial theory," *Marconi Rev.*, Part III, vol. 10, pp. 41–58; April/June, 1947.

[8] G. Millington, "The Fourier transform of the incomplete Gaussian function," *Marconi Rev.*, vol. 11, pp 17–30; January/March, 1948.

[9] E. F. Bolinder, "Fourier transforms and tapered transmission lines," PROC. IRE, vol. 44, p. 557; April, 1956.

Rebuttal[10]

From a theoretical point of view the assumption of a continuous taper is not necessary and, as pointed out by Dr. Klopfenstein, does result in a simpler analysis. The approach to the problem as given in my paper does, however, give additional useful information such as:[1]

1) An estimate of the range of variation in the load impedance and pass band tolerance for which the theory can be expected to give an accurate result.
2) Design information for a taper which departs only slightly from the ideal taper and does not have a discontinuity at each end. A discontinuity in characteristic impedance may be undesirable in certain cases as for example when it leads to a sharp discontinuity in the physical structure of the line with its consequent limitation on the power handling capability of the line.

The tabulated data given in the paper by Klopfenstein is of considerable value.[2] The results of my analysis could of course be tabulated in a similar form. The difference in the two taper designs (continuous taper and taper with a discontinuity at each end) is negligible in practice (the continuous taper is 5–10 per cent longer) considering that either design is only approximate since it neglects the square of the reflection coefficient as compared to unity in the differential equation governing the system.

The terminology introduced by Mr. Bolinder has its own merits but does not appear to be in accordance with the usual terminology used in regard to tapered transmission lines. Southworth mentions the use of the word "Gaussian" to describe a taper with a characteristic impedance varying as an "incomplete" Gaussian function.[11] Other tapered transmission lines have been named after the type of function that describes the characteristic impedance along the taper, *e.g.*, exponential,[12] hyperbolic,[13] parabolic,[14] etc. If the terminology introduced by Mr. Bolinder is used, then, in order to be consistent, one should refer to the exponential taper for which

$$\frac{d}{dx} \ln Z_0(x) = \text{constant},$$

as a constant taper. At the same time one would be somewhat at a loss for a suitable name for the linear taper for which

$$\frac{d}{dx} \ln Z_0(x) = \frac{A}{Ax + B}$$

(A and B suitable constants).

I regret to say that I have not yet read the "extensive Fourier transform work" of Mr. Bolinder, although I have been aware of its existence for some time.[6] In fact I wrote a letter requesting some information regarding this work to the author addressed to the Royal Institute of Technology in Stockholm before submitting the manuscript of my paper for publication but, for some unknown cause, did not receive a reply.

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[10] Received by the IRE, May 14, 1956.

[11] G. C. Southworth, "Principles and Applications of Waveguide Transmission," D. Van Nostrand Co., Inc., New York, N.Y., sec. 9.1; 1950.

[12] H. A. Wheeler, "Transmission lines with exponential taper," PROC. IRE, vol. 27, pp. 65–71; January, 1939.

[13] H. J. Scott, "The hyperbolic transmission line as a matching section," PROC. IRE, vol. 41, pp. 1654–1657; November, 1953.

[14] R. F. H. Yang, "Parabolic transmission line," PROC. IRE, vol. 43, p. 1010; August, 1955.

Marconi's Last Paper, "On the Propagation of Microwaves over Considerable Distances"*

Recently Professor R. M. Fano translated from Italian into English Marconi's last paper (1933), only thirteen sentences long. I have never seen any reference to this paper in any English language periodical, not even in the moving tribute to Marconi by fellow inventor E. H. Armstrong.[1] Fano's complete translation of the text, which follows this letter, will interest readers of PROCEEDINGS, in view of the current revival of interest in this subject.

Considered as a sequel to Marconi's longer paper[2] (also rarely mentioned or read now, alas), this remarkable paper indicates almost certainly that Marconi discovered experimentally the existence of useful microwave radio propagation into the twilight

* Received by the IRE, May 4, 1956.

[1] E. H. Armstrong, "The spirit of discovery: an appreciation of the work of Marconi," *Elect. Engrg.*, vol. 72, pp. 670–676; August, 1953.

[2] G. Marconi, "Radio communication by means of very short electric waves," *Proc. Roy. Inst. G. Brit.*, vol. 27, pp. 509–544; 1933. This paper is expected to be republished in an early issue of the IRE TRANS. ON ANTENNAS AND PROPAGATION.

region beyond the horizon which could not be explained theoretically by diffraction and refraction. The increase in range from 52 to 150 km, from 1.7 to 5 times geometrical optical, between the experiments of 1932 and 1933, with only transmitter power increase from about 10 to 25 watts, and improvements in receiving equipment and reflectors, seems unmistakably to indicate he was experimenting in the twilight region where the field strength attenuation rate is now known to be roughly 0.1 db/km, instead of the value about 10 times higher calculated from diffraction theory with a refraction correction. The skepticism of the experts concerning this last great propagation discovery by Marconi clearly parallels the similar history of his 1901 discovery of transatlantic wireless and his short wave revolution of the early nineteen twenties, except that this time his ill health and death in 1937 caused the skeptical radio world to wait a generation before rediscovering the truth and importance of the phenomenon. Although Hershberger's experiments[3] and those of Trevor and George[4] soon afterwards in the same frequency band apparently agreed with Marconi's observations well beyond the horizon in 1932 and 1933, radio science generally came to rely instead on calculations unchecked by experiment and to delay to the last few years the exploitation of Marconi's vision, heedless of the warning so well phrased in his 1932 paper: "Long experience has, however, taught me not always to believe in the limitations indicated by purely theoretical considerations, or even by calculations. These—as we well know—are often based on insufficient knowledge of all the relevant factors. I believe, in spite of adverse forecasts, in trying new lines of research, however unpromising they may seem at first sight."

While the current controversy over the explanation of twilight region propagation still rages, the importance of checking theory by experiment and vice versa is a lesson retaught by this bit of radio history. Considering the enormous amount of effort which has gone into research and production of microwave systems since 1933, certainly we should ask ourselves how these last two papers on microwaves by so successful a discoverer as Marconi could go practically unread, even granting that one was published in a foreign language and the other in an inaccessible journal. Renewed scholarly attention to this phase of Marconi's career seems unquestionably to be called for, with belated homage to the last neglected discovery of a great radio pioneer, which seems to me to illustrate, in Debye's memorable phrase[5] that: "... our science is essentially an art which could not live without the occasional flash of genius in the mind of some sensitive man, who, alive to the smallest of indications, knows the truth before he has the proof." The practical consequences of this last propagation discovery of Marconi's could well rival in importance his discoveries of transatlantic wireless and the short wave revolution.

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[3] W. D. Hershberger, "Seventy-five centimeter radio communication tests," PROC. IRE, vol. 22, pp. 870–877; July, 1934.
[4] B. Trevor and R. W. George, "Notes on propagation at a wavelength of seventy-three centimeters," PROC. IRE, vol. 23, pp. 461–469; May, 1935.
[5] "The Collected Papers of Peter J. W. Debye," Interscience Publishers, Inc., New York, N. Y., p. xxi, 1954.

*On the Propagation of Microwaves over Considerable Distances**

(Professor R. M. Fano of the Department of Electrical Engineering and Research Laboratory of Electronics, M.I.T., has made the following translation into English of the Italian original "Sulla Propagazione Di Micro-Onde A Notevole Distanza," as it appeared in the volume, "Scritti di Guglielmo Marconi," "Collected Papers of Guglielmo Marconi," published by the Royal Academy of Italy, Rome, 1941, pp. 447–449.)

Electromagnetic waves of wavelengths smaller than one meter are commonly known as microwaves; they are also called quasi-optical waves because it was generally believed that with them radio-telegraphic communication would have been possible only when the transmitting equipment and the receiving equipment were within line-of-sight: their practical utility would then have been limited by such restriction.

In the course of experiments carried out in the months of July and August of last year, I was able to discover that the useful range of these waves was not at all limited to the optical distance—depending, in the main, on the height of the equipment—but, that these waves could be received and detected beyond the horizon up to a distance approximately twice the optical one and also between stations shielded from each other by hills.[1]

Between the second and the sixth of this month I was able to carry out further radio-telegraphic and radio-telephonic transmission tests by means of microwaves of approximately 60 centimeter wavelength (500 megacycles) from a transmitter located at Santa Margherita Ligure and a receiver installed on the yacht Elettra which was sailing along the coast of the Tirreno.

The transmitting dipole which was radiating approximately 25 watts was located on the hotel Miramare at Santa Margherita at a height of 38 meters above sea level and was placed near the focus of a parabolic reflector having an aperture of 2 meters.

The receiving dipole was in a similar reflector located on the yacht Elettra at a height of 5 meters above sea level.

In spite of the fact that the optical distance was only 30 kilometers, the radio-telephonic and radio-telegraphic signals sent by the transmitting station were received on the yacht with clarity, great strength, and regularity at a distance of 150 kilometers, that is, at five times the optical distance; during the last year's tests, on the other hand, although the height above sea level of the transmitter at Santa Margherita was greater (50 meters), the maximum distance at which Morse signals were detected feebly was 52 kilometers.

It was not possible during these tests to make continuous observations beyond the above mentioned distance of 150 kilometers because the navigation requirements specified by the configuration of the coast did not permit maintaining the reflector of the Elettra pointing at all times toward the transmitting station. The Morse signals were nevertheless detected very feebly and with slight fading, but often legible, up to the mooring at St. Stefano Harbor, at a distance of 258 kilometers from Santa Margherita—that is, at almost nine times the optical distance—although, in this case, on the straight path joining the two stations there was land broken by high hills for almost 17 kilometers: the promontory of Piombino for 11.482 kilometers and the Punta Troja for 5.566 kilometers.

