Correspondence

Amplification by Acceleration and Deceleration of a Single-Velocity Stream*

A method of amplification at microwave frequencies based upon the growth of space-charge waves in a decelerating stream of electrons has recently come to our notice. This mechanism became evident during a study of the type of waves described by Hahn[1] and Ramo.[2] It was found here that these waves not only change in length as the stream velocity changes, but also change in amplitude.

By a suitable combination of gradual decelerations and sudden accelerations, the amplitude of the space-charge wave may be essentially arbitrarily increased without the necessity of either wave carrying circuits, additional ions, or electron streams with different or distributed velocities, or space-charge-produced differences of velocity in a single stream.

An even simpler mechanism of amplification involving only short accelerating and decelerating gaps and constant potential drift regions exists which is closely related to the one just described. Consider a space-charge wave with ac velocity v_1 and an ac convection current density i_1 on a stream of electrons at a dc velocity u_1, described by

$$v = v_{1m} \cos\left(\frac{\omega_{p1}}{u_1} z\right) e^{j(\omega t - \omega z/u_1)} \qquad (1)$$

$$i_1 = j v_{1m} \frac{\omega}{\omega_{p1}} \frac{I_0}{u_1} \sin\left(\frac{\omega_{p1}}{u_1} z\right) e^{j(\omega t - \omega z/u_1)}, \qquad (2)$$

where $\omega_{p1}{}^2 = \eta I_0/\epsilon u_1$ and I_0 is the dc beam-current density. Now if at a position along the stream at which the ac velocity reaches its maximum value v_{1m}, the dc velocity is suddenly changed from u_1 to a lower value u_2, the ac velocity will increase from v_{1m} to v_{2m} such that

$$v_{2m} = v_{1m} \frac{u_1}{u_2}, \qquad (3)$$

provided only that the dc velocity change occurs in a distance which is short compared with a quarter space-charge wavelength at the lower velocity. That this is so can be demonstrated by simple kinematics or by application of the Llewellyn-Peterson diode equations.[3]

If the beam is then allowed to drift at the low velocity u_2 for an odd number of quarter space-charge wavelengths, that is, until the ac velocity has disappeared and the ac convection current which it produces is a maximum, this current i_2 will be

$$i_2 = i_{2m} = i_{1m} \left(\frac{u_1}{u_2}\right)^{3/2}, \qquad (4)$$

in which i_{1m} is the maximum ac convection-current density which would have been produced by the velocity modulation v_{1m}, if the stream had remained at the velocity u_1. At this point the stream may be suddenly returned to the dc velocity u_1. If this is again done in a distance which is short compared with the quarter space-charge wavelength at the lower velocity, the ac convection current is continuous across the gap, and the stream has returned to the original dc velocity with an ac current modulation which has been amplified from its original value by $(u_1/u_2)^{3/2}$. If the beam is again allowed to drift an odd number of quarter space-charge wavelengths, this current will convert to an ac velocity which is also $(u_1/u_2)^{3/2}$ times its original maximum value.

The dc velocity may be suddenly dropped again and the whole process repeated. Thus each stage consisting of one short low-velocity drift space and one long high-velocity drift space will provide an ac power amplification of $(V_1/V_2)^{3/2}$, where V_1 and V_2 are the dc voltages in the high- and the low-velocity drift spaces, respectively.

Amplification appears to be essentially independent of beam-current density, although the density determines the required lengths of the drift spaces, and the total beam current determines the maximum obtainable ac convection current, and hence the large signal saturation level.

An amplifier based on the above principles has been constructed and has provided a net power gain of 22 db at 3,000 Mc, using a single low-voltage drift region at 51 volts and two helices at 1,900 volts for modulation and demodulation of the stream. With the potential of the center drift region raised to 1,900 volts, the gain changed to zero db. Gain of progressively larger amounts was observed at drift region voltages of 178 volts, 117 volts, 78 volts, and 51 volts corresponding to n quarter space-charge wavelengths in the 5-cm drift space where n was 5, 7, 9, and 11, respectively. The total beam current was 0.7 ma and the approximate beam diameter, 0.15 cm.

At sufficiently low signal frequencies, the effective plasma frequency in the stream is reduced because of the finite beam size, and consequently the gain is reduced. At very high frequencies, it becomes difficult to excite the first-order plasma waves used in the above discussion, and higher-order space-charge waves will appear. It seems that they can be used to give gain, but require longer drift spaces and will saturate at lower power levels.

Finally, it might be mentioned that space-charge waves may be decreased as well as amplified by using similar principles, and where noise exists on the stream in the form of space-charge waves, the noise content in a limited frequency range may be reduced in this fashion. This has been experimentally verified in some low-noise traveling-wave amplifiers.

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* Received by the Institute, November 29, 1950. The research necessary for this correspondence was sponsored by the Office of Naval Research, the United States Army Signal Corps and the United States Air Force.