The greater range obtained in these experiments seemed due to the improved efficiency of the transmitting and receiving equipment and of the reflectors employed.

In these experiments, as in those of last year, I was effectively assisted by Mr. G. A. Matthieu who has personally taken care of the construction and of the initial tests of the new equipment and also by technicians of the Marconi Company.

The theoretical explanation of the results obtained when the wavelength employed is taken into account presents, in my view, serious difficulties even when using the calculations involving diffraction and refraction indicated by Pession in his paper "Considerations on the Propagation of Ultra-Short Waves and of Mircowaves."[2]

The speculations that may arise from such results concern the entire theory of radio transmission over distances greater than the optical one.

After further, more complete, and more extended experiments, I am planning to publish a detailed paper on the methods employed and on the results obtained and I express the hope that in addition to theoretical speculations that might be of scientific interest, the present results will lead to new and substantial progress in the field of radio communications.

* Roy. Acad. of Italy, "Proceedings of the section on physical, mathematical and natural sciences," vol. IV, 1933; paper no. 16, presented at the special meeting of August 14, 1933.
[1] Guglielmo Marconi, paper presented on December 2 1932 at the Roy. Inst. of Gr. Brit., London.
[2] G. Pession, "Alta frequenza," vol. I, no. 4; December, 1932.

The Statistics of Combiner Diversity*

Several papers[1–3] have recently appeared pointing out the advantages of combiner diversity over switch diversity both from a theoretical and practical point of view. Mack,[3] using numerical techniques, presented curves of the statistical distribution of the combined signal for dual and triple

* Received by the IRE, May 23, 1956.
[1] L. R. Kahn, "Ratio squarer," PROC. IRE, vol. 42, p. 1704, November, 1954.
[2] D. G. Brennan, "On the maximum signal-to-noise realizable from several noisy signals," PROC. IRE, vol. 43, p. 1530; October, 1955.
[3] C. L. Mack, "Diversity reception in uhf long-range communications," PROC. IRE, vol. 43, pp. 1281–1289; October, 1955.

diversity. It is the purpose of this letter to point out that an analytic evaluation of the combined statistical distribution is available in terms of a well tabulated function and to present the statistics of the combined signal up to ten-fold diversity. This author has already participated in a systems engineering study utilizing quadruple diversity and feels that even higher order diversity might be used for future longer-range tropospheric scatter circuits.

We begin with Brennan's[2] generalized formula that the optimum combination of many signals provides

$$P = \sum_{i=1}^{n} p_i \qquad (1)$$

where P=combined output S/N power ratio, p_i=output S/N power ratio of receiver i, and n=order of diversity. It is usually assumed that the individual S/N ratios are Rayleigh distributed whose statistical distribution is of the form

$$\frac{R}{\psi_0} e^{-R^2/2\psi_0} dR. \qquad (2)$$

Actually, it is the voltage S/N ratio which is Rayleigh distributed while the power S/N ratio is exponentially distributed. Since p is proportional to R^2, its statistical distribution is of the form

$$\frac{1}{p_0} e^{-p/p_0} dp. \qquad (3)$$

If each of the individual p_i have the statistical distribution given by (3), then $P=\sum_{i=1}^{n} p_i$ has the distribution (assuming that the p_i are independent)

$$\frac{1}{(n-1)!}\left(\frac{P}{p_0}\right)^{n-1} e^{-P/p_0} \frac{dP}{p_0}. \qquad (4)$$

This is well-known and is discussed in almost any mathematical statistics book under the topic Chi-Square distribution. To the radar engineer, the same distribution is known as the output of a square law detector and integrator. It should be quite clear that p_0 is simply related to the median of the distribution of (3). Since it is convenient to present our data relative to the median of (3), it is only necessary to evaluate the cumulative probability distribution of $y = p/p_0$. The probability that y will exceed any particular value, y_0, is given by

$$\frac{1}{(n-1)!}\int_{y_0}^{\infty} y^{n-1}e^{-y}dy = \left[1 + y_0 + \frac{1}{2!}y_0^2 + \frac{1}{3!}y_0^3 + \cdots + \frac{1}{(n-1)!}y_0^{n-1}\right]e^{-y}. \qquad (5)$$

It is also well-tabulated as the incomplete gamma function.[4] Eq. (5) for n=1, 2, 3, 4, 6, 8, 10 is shown in Fig. 1. When compared with Mack's estimates for dual and triple combiner diversity, differences of about one or two db between our estimate and his appears. This presumably is due to the difference between our exact integration in closed form and his numerical integration.

[4] "Tables of the Incomplete Gamma Function" edited by K. Pearson, Cambridge Univ. Press, Cambridge, Eng.; 1946.

In designing a scatter circuit (either tropospheric or ionospheric) one must be able to override the expected fading range to some predetermined degree of reliability (*e.g.*, 99 or 99.9 per cent). In another paper,[5] curves are presented for system gain for various orders of diversity and different degrees of reliability. Unfortunately, the curves there assumed that only independent Rayleigh fading occurred at all receivers. As is quite well known this is not true. There are, to a fair degree of approximation, two types of fading. One is referred to as slow fading which describes the variability of hourly medians throughout the whole year and which is statistically described by a db Gaussian distribution. An estimate of this variability on a tropospheric scatter link can be obtained from Fig. 3 of a Lincoln Laboratory paper.[6] The second type of fading is referred to as fast fading and its statistical distribution is described in this letter for various orders of diversity. Diversity action has two effects. It reduces the median scatter loss by the increase of the median signal and it reduces the range of the fast fading only. Therefore, to estimate the required power for a scatter link requiring 99 per cent reliability and utilizing quadruple diversity, we may proceed as follows. We decrease the yearly median scatter loss estimate obtained from the Mellen paper[6] by 7.2 db (as shown in Fig. 1). The total fading range

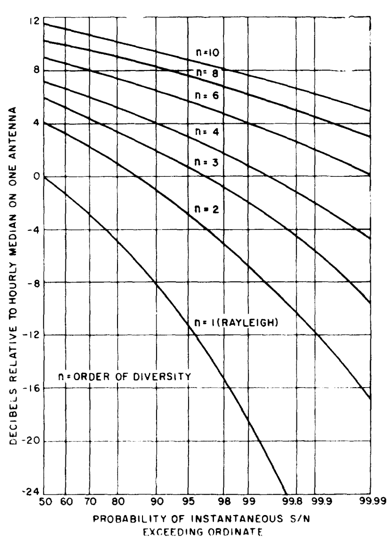

Fig. 1.

to be expected can be taken as the root sum square of the fast fades (6.8 db in this example) and the slow fades as shown by Mellen (this is a function of distance). The procedure just described would be exact if both the fast fades and the slow fades were db Gaussian. For diversity systems even the fast fades seem to become approximately db Gaussian which would be a straight line on the accompanying figure.

It should be pointed out, however, that for an fm system, the fast fade statistics are really a cross between the combiner statistics just discussed and the statistics appropriate to switch diversity which is already well understood by systems engineers. The reason for this lies in the fm threshold effect in that combiner diversity does not operate effectively on signals that are below the fm threshold. Therefore, the statistics of combiner diversity of an fm system are a function of median signal level. A complete analysis of the statistics appropriate to an fm system is a very interesting mathematical statistics problem; however, from a practical point of view, such a complex analysis is not justified at the present state of the art.

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[5] F. J. Altman and W. Sichak, "A simplified diversity communication system for beyond-the-horizon links," IRE TRANS., vol. CS-4, pp. 50–56; March, 1956.

[6] G. L. Mellen, W. E. Morrow, Jr., A. J. Poté, W. H. Radford, and J. B. Wiesner, "UHF long-range communication systems," PROC. IRE, vol. 43, pp. 1269–1281; October, 1955.

A Note Concerning the Dirac Delta Function*

In most discussions of the impulse or Dirac Delta function, the authors give several different definitions of the function which they claim to be equivalent.[1]

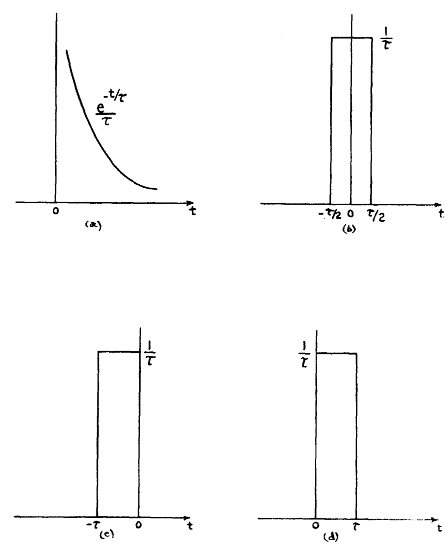

Fig. 1.

Usually the forms given are as shown in Fig. 1 and are defined as:

$$\text{(a)}\quad \delta(t) = \lim_{\tau\to 0}\frac{e^{-t/\tau}}{\tau} \qquad t > 0$$
$$= 0 \qquad t \leqq 0$$

$$\text{(b)}\quad \delta(t) = \lim_{\tau\to 0}\frac{U(t+\tau/2) - U(t-\tau/2)}{\tau}$$

$$\text{(c)}\quad \delta(t) = \lim_{\tau\to 0}\frac{U(t+\tau) - U(t)}{\tau}$$

$$\text{(d)}\quad \delta(t) = \lim_{\tau\to 0}\frac{U(t) - U(t-\tau)}{\tau}.$$

It can be shown however that the Laplace transforms of the above functions are

* Received by the IRE, May 21, 1956.

[1] See: S. Goldman, "Transformation Calculus and Electrical Transients," Prentice-Hall, Inc., New York, N. Y.; 1953.

different and hence that the definitions are not equivalent, if it is remembered that in dealing with the δ function the limiting process should be carried out after all other manipulations.[2]

Consider the definition given in (a) above. The Laplace transform is written:

$$L[\delta(t)] = \lim_{\tau\to 0} \int_0^\infty \frac{e^{-t/\tau}}{\tau} e^{-st}dt$$

$$= \lim_{\tau\to 0} \lim_{\substack{T\to\infty \\ a\to 0}} \frac{1}{\tau} \int_a^T e^{-(s+1/\tau)t}dt$$

$$= \lim_{\tau\to 0} \lim_{\substack{T\to\infty \\ a\to 0}} \frac{-1}{\tau\left(s+\dfrac{1}{\tau}\right)} e^{-(s+1/\tau)t} \Big|_a^T$$

$$= \lim_{\tau\to 0} \frac{1}{\tau s + 1}(1-0) = 1$$

which is the value normally given.

The other three cases are specific examples of the general case in which:

$$\delta(t) = \lim_{\tau\to 0} \frac{U[t+(1-\rho)\tau 1] - U(t-\rho\tau)}{\tau},$$

where $0 \leqq \rho \leqq 1$, the equalities holding for cases (c) and (d) and $\rho = 1/2$ for case (b).

The Laplace transform then becomes:

$$L[\delta(t)] = \lim_{\tau\to 0} \frac{1}{\tau} \int_0^\infty \{U[t+(1-\rho)\tau] - U(t-\rho\tau)\} e^{-st}dt$$

$$= \lim_{\tau\to 0} \frac{1}{\tau} \left\{ \int_0^\infty U[t+(1-\rho)\tau]e^{-st}dt - \int_0^\infty U(t-\rho\tau)e^{-st}dt \right\}$$

$$= \lim_{\tau\to 0} \frac{1}{\tau} \left\{ \int_0^\infty e^{-st}dt - \int_{\rho\tau}^\infty e^{-st}dt \right\}$$

since

$$U[t+(1-\rho)\tau] = 1 \text{ for } 0 \leq t$$

and

$$U(t-\rho\tau) = 0 \quad \text{for} \quad t < \rho\tau.$$

Thence,

$$L[\delta(t)] = \lim_{\tau\to 0} \frac{1}{\tau} \int_0^{\rho\tau} e^{-st}dt = \rho$$

which can take on any value from 0 to 1 inclusive.

The symmetrical definition of case (b) has a Laplace transform of $\frac{1}{2}$, a fact which has been used by Spiegal[3] to evaluate certain integrals.

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[2] H. Weyl, "The Theory of Groups and Quantum Mechanics," E. P. Dutton and Co., Inc., New York, N. Y., 1931.
T. Lewis, "Some applications of the Dirac Delta function;" *Phil. Mag. Ser. 7*, vol. 24, p. 329; September, 1937.

[3] M. R. Spiegal, "Applications of the Dirac Delta function to the evaluation of certain integrals," *J. Appl. Phys.*, vol. 25, p. 1302 [see (20)]; October, 1954.

A Sensitive Method for the Measurement of Amplitude Linearity*

Virtually all electronic devices and components exhibit variations in their characteristics which vary with frequency and/or amplitude. The square wave generator is a useful tool for the measurement of frequency response but no comparable expedient is in general use for the measurement of amplitude characteristics. The method proposed below permits the rapid determination of the amplitude response of a system or device with considerable sensitivity and extremely simple apparatus. It consists of the application of a linear sawtooth waveform to the input terminals of the device under test and a differentiator through which the output is passed. The differentiated waveform is then observed on an oscilloscope as a function of the input, output, or other variable. If the device is linear over the dynamic range encompassed by the sawtooth, the derivative, with respect to time, will be a constant proportional to the slope of the input and the gain of the device. Any departure from linearity will result in a corresponding departure of the derivative from the horizontal which can be readily detected and measured. This method has the virtue of practically separating the linear and nonlinear components since the base line can be translated to the horizontal axis by adjustment of the oscilloscope centering control. The more conventional method of harmonic analysis shares this feature of separation (by means of filters) but does not provide an instantaneous display as a function of amplitude and is inherently not as sensitive, as will be shown.

The equipment required is shown in block form in Fig. 1. The sawtooth generator

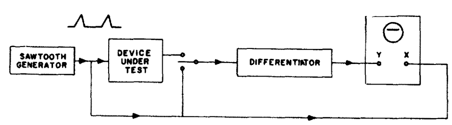

Fig. 1—Block diagram of system to measure linearity.

output consists of repetitive waveforms for convenient display. The amplitude must cover the dynamic range of interest and the linearity should be as high as possible; certainly high compared with the device to be tested. The low duty cycle which can be achieved offers an additional feature in permitting measurements to be made using peak amplitudes which might cause excessive dissipation under steady state conditions.

The differentiator may be of the electronic feedback type but for most applications a simple RC differentiator having a short time constant is satisfactory. The oscilloscope should have a dc amplifier and the sweep can be supplied by the sawtooth generator. Loading of the sawtooth generator by the device under test can be checked by examining the derivative of the input, with the load connected, and this comparison can be used as a basis for gain measurement.

* Received by the IRE, January 23, 1956; revised manuscript received, June 7, 1956.

Since the concept of "degree of nonlinearity" is not as familiar as that of harmonic generation, a quantitative comparison is given below between the results obtained with a generalized nonlinear device, using the proposed method, and the method of harmonic analysis. Consider a nonlinear device whose characteristics can be expressed by the power series

$$y = Ax + Bx^2 + Cx^3 + Dx^4 + \cdots \quad (1)$$

where

$$y = \text{output}$$
$$x = \text{input}.$$

If the input signal is a ramp function then

$$x = kt \quad (2)$$

and

$$y = Akt + Bk^2t^2 + Ck^3t^3 + Dk^4t^4 + \cdots \quad (3)$$

and the derivative of the output with respect to t is

$$\frac{dy}{dt} = Ak + 2Bk^2t + 3Ck^3t^2 + 4Dk^4t^3 + \cdots. \quad (4)$$

A measure of the linearity can be made by comparing the magnitude of the first term with the remaining terms. This is done by taking the ratio of the departure from the initial value, at time T, to the initial value. (See Fig. 2.)

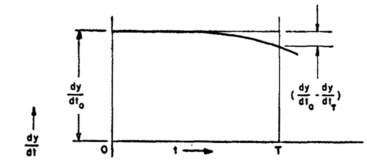

Fig. 2—Curve showing derivative as $f(t)$ with ramp input.

Thus

$$\frac{\dfrac{dy}{dt_0} - \dfrac{dy}{dt_T}}{\dfrac{dy}{dt_0}} = -\left(\frac{2Bk^2T + 3Ck^3T^2 + 4Dk^4T^3}{Ak} + \cdots\right) \quad (5)$$

where

$$\frac{dy}{dt_0} = \frac{dy}{dt} \text{ at } t = 0, \quad \frac{dy}{dt_T} = \frac{dy}{dt} \text{ at } t = T.$$

Eq. (5) shows that the total departure at $t = T$ is a weighted sum of the distortion components whose coefficients are B, C, D, etc. Theoretically, the magnitude of each coefficient can be computed by deriving the equation of the curve of Fig. 2. Practically, this becomes increasingly difficult as the order of the curvature increases and it is proposed that a "figure of demerit," which weighs the various distortion coefficients, be used as a measure of nonlinearity. The relation between this "figure of demerit" and intermodulation distortion should be apparent.

If a sinusoidal waveform, having a peak amplitude of kT, is introduced into the same device the input is given by

$$x = Kt \sin \omega t \quad (6)$$

and the output is

$$y = Akt \sin \omega t + Bk^2T^2 \sin^2 \omega + Ck^3T^3 \sin^3 \omega t + Dk^4T^4 \sin^4 \omega t + \cdots. \quad (7)$$

Expanding each term gives

$$y = AkT \sin \omega t + \frac{Bk^2T^2}{2}(1 - \cos 2\omega t) + \left(\frac{Ck^3T^3}{4} 3 \sin \omega t - \sin 3\omega t\right) + \frac{Dk^4T^4}{8}(3 - 4\cos 2\omega t + \cos 4\omega t) + \cdots. \quad (8)$$

A direct comparison with (5) can now be made on a term by term basis. Considering the second harmonic [in the second term of (8)], the ratio of the coefficient to that of the fundamental is

$$\frac{\text{Second harmonic}}{\text{fundamental}} = \frac{BkT}{2A}. \quad (9)$$

From (5), the departure from the initial value at $t = T$ divided by the initial value is

$$\frac{\dfrac{dy}{dt_0} - \dfrac{dy}{dt_T}}{\dfrac{dy}{dt_0}} = \frac{2BkT}{A} \quad (10)$$

for second order term only.

As an example, if only a second harmonic is present whose amplitude is 1 per cent of the fundamental, $Bkt/2A = 0.01$, then the departure of the sawtooth derivative from the horizontal at $t = T$ will be 4 per cent of the initial value, $2BkT/A = 0.04$. From actual operating data, it has been observed that a departure of 0.1 per cent is readily detected making it completely feasible to detect a second harmonic whose amplitude is only 0.025 per cent of the fundamental.

A comparison of the high order terms shows an even greater sensitivity advantage for this method. For the third and fourth terms the ratio of linearity departure to harmonic amplitude is 12 and 32, respectively.

Some approximations have been made in obtaining these figures, which include neglecting the contributions to the fundamental by the third term, and to the second harmonic by the fourth term, etc. (8). If it is desired to correlate the differentiated sawtooth signal with the distortion coefficients this can be done by a simple comparison between (1) and (4). Thus the ratios B/A, C/A, D/A, etc., are multiplied by 2, 3, 4, etc. A device having a 2 per cent second order nonlinearity ($B/A = 0.02$) would give a departure from the horizontal of 0.04 at $t = T$.

The discussion thus far has been limited to a nonlinear device which exhibits no frequency distortion. In practical applications of this method it is desirable to use a repetitive waveform and it is necessary to know the bandwidth requirements for the particular waveform selected. This bandwidth will be a function of the degree of precision which is sought.

A simple Fourier analysis of the repetitive sawtooth is not applicable because much of the harmonic content is associated with the sharp trailing edge which does not contribute to the measurement. Therefore, the analysis is made using a ramp function in conjunction with several elementary low- and high-pass filters. Consider first an elementary low-pass RC network whose transfer function is $1/(Ts+1)$ where $T = RC$ and the transform of the ramp function is k/s^2. Thus the transform of the output is

$$\mathcal{L}e_{\text{out}} = \frac{k}{s^2(Ts + 1)} \quad (11)$$

and

$$e_{\text{out}} = k(t - T(1 - e^{-t/T})). \quad (12)$$

When $t \geqq 3.4\ T$, the departure from linearity of the sawtooth is less than 1 per cent. If the initial portion of the sweep is of interest, it is necessary to use a sweep of sufficient duration to permit the error to drop to the required limit during the early portion of the sweep.

For the corresponding high-pass network, the transfer function is given by $TS/(1+Ts)$ and the transform of the output is

$$\mathcal{L}e_{\text{out}} = \frac{kTs}{s^2(1 + Ts)}, \quad (13)$$

and

$$e_{\text{out}} = kT(1 - e^{-t/T}). \quad (14)$$

Eq. (14) is similar to the familiar expression for the current in an RL circuit with a step function input. If $t = 0.02\ T$ the departure from linearity is less than 1 per cent.

This method, where applicable, provides directly the same information which is obtained indirectly by intermodulation or harmonic distortion measurements. In addition, the information is displayed as a function of amplitude and with extremely modest equipment. The bandwidth requirements will prove a drawback for some measurements.

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When is a Backward Wave Not a Backward Wave?*

The following remarks have reference to a number of studies of the large-signal (nonlinear) performance of traveling-wave amplifiers.[1-4] There seems to be some difference of opinion regarding the treatment of the transmission-line theory for the helix. The procedure of Nordsieck and Rowe is essentially to integrate numerically the second-order differential equation for the helix with the proper boundary conditions. Clearly the result is the complete and unique solution of the problem. Poulter and Tien prefer to start the numerical work from the general solution of the differential equation written in semiclosed form. While also formally correct, their procedure is rather involved and leads to some intuitive difficulties. Tien's claim that Rowe omits part of the solution is a direct consequence of this confusion.

The Poulter-Tien general solution referred to above contains an integral which can be considered as the convolution integral of the spatial impulse response of the helix with the current induced in the helix by the beam. The *impulse response* has two components, one of which can be interpreted as a forward wave, the other as a backward wave. However, to consider the corresponding *component integrals* as independent forward and backward waves, respectively, has very serious consequences.

Take a small-signal growing-wave solution as an illustration. Referred to the system *helix-beam* as a whole, this wave constitutes a single component of the complementary function of the differential equation, but referred to the *helix alone* this wave, according to the Poulter-Tien convolution terminology, breaks up into one "forward" and one "backward" wave with the *identical (forward) propagation constant.* Actually these two "waves" have no separate physical existence, and it is hard to see that their mathematical separation serves any useful purpose. Tien is stretching the concept "backward wave" to the point where it becomes very nearly useless. He is making one particular method of derivation rather than the actual space-time variation the criterion for "backward" and "forward" waves.

There is no contradiction in the circumstance that the differential equations for Rowe's $A(y)$ and Tien's $a_1(y)$ and $a_2(y)$ are different. The differential equation for $A(y)$ must necessarily be of the second order; its numerical solution must include all the components as required by the boundary conditions. Tien's variables, on the other hand, do not represent the complete solution but only his "forward wave"; they consequently satisfy first-order differential equations.

The Nordsieck-Rowe procedure is the straightforward approach to the numerical solution of a nonlinear problem. Since superposition of forward and backward waves does not hold for the system as a whole, it seems artificial and pointless to break up the phenomena on the linear part of the system into forward and backward waves, as Poulter and Tien do.

While the amplitude function $A(y)$ represents the complete solution, it should be noted that Rowe's (45) for the power along the helix

$$P \simeq \left[\frac{\overline{V}^2(z, t)}{Z_0}\right]_{\text{avg}} = 2CI_0V_0A^2(y) \quad (1)$$

involves an approximation that was unfortunately not discussed in his paper.[2] We shall calculate here the magnitude of this approximation.

* Received by the IRE, May 28, 1956.
[1] A. Nordsieck, "Theory of the large-signal behavior of traveling-wave amplifiers," PROC. IRE, vol. 41, pp. 630–637; May, 1953.
[2] J. E. Rowe, "A large-signal analysis of the traveling-wave amplifier; theory and general results," TRANS. IRE, vol. ED-3, pp. 39–57; January, 1956.
[3] H. C. Poulter, "Large signal theory of the traveling-wave tube," Tech. Rep. No. 73, Electr. Res. Lab., Stanford Univ., Stanford, California; January, 1954.
[4] P. K. Tien, "A large signal theory of traveling-wave amplifiers," *Bell Syst. Tech. J.*, vol. 35, pp. 349–374; March, 1956.

The total power on the helix in the forward direction is exactly

$$P = \tfrac{1}{2}\,\mathrm{Re}\,[V^*I]. \tag{2}$$

One of the two first-order transmission-line equations may be written as

$$\frac{C\omega}{u_0}\,\frac{\partial V(y,\phi)}{\partial y} + \frac{Z_0}{v_0}\,\frac{\partial I}{\partial \phi}\,\frac{\partial \phi}{\partial t} = 0 \tag{3}$$

where

$$y = \frac{C\omega}{u_0} z \tag{4}$$

$$\phi(y, \phi_0) = y/C - \theta(y) - \omega t \tag{5}$$

and

$$V(y, \phi) = \mathrm{Re}\left[\frac{Z_0 I_0}{C} A(y) e^{-j\phi}\right]. \tag{6}$$

When (5) and (6) are used in the transmission-line equation, (3), the current on the helix is given by

$$I = \frac{I_0}{1 + Cb}\left[\left(\frac{1}{C} - \frac{d\theta(y)}{dy}\right) A(y) e^{-j\phi} + j\,\frac{dA(Y)}{dy}\,e^{-j\phi}\right]. \tag{7}$$

Substituting (6) and (7) into (2) and simplifying gives for the power along the helix

$$P = 2CI_0V_0A^2(y)\,\frac{\left(1 - C\,\dfrac{d\theta(y)}{dy}\right)}{(1 + Cb)}. \tag{8}$$

The efficiency is then given by

$$\eta = \frac{P}{I_0V_0} = 2CA^2(y)\,\frac{\left(1 - C\,\dfrac{d\theta(y)}{dy}\right)}{(1 + Cb)}. \tag{9}$$

It should be noted that since $\theta(y)$ is a negative function the numerator as well as the denominator of the correction factor is always greater than unity. It was found that for all cases of interest the correction factor varies between zero and four per cent, justifying the use of the approximate relation (1).

A further insight into the equivalence of the differential-equation approach and the convolution-integral approach may be seen in Fig. 1 where Poulter's solution is compared with Rowe's. The case selected for comparison is one for which $QC = 0$ and large $C (C = 0.2)$ so that the effect of the so-called "backward wave" should be large.

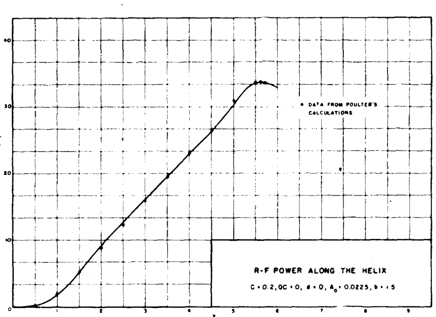

Fig. 1.

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The Noise Factor of Traveling-Wave Tubes*

Pierce, Watkins, Bloom, Peter, and others, have presented various approximate figures for the minimum value of the noise-factor of traveling-wave tubes with an infinite confining magnetic field. The purpose of this letter is to direct attention to the fact that no obvious theoretical lower limit for the noise factor is found when a simple mathematical model with different means for shaping the beam is chosen.

We shall in this discussion assume a positive space charge with infinite mass and with a density exactly neutralizing the dc negative charge of the beam. It appears that any way of shaping the beam that is based on balancing out the dc lateral space-charge forces rather than on brute-force suppression of all lateral motion will give substantially the same result. Other examples of such methods are the E-beam flow and the Brillouin flow. The beam is treated as a continuous fluid transmitting shot noise and velocity fluctuations from a cathode.

Inside the beam, where the dc velocity and space-charge density are uniform and constant, the Maxwell-Lorentz equations have two classes of small-signal solutions.

1) "Space-charge Waves," or "Plasma Waves." In a reference frame moving with the beam velocity these waves are irrotational; the total current density is everywhere zero. The divergence of the rf electric field is *not* in general zero. The shot noise generated by thermionic emission is primarily of this nature.

2) Solenoidal Waves ($\mathrm{div}\, E = 0$).

If a step-function beam edge is postulated and Hahn's method for satisfying the boundary conditions is used the following results are obtained. A space-charge wave with components of its rf velocity and electric field perpendicular to the boundary surface produces a ripple along the edge of the beam. The equivalent surface charge of this ripple is equal and opposite to the normal component of the displacement vector. As far as the velocity and electric-field components parallel to the beam edge are concerned, the boundary conditions require the addition of a solenoidal surface wave inside the beam edge. When the resulting additional ripple is accounted for, it is found that the field external to the beam is zero. The space-charge waves as defined above simply do not interact with external fields.[1] For an ideal model of a traveling-wave tube using the kind of beam discussed here, noise waves of the first class produce no noise in the external circuit.

The study of the matching of the external fields by solenoidal beam waves leads to a theory of traveling-wave amplification very closely analogous to the conventional theory. In this theory, the ripple of the beam boundary plays the same part as the space-charge density fluctuations do in the confining-field type of tubes.

Since the beam in this 0-db noise-factor model of a traveling-wave tube is in a state very far from thermal equilibrium, there is no thermodynamic paradox in the behavior just described.

The important question is of course what obstacles may impede the approximate realization of this noise factor. The three idealizations on which the analysis is based are: homogeneous fluid; ideal flow ($\vec{u}_0 =$ const, $\rho_0 =$ const); step-function beam edge.

The first assumption probably does not produce an appreciable error in the analysis. Most likely the second point is the crucial one in a Brillouin-flow or E-beam tube. It has been established experimentally that the space-charge distribution in a cutoff dc magnetron is very far from the Brillouin solution; by analogy it may be concluded that a beam will also tend to assume a different distribution. However, the means available for control of the space-charge distribution in an electron gun is far superior, and once the desired flow pattern has been established, it can be expected not to change materially during the relatively short time of transit through the tube. For the same reason the third point will probably not cause any additional difficulty; a simple collimation scheme will most likely give a sufficiently sharp beam edge. However, everything else being ideal, a small but finite limit for the noise factor is certainly set by the blurring of the beam edge by the thermal velocities of the electrons. In the Maser, which has been hailed as the 0-db noise-factor microwave amplifier, the beam entropy is also growing and a small amount of noise generated; even if the *mean* life of the active energy state of the molecules is considerably longer than the transit time through the cavity, a small number of spontaneous transitions will necessarily occur. As long as this additional noise is small compared to the thermal noise in the input circuit, it will have a negligible effect on the noise factor.

It is conceivable that also solenoidal noise waves are generated in the electron gun, but their phase velocity is so high that the gun itself or a short subsequent drift tube can be designed as a waveguide below cutoff and serve as an effective attenuator of these waves.

On the basis of some theoretical arguments and rough estimates of the consequences of the oversimplification involved, the prospects of improving the noise factor of a traveling-wave tube consequently appear rather promising.

Once the strong coupling is removed between external "circuit waves" and the noise "space-charge waves" there seems to be no reason why the conventional traveling-wave tube should not be able to hold its own in the competition with the so-called transverse-field tube as far as noise factor is concerned.

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* Received by the IRE, April 20, 1956.

[1] In their original study of plasma oscillations, Tonks and Langmuir pointed out the difficulty of observing the oscillations because of the absence of external fields.

Geophysical Prospection of Underground Water in the Desert by Means of Electromagnetic Interference Fringes*

In his above paper[1] Prof. El-Said writes: "The author wishes also to acknowledge the early work by Dr. H. Löwy under the supervision of Prof. H. M. Mahmoud, Professor of Electrical Engineering, Cairo University, for the determination of the degree of transparency of desert rocks to electromagnetic waves."

To this I note that my early work concerning the electric transparency of desert rocks begins (without "supervision") in the year 1911, with my paper in the *Annalen der Physik*, vol. 36, p. 125. It is the first systematic treatment of the problem, as can be seen from the references of R. L. Smith Rose.[2]

Concerning the method by means of which he succeeded to measure groundwater depths of about 800m, Prof. El-Said quotes Appleton and Barnett, *Nature*, vol. 115, p. 333; 1925. Missing here is the reference to another paper of mine, published in 1912 in the *Physikalische Zeitschrift*, vol. 13, p. 397, concerning the ground-water interference experiments which I made in German potash mines. Reference is missing also to my recent paper in the *Bulletin de l'Institut d'Egypte*, vol. 35, p. 103, 1953, in which I have developed the theory used by Prof. El-Said for calculating the groundwater depth.

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* Received by the IRE, May 8, 1956.
[1] Proc. IRE, vol. 44, pp. 24–30; January 1956.
[2] *Jour. IEE*, vol. 75, p. 221; 1934.

Increasing the Accuracy of CRO Measurements*

We have developed several convenient ways of increasing the accuracy of measurements made on a cathode ray oscillograph. Because of deflection nonlinearities, curvature of the face plates, the time lag between measurement and calibration, and other factors, amplitude measurements made on the face of a cathode ray oscillograph are, as a rough rule of thumb, considered accurate to ±5 per cent. With the techniques to be described accuracies of ±0.1 per cent are possible.

An essential feature of the technique is that the oscillograph is used as a null detector. The unknown and a standard voltage are displayed simultaneously and the standard is adjusted until it equals the unknown. The standard is usually a dc voltage obtained from a battery and checked occasionally against a standard cell. A precision attenuator is used with the standard. The simultaneous display is obtained by means of a relay, usually operated directly from the 60 cycle line, which alternately connects the standard and unknown to the input of the oscillograph. Electronic switching means could also be used. One observes the unknown waveform and a straight line due to the standard. It is simple to adjust the standard until it coincides with unknown waveform at the point of measurement. With instruments that have dc amplifiers which have a wide dynamic range, such as the DuMont type 304, the gain may be increased so that the zero is far off scale and high sensitivity and repeatability in setting the null are obtained. With that instrument the gain can be adjusted so that the pattern has a height six times the tube diameter and the centering controls can be adjusted so that any portion of the waveform can be seen. On other instruments the same effect can be obtained by introducing a bucking voltage in series with both the unknown and the standard.

Details of the application of the principle vary with the particular situation and there is considerable room for exercise of ingenuity. Several other examples will be given:

1) In measuring 60 cycle hysteresis loops[1] it is desirable to measure the peak drive current. Distortion in the waveform which may be caused by the nonlinear core can make meter measurements inaccurate. In this case we switch both the X and Y axes of the oscillograph. During one period the loop is displayed. During the next 1/120 second the X axis is switched to the standard and the Y axis to 3 kc relaxation oscillator. One sees the right half of a hysteresis loop and a vertical line. The line is adjusted until it touches the tip of the loop.
2) In the application above it is necessary to measure the coercive force of the material. Here the hysteresis loop is reflected about the Y axis by passing the X axis signal through a full wave rectifier. Now on alternate 1/120 second periods one displays the right half of the hysteresis loop and a straight line, which is obtained by shorting the Y axis and passing the X-axis signal through a precision 10-turn window reading potentiometer. The potentiometer is adjusted until the line touches the side of the loop. The potentiometer than reads the ratio of the coercive force to the peak magnetic field.
3) To measure the gain of an amplifier to a repetitive signal the output and input could be displayed simultaneously. In many cases it is most convenient to obtain the input from a standard attenuator which is connected to a signal generator. The attenuator is adjusted until its attenuation is equal to the gain of the amplifier. With both traces present distortion is quite evident.
4) To measure the percentage droop in pulse transmission the synchronization of the oscillograph could be switched so that the leading edge of the input lined up with the trailing edge of the output when input and output are simultaneously displayed. The input is sent through a precision attenuator which is adjusted until the leading and trailing edges are equal in amplitude.

The possibilities of the technique should be apparent. Its advantages are:

1) The deflections for both measurement and calibration are identical, eliminating errors due to lack of linearity in the system.
2) Measurement and calibration are made simultaneously. The possibility of error due to drift in gain or unintentional change in settings is eliminated.
3) The accuracy can be made dependent on a dc source and a calibrated attenuator, two components which can readily be made accurate and stable.

William J. Bartik and Fred Bernstein contributed to these techniques. We have observed a pulse amplitude calibration method similar to the one described here in use at the Digital Computer Laboratory, Massachusetts Institute of Technology.

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* Received by the IRE, May 31, 1956.

[1] F. Bernstein, T. H. Bonn, and R. D. Torrey, "A sensitive single-turn hysteresis-loop tester," presented at the Special Magnetic Amplifier Conf., Syracuse, N.Y.; April 5, 1956.

Optimum Slicing Level in a Noisy Binary Channel*

Suppose that a binary signal, consisting of the familar "on-off" type in radio telegraphy, is transmitted over a noisy channel. At the receiver a very small amplitude slice of the envelope is selected and amplified, as is shown in Fig. 1.

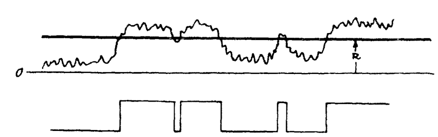

Fig. 1.

If the sine wave frequency is located at the center of a flat noise spectrum of width β, the expected number N_R of crossings with positive slope at the level R by the envelope is given by Rice[1] as

$$N_R = \left(\frac{\pi}{6b_0}\right)^{1/2} \beta R I_0\left(\frac{RQ}{b_0}\right) \cdot \exp\left(-\frac{R^2+Q^2}{2b_0}\right) \qquad (1)$$

* Received by the IRE, April 11, 1956; revised manuscript received May 28, 1956.

[1] S. O. Rice, "Statistical properties of a sine wave plus random noise," *Bell Syst. Tech. J.*, vol. 27, pp. 125–126; January, 1948.

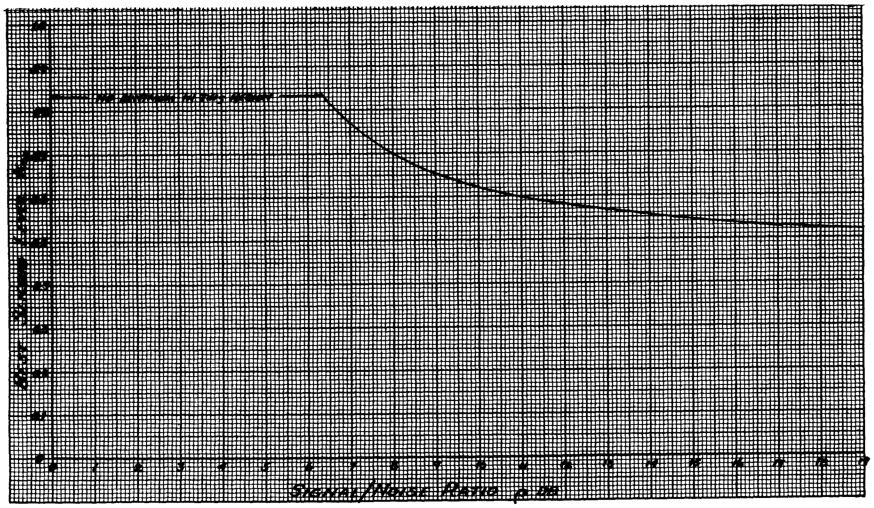

Fig. 2—Best slicing level for various signal/noise ratios.

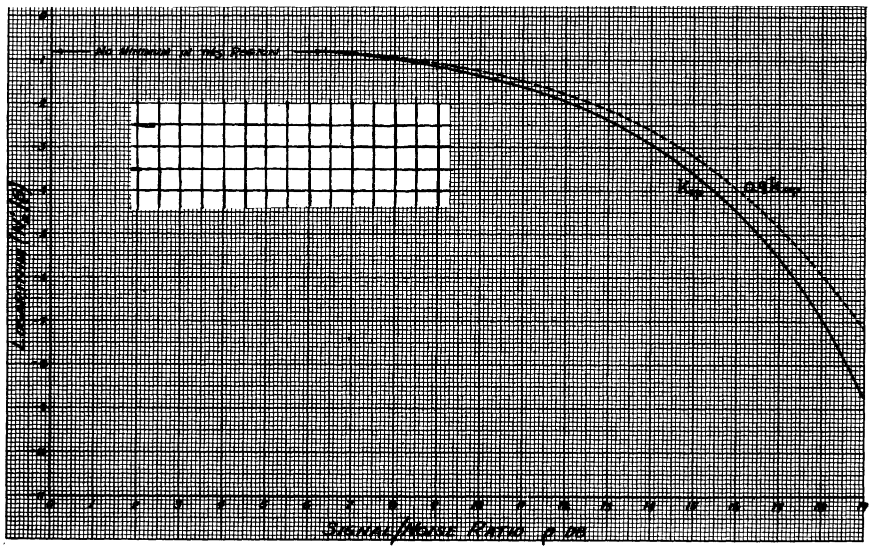

Fig. 3—Common logarithm of number of expected positive crossings per second per cycle bandwidth.

where b_0 is the mean noise power and Q is the peak amplitude of the sine wave. In many cases of interest the total duration of "signal" about equals that of "no signal." The crossings that are considered herein are only those caused by noise and are not due to the changes in the signal amplitude. Assuming this to be true the expected number of positive crossings is

$$N_R' = \left(\frac{\pi}{6b_0}\right)^{1/2} \cdot \frac{\beta}{2} \cdot R \exp\left(-\frac{R^2}{2b_0}\right) \cdot \left\{1 + I_0\left(\frac{RQ}{b_0}\right) \exp\left(-\frac{Q^2}{2b_0}\right)\right\}. \quad (2a)$$

We set the slicing level R at some fraction $k(0<k<1)$ of the peak signal amplitude Q so that $R=kQ$ and let ρ equal the power signal/noise ratio $Q^2/2b_0$. Eq. (2a) becomes

$$N_{R'} = \left(\frac{\pi}{3}\right)^{1/2} \cdot \frac{\beta}{2} \cdot k\sqrt{\rho} \exp(-k^2\rho) \cdot \{1 + I_0(2k\rho) \exp(-\rho)\}. \quad (2b)$$

The derivative of $N_{R'}$ with respect to k, when set equal to zero, yields the necessary condition which stationary points must obey. This condition is

$$2k \exp(-\rho) I_1(2k\rho) = \left(2k^2 - \frac{1}{\rho}\right) \cdot \{1 + I_0(2k\rho) \exp(-\rho)\}. \quad (3)$$

The best slicing level k_{0p} occurs when the expected number of crossings is a minimum. For ρ less than about 6.4 db the curve (2b) has no minimum over the range of k of interest. For values of ρ up to 19 db, this optimum level has been computed and is plotted in Fig. 2. As ρ increases indefinitely, (3) shows that k_{0p} approaches 1/2 as one would expect. Fig. 3 is a graph of the logarithm of the minimum number of expected crossings, corresponding to the values of k_{0p}, against the signal/noise ratio. For large ρ the curve varies like the function -0.11ρ (ρ a ratio). For a nonoptimum slicing level ($k=0.9\ k_{0p}$) the logarithm of the expected crossing rate is shown as a dashed curve. A value of $k=1.1\ k_{0p}$ yields closely the same curve.

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Power Transfer in Double-Tuned Coupling Networks*

In tuned amplifiers using amplifying elements with finite power gain (such as transistors) the interstage coupling network must, in addition to the required selectivity, provide optimum power transfer from driving to driven stage.

The design of single-tuned and transitionally coupled double-tuned networks for tuned transistor amplifiers has been described elsewhere.[1] In particular, it is known that good power transfer and narrow bandwidth are conflicting requirements which can be satisfied only by the use of inductances having very high *unloaded* Q's. The purpose of this note is to present considerations applicable to the general double-tuned case (with inductive coupling).

In the circuit of Fig. 1 transistor T_1 can be represented by a current source I_1 and its output admittance (g_0 and C_0) and the second transistor T_2 by its input admittance (g_i and C_i), resulting in the equivalent circuit of Fig. 2. g_1 and g_2 are the parallel loss conductances of the transformer windings. Lumping the capacitances and conductances together, we obtain the circuit of Fig. 3.

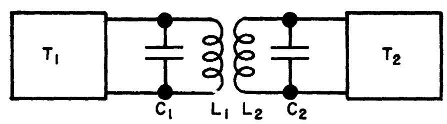

Fig. 1—Transistor amplifier stages coupled by double-tuned network.

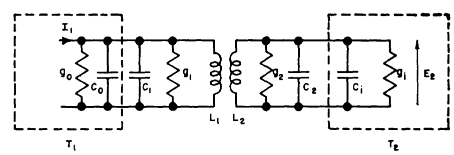

Fig. 2—Equivalent of Fig. 1.

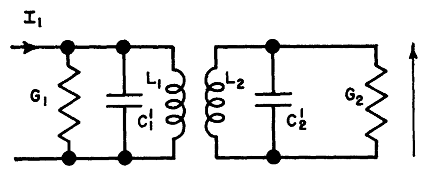

Fig. 3—Simplified equivalent of Fig. 1.

If Q_0 is the *unloaded* Q of the transformer windings (primary and secondary unloaded Q's are assumed equal) the *loaded* Q's are:

$$Q_1 = Q_0 g_1/G_1 = Q_0/x \quad (1)$$

$$Q_2 = Q_0 g_2/G_2 = Q_0/y. \quad (2)$$

The absolute value of the transfer impedance z_{21} is given by[2,3]

$$z_{21} = \frac{s}{(1+s^2)\sqrt{G_1G_2}} \times \frac{1}{\sqrt{1 - \alpha v^2/(1+s^2) + v^4/(1+s^2)^2}} \quad (3)$$

* Received by the IRE, March 20, 1956.

[1] R. R. Webster, "How to design IF transistor transformers," *Electronics*, vol. 28, pp. 156–160; August, 1955.

[2] C. B. Aiken, "Two mesh-tuned coupled circuit filters," Proc. IRE, vol. 25, pp. 230–272; February, 1937.

[3] G. E. Valley and H. Wallman, "Vacuum Tube Amplifiers," McGraw-Hill Book Co., Inc., New York, N.Y.; 1948.

with

$$\alpha = \frac{2(s^2 - b/2)}{1 + s^2} \cdot \quad (4)$$

The symbols in (3) and (4) have the following significance: $s = k\sqrt{Q_1Q_2}$, k being the coupling coefficient; $b = Q_1/Q_2 + Q_2/Q_1$; $v = \sqrt{Q_1Q_2}\ (\omega/\omega_0 - \omega_0/\omega) \cong Q_1Q_2\ (2\Delta\omega/\omega_0)$, where $\Delta\omega = (\omega - \omega_0)$.

α can be called the *shape factor* of the coupled-tuned circuit response: $\alpha = 0$ corresponds to the case of transitional coupling (maximum flat response), whereas $\alpha < 0$ and $\alpha > 0$ are characteristic of the undercoupled and overcoupled cases respectively.

Using the above approximation for v, the fractional bandwidth[4] B_n is determined by

$$Q_1^2Q_2^2B_n^4/(1 + s^2)^2 - \alpha Q_1Q_2B_n^2/(1 + s^2) - n = 0. \quad (5)$$

At the center frequency, the power consumed by the load g_i is

$$P_d = E_2^2 g_i = |I_1|^2[s^2/(1 + s^2)^2](g_i/G_1G_2) \quad (6)$$

and dividing P_d by the available power $(|I_1|^2/4g_0)$, we find the power transfer efficiency from T_1 to T_2

$$\eta = 4[s^2/(1 + s^2)^2](g_0g_i/G_1G_2). \quad (7)$$

Using the variables x and y, (5) and (7) can be written:

$$F(x, y) = Q_0^4B_n^4 - [\alpha/(2 - \alpha)]Q_0^2B_n^2(x + y)^2 - [n/(2 - \alpha)^2](x + y)^4 = 0 \quad (8)$$

$$\eta = 4(2 - \alpha)(x^2 + y^2 + \alpha xy)(x - 1)(y - 1)/(x + y)^4. \quad (9)$$

The maximum value of η can be found by setting

$$\partial\eta/\partial x + \lambda\partial F/\partial x = 0 \quad (10)$$

$$\partial\eta/\partial y + \lambda\partial F/\partial y = 0. \quad (11)$$

This results in the simple condition for η_{max}

$$x = y. \quad (12)$$

Substituting (12) into (8) and solving for x(or y), we find

$$x = y = (Q_0B_n\sqrt{2 - \alpha}/2\sqrt[4]{n}) \times\sqrt{\sqrt{1 + \alpha^2/4n} - \alpha/2\sqrt{n}}. \quad (13)$$

With (13) the circuit elements and coupling coefficient required for maximum power transfer can be calculated

$$L_1 = (x - 1)/\omega_0Q_0g_0 \quad (14)$$

$$L_2 = (x - 1)/\omega_0Q_0g_i \quad (15)$$

$$k = (x/Q_0)\sqrt{(2 + \alpha)/2 - \alpha)}. \quad (16)$$

In the case of *transitional coupling* $\alpha = 0$ and, therefore, (13) becomes

$$x = Q_0B_n/\sqrt{2}\sqrt[4]{n}. \quad (17)$$

Substituting (12) and (13) into (9) we find the maximum power transfer efficiency

$$\eta_{max} = (1 - \alpha^2/4)(1 - 1/x)^2 = (1 - \alpha^2/4) \times \left[1 - \frac{2\sqrt[4]{n}}{Q_0B_n\sqrt{2 - \alpha}\sqrt{\sqrt{1 + \alpha^2/4n} - \alpha/2\sqrt{n}}}\right]^2 \quad (18)$$

For *transitional coupling*:

$$\eta_{max} = (1 - \sqrt{2}\sqrt[4]{n}/Q_0B_n)^2. \quad (19)$$

In solving practical design problems n, B_n, and Q_0 are usually known. The shape of the response curve is often of minor importance and the question may arise: What value of α should be chosen for maximum power transfer efficiency?

The optimum α can be determined from (18) by setting $d\eta_{max}/d\alpha$ equal to zero. This procedure leads to computational difficulties and, in general, one can only state that maximum power transfer efficiency occurs for $\alpha < 0$, *i.e.*, in the case of undercoupled circuits. Transitionally coupled circuits lead to less than maximum power transfer efficiency. The loss in the case of transitional coupling is very small for large values of B_nQ_0 but significant if B_nQ_0 is small. The situation is illustrated by Fig. 4. Undercoupled circuits should be used for small values of B_nQ_0.

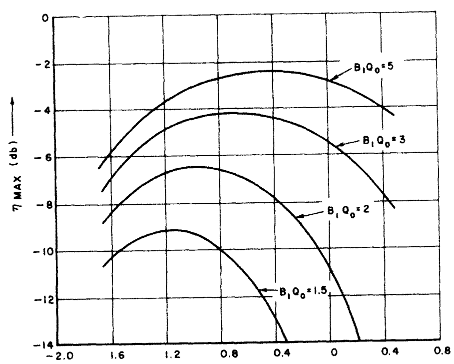

Fig. 4—η_{max} as a function of α with B_1Q_0 as parameter.

ACKNOWLEDGMENT

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[4] The bandwidth B_n' is defined as the interval between frequencies at which the power response is $1/(n+1)$ times the response at the center of the band. The fractional bandwidth B_n is $2\pi B_n'/\omega_0$.

Time Signals for the Determination of Longitude*

During the forthcoming International Geophysical Year, it is proposed to determine the relative positions of certain points on the earth's surface with increased precision.[1] At these points astronomical observations are to be made at times that are closely related to the readings of clocks at Greenwich.

This raises afresh the problem of defining simultaneity at distant points. Radio time signals are propagated from point-to-point at velocities that are less than that in free space, and that are not known with complete certainty.[2]

To judge by preliminary papers issued by the Bureau de l'Heure, the velocities that are considered normal are: 252,000 km per sec for frequencies of the order of 16 kc; 274,000 km per sec for frequencies of the order of 10 mc, by the short path, and 286,000 km per sec for frequencies of the order of 10 mc, by the long path. For single observations, the spread from these values may be considerable.

For a path such as that from Europe to New Zealand, which are approximately antipodal, the distinction between long and short path is by no means clear, and it may be difficult to tell whether a short-wave time signal has been 70 msec on the way at the *short path* velocity, or 67 msec at the *long path* velocity. Since OSAGI recommends that times should be established to within 1 msec for longitude determinations, and to "very high precision" for propagation studies,[3] the uncertainty of velocity is serious.

OSAGI suggests that the velocity of propagation may be investigated from signals initiated by a quartz clock at each end of a transmission path, so that each terminal station both transmits and receives a signal. The relative short-term instabilities of two crystal clocks enter into this experiment, and the measurement can be successful only if cooperation between distant points is uniformly good.

The experiment can be freed from all reliance on crystal clocks, and the radio measurements can be made entirely at one end of the transmission path if a respondor is installed at the remote end. In the simplest case, the respondor consists of a receiver which energizes a transmitter within microseconds of the receipt of a signal. The time between the transmission from the master end of the path and the receipt of the return signal is the time of a double traverse of the path. To a first approximation the midpoint of this interval is the time of arrival of the signal at the remote end of the path.

So simple a respondor is impractical except for short distances, but it may be replaced by existing communication receiving and transmitting stations, as shown in Fig. 1.

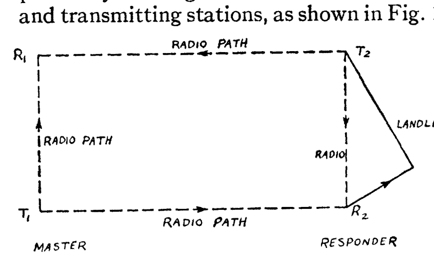

Fig. 1.

* Received by the IRE, April 3, 1956.

[1] 2nd Assembly of OSAGI, Rome, 1954, VIII, p. 1.

[2] See, for example, Mme. A. Stoyko, "Sur la variation de la vitesse de propagation des oudes radio electroniques," *Compt. Rend.* (Paris), vol. 232, pp. 1916–1920; May, 1951.

[3] 2nd Assembly of OSAGI, Rome, 1954, VIII, 2.4.1 and 2.4.4.

T_1 transmits short (1 msec or less) pulses which are received by R_2, some thousands of kilometers away, and are passed on to T_2 over an existing land line. T_2 retransmits at the same frequency as T_1 and is received at R_1, where the total travel time is measured. For the present purpose each transmitter-receiver pair should be as close as possible, but since existing stations are to be used, the existing distances (up to 100 or so km) must be accepted. The connection from R_2 to T_2 will normally be by a devious land line, and the travel time in it must be determined. There may also be instrumental delays in R_2 and T_2. If T_1 and T_2 operate at the same frequency the transmission from T_2 will be received by R_2 which will re-energize T_2, so leading to a train of pulses with a period of the total time of travel from R_2 to T_2 and back to R_2.

This train, received at R_1, will show how much of the total travel time is occupied between R_2 and T_2, and so will lead to the time taken for the two main radio paths $T_1 \cdots R_2$ and $T_2 \cdots R_1$. It will be expedient to interrupt the train of pulses after say 0.1 second. The times for the local radio transmissions, $T_2 \cdots R_2$ and $T_1 \cdots R_1$, will be so short that they may be computed. The times to be expected are:

For the main radio path (double journey) for intercontinental distances.	10–140 msec.
For the land line	up to 1 msec.
For the local radio paths	up to $\frac{1}{4}$ msec.

High-power very low frequency transmitters are so rare that few responding circuits using them can be arranged, but the transmission time for these frequencies can be found by using short waves for the return path. If these measurements are made immediately after those described above, or, better still, interposed among a series of high frequency determinations, the difference between the total travel times will be the difference between the single transmission time at high frequency and by very low frequency. Since the shape of the pulse received at R_1 can be matched against the shape of the pulse transmitted from T_1, there should be little trouble from the slow rise-time of the very low frequency signal.

Measurements, of course, will need to be made to determine the variation of apparent velocity during the day, and during the year.

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Application of Equipartition Theory to Electric Circuits*

In the derivation of a theoretical formula for Johnson noise Nyquist[1] said: "To each degree of freedom there corresponds an energy equal to kT on the average, on the basis of the equipartition law, where k is the Boltzmann constant." This formulation could cause difficulty because the usual statement on equipartition is that with each degree of freedom there is associated *kinetic* energy of mean value *one half* of kT, and in lumped-circuit applications we do in fact find an energy of $\frac{1}{2}kT$ per degree of freedom.

Since equipartition is founded in the science of mechanics, we start with the definition that the number of degrees of freedom of a mechanical system is equal to the number of coordinates which must be specified in order to define the *position* of the system; but the full specification of a mechanical system requires in general a knowledge of *two coordinates* for each *degree of freedom, e.g.*, position and momentum as used in the Hamiltonian form of the equations of motion. The equipartition theorem then says that if there is associated with a *coordinate* an energy proportional to the square of that coordinate, the mean-square value of that energy will be $\frac{1}{2}kT$. If one of the coordinates is momentum and the mass is constant, the kinetic energy being proportional to the square of momentum must have the mean value $\frac{1}{2} kT$, as assumed above. The magnitude of any potential energy which may exist depends on the law of the field of force to which it is due, but in the special case of a harmonic oscillator the mean potential energy is equal to the mean kinetic energy. The total energy associated with the harmonic oscillator is thus kT, but any experiment in which one observes a single coordinate gives a direct measure of only $\frac{1}{2} kT$.

For example, in deriving from the Nyquist formula for noise in a narrow bandwidth the expected total (all-frequency) thermal energy in both a CR circuit and an LCR circuit, Moullin[2] arrived at a value of $\frac{1}{2} kT$. By applying the well-known technique of contour integration it can be shown[3] that this value of $\frac{1}{2} kT$ will be found for any two-terminal lumped network, however complicated its internal structure. Yet we can correctly assume the "available noise power" in a channel of bandwidth B to be kTB. This apparent paradox is eliminated if in Nyquist's formulation one replaces "degree of freedom" by "mode of oscillation," since a harmonic oscillator is known to have mean potential energy equal to its mean kinetic energy and hence to be entitled to a total of kT.

It remains to show how we reconcile $\frac{1}{2} kT$ total for a two-terminal lumped circuit with kT for a transmission line or distributed circuit. (A circuit having a truly flat response over bandwidth B is not physically realizable with lumped elements.) One normally equates magnetic and electrostatic energies to kinetic and potential energies respectively, and each mode of oscillation of a transmission line will have associated with it a total energy kT, made up of $\frac{1}{2} kT$ of electric-field energy and $\frac{1}{2} kT$ of magnetic-field energy. Only the former half should be relevant to the measurement and exchange of energy through the mechanism of the potential difference between a pair of fixed points on the line. In a line which is open-circuited at both ends there is a total standing-wave energy $(kTl/v)d\nu$ which is equivalent to pairs of traveling waves with velocity $\pm v$ in the forward and backward directions. Assuming for the moment that the total energy can simply be halved to find the energy associated with waves traveling in one direction, this is equivalent to a power flow of $\frac{1}{2} kTd\nu$ in each direction. Since this is independent of line length, the latter can now be allowed to tend to infinity. On subsequently cutting the infinite line at an arbitrary point in the neighborhood of the observer, a single matching resistance R can be connected to one of the remaining parts of the line, whereupon conditions in the line will be the same as when the line was of infinite length. The power exchange between this one resistor and the line should then, according to the above argument, be $\frac{1}{2} kTd\nu$, which is only half that required by the Nyquist formula.

The fallacy in the above arguments seems to be in the assumptions about the energy of a standing wave. If the voltage standing wave is to be described as the resultant of two traveling waves of amplitude A each, the formula may be written.

$$\begin{aligned} V &= A \cos(\omega t + \beta x) + A \cos(\omega t - \beta x) \\ &= 2A \cos \omega t \cos \beta x \end{aligned} \tag{1}$$

where x is distance from the end of the line and β is the phase constant of the line. Now the total mean energy stored in the line is proportional to $\overline{\overline{V^2}}$ where the double bar indicates that an average must be taken over the length of the line, as well as a mean in time. From (1) it follows that

$$\begin{aligned} V^2 &= 4A^2 \cos^2 \omega t \cos^2 \beta x \\ \therefore \overline{\overline{V^2}} &= A^2 \end{aligned} \tag{2}$$

and therefore A^2 is proportional to $\frac{1}{2} kT$. But at the input end of the line where $x=0$,

$$\begin{aligned} V|_{x=0} &= 2A \cos \omega t \\ \overline{V^2}|_{x=0} &= 2A^2. \end{aligned} \tag{3}$$

Thus the time average of the squared-voltage seen at the end of the line is twice as great as the over-all average-squared-voltage which corresponds to the equipartition energy, and viewed from its terminals the line behaves as though it had capacitive energy kT per degree of freedom instead of the equipartition value $\frac{1}{2} kT$ per degree of freedom.

Finally, the writer wishes to emphasize that although he criticizes some details of the original proof, he has always strongly supported the importance and validity of Nyquist's theoretical derivation of the thermal-noise theorem.[4]

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[1] H. Nyquist, "Thermal agitation of electric charge in conductors," *Phys. Rev.*, vol. 32, p. 110; July, 1928.

[2] E. B. Moullin, "Spontaneous fluctuations of voltage," Oxford Univ. Press, New York, N.Y.; 1938.

[3] D. A. Bell, "Johnson Noise and equipartition," *Proc. Phys. Soc. B*, Vol. 66, p. 714; August, 1953.

[4] D. A. Bell, "On the general validity of Nyquist's theorem," *Phil. Mag.*, Ser. 7, vol. 27, p. 645; June, 1939.

Russian Condenser Terminology*

The Russian word for condenser, конденсатор is simply a transliteration of the German *Kondensator;* Russian has no word equivalent to the English capacitor, although емкость, capacitance is sometimes used in this respect. The adjective эталонный is derived from the French *étalon*, standard.

adjustable c	регулируемый к.
air c	воздушный к.
air-dielectric c	к. с воздушным диэлектриком
antenna c	к. антенны
automobile c	автомобильный к.
bakelite c	к. с диэлектриком из бакелита
balancing c	уравновешивающий к.
band-spread c	к. растянутого диапазона
blocking c	блокировочный к.
buffer c	буферный к.
bypass c	пропускающий к.
ceramic c	керамический к.
charged c	заряженный к.
charging c	зарядный к.
compressed-air c	к. со сжатым воздухом в качестве диэлектрик
compression-type c	к. емкость которого регулируется изменением сжатия пластин
continuously adjustable c	плавно-переменный к.
coupling c	к. связи
cylindrical c	цилиндрический к.
dual c	двойной к.
electric c	электрический к.
electrolytic c	электролитический к.
dry	сухой
semidry	полусухой
wet	жидкостный
experimental	экспериментальный к.
filter	к. фильтра
fixed c	постоянный к.
fixed-capacity c	к. постоянной емкости
flat-plate c	1 плоский к.
	2 к. с плоскими пластинами
gang c	конденсаторный агрегат (блок)
two-gang c	агрегат двух к—ров
three-gang c	агрегат трех к—ров
general-purpose c	универсальный к.
glass c	стеклянный к.
glass-plate c	к. со стеклянными пластинами
grid c	сеточный к.
high-voltage c	к. высокого напряжения
insulating c	изолирующий к.
inverse-square-law c	обратно-квадратический к.
leaky c	к. с утечкой
Leyden jar	лейденская банка
low-capacity c	к. небольшой емкости
low-loss c	1 малопотерьный к.
	2 к.с малыми потерями
metallic-plate c	к. с металлическими пластинами
metallized-paper c	к. из металлизованной бумаги
mica c	слюдяной к.
microfarad c	микрофарадный к.
micromicrofarad c	пикофарадный к.
midget c	крохотный к.
mid-line c	среднелинейный к.
miniature c	миниатюрный к.
molded c	формованный к.
multiple c	многократный к.
neutralizing c	нейтрализующий к.
noninductive c	безындукционный к.
oil c	масляный к.
oil-impregnated-paper-dielectric c	к. с диэлектриком из пропитанной маслом бумаги
paper c	бумажный к.
paper-dielectric c	к. с бумажным диэлектриком
parallel c	параллельный к.
perfect c	совершенный к.
pigtail c	к. с гибкими электродами
plate c	пластинчатый к.
plate (anode) c	анодный к.
plug-in c	сменный к.
polarized c	поляризованный к.
porcelain c	фарфоровный к.
power c	силовой к.
primary c	первичный к.
protective c	предохранительный к.
punctured c	к. с пробой изоляции
receiving c	приемный к.
root-mean-square c	среднеквадратичный к.
series-gap c	к. представляющий собой последовательное соединение
semiadjustable c	секционно-переменный к.
semivariable c	полупеременный к.
shielded c	экранированный к.
short-circuited c	короткозамкнутый к.
shortening c	укорачивающий к.
short-wave c	к. коротких волн
single-section c	к. одной секции
smoothing c	сглаживающий к.
spherical c	сферический к.
square-law c	квадратический к.
standard c	эталонный к.
steatite c	стеатитовый к.
stopping c	преграждающий к.
straight-line-capacity c	прямоемкостный к.
straight-line-frequency c	прямочастотный к.
straight-line-wavelength c	прямоволновый к.
subdivided (sectionalized) c	секционированный к.
synchronized c	синхронизированный к.
taper-plate c	к. с клинообразными пластинами
telephone c	телефонный к.
trimmer	равнитель
trimming c	подстроечный к.
transmitting c	передающий к.
tubular c	трубчатый к.
tuning c	к. настройки
twin c	сдвоенный к.
unshielded c	неэкранированный к.
vacuum c	вакуумный к.
vane c	к. с крыльчатыми пластинами
variable c	1 переменный к.
	2 к. переменной емкости
vernier c	1 верньерный к.
	2 к. точной настройки

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