[1] W. C. Hahn, "Small signal theory of velocity modulated electron beams," *Gen. Elec. Rev.*, vol. 42, pp. 258–270; June, 1939.

[2] S. Ramo, "Space charge and field waves in an electron beam," *Phys. Rev.*, vol. 56, p. 276; August, 1939.

[3] F. B. Llewellyn and L. C. Peterson, "Vacuum-tube networks," PROC. I.R.E., vol. 32, pp. 144–166; March, 1944.

The Traveling-Wave Cathode-Ray Tube*

The paper by K. Owaki, S. Terahata, T. Hada, and T. Nakamura on "The Traveling-Wave Cathode-Ray Tube," in the October, 1950, issue of the PROCEEDINGS OF THE I.R.E. reveals significant progress in the field of microwave oscillography. In order to establish the development of this art—for 20 years one of the writer's hobbies—the following comments may be of some interest.

The prototype of the traveling-wave deflecting system are the multiphase deflecting plates.[1-3] According to Fig. 1(b) and (c), they consist of subsequent pairs of deflecting plates exhibiting alternate polarity due to their criss-cross connections. Maximum

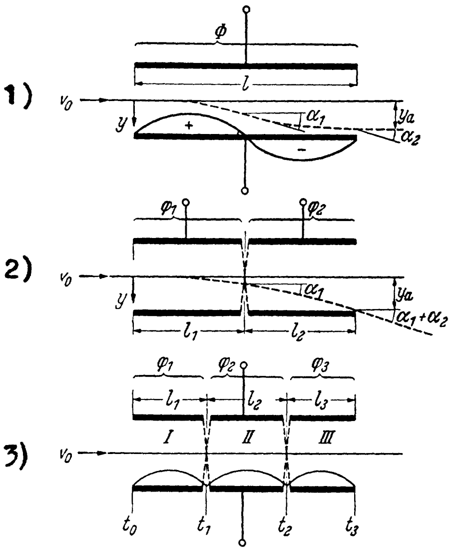

Fig. 1—(1) Single-, (2) two-, and (3) three-phase deflecting field.

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* Received by the Institute, November 8, 1950.

[1] H. E. Hollmann, "Die Quersteuerung, eines Kathodenstrahls in Mehrphasenfeldern," (Deflection of an electron beam in multiphase fields). *Elek. Nach. Tech.*, vol. 15, p. 336; 1938.

[2] H. E. Hollmann, "Das Verhalten der Kathodenstrahlröhre im Laufzeitgebiet," (The behavior of the cathode ray tube in the transit-time range), *Forts. der Hochfrequenz.*, vol. 1, p. 453; 1941.

[3] H. E. Hollmann, "Physik und Technik der Ultrakurzen Wellen," (Physics and technique of vhf), vol. 2, chapter 6, section 3, d; Berlin, 1936.

sensitivity occurs, of course, if frequency and beam velocity are matched in such a manner that the traveling electrons pass the partial fields always whenever they have the same polarity.

The improvement caused by the multiphase deflection, as compared with a single field (Fig. 1(a)) under dc operation, can be expressed by means of the multiphase inversion formulas:

$$P_1 = \frac{\sin\frac{\Phi}{2}}{\frac{\Phi}{2}} = \frac{1}{\Phi}\sqrt{2(1-\cos\Phi)}$$

$$P_2 = \frac{\sin^2\frac{\Phi}{4}}{\frac{\Phi}{4}} = \frac{2}{\Phi}\left(1-\cos\frac{\Phi}{2}\right)$$

$$P_3 = \frac{\sin\frac{\Phi}{2} - 2\sin\frac{\Phi}{6}}{\frac{\Phi}{2}} = P_1 - \frac{4}{\Phi}\sin\frac{\Phi}{6},$$

wherein Φ denotes the transit-time angle over the total deflecting system:

$$\Phi = \frac{\omega l}{v_0} = 2\pi\frac{lc}{\lambda v_0} = \frac{\pi l}{\lambda\sqrt{V_{p\,\text{volts}}}} \times 10^3.$$

(c=velocity of light; v_0=beam velocity; V_p=plate voltage). The function P_1 is the almost classic inversion factor of a single field,[2-6] i.e., the dynamic sensitivity at any vhf referred to the static sensitivity. The functions P_2 and P_3 are the two- and three-phase versions. All three functions are diagrammed in Fig. 2. The two-phase system for dc produces no deflection whatsoever because the first partial field compensates the second field. The curve of the three-phase system starts at $\frac{1}{3}$ because only one partial field remains effective. The loss of static sensitivity, however, is compensated for by the shifting of the dynamic maxima towards higher Φ-values or higher frequencies, respectively. The first P_2-maximum occurs in the vicinity of 2π and P_3 in the vicinity of 3π which, in terms of present-day language, means accord between phase and beam velocity.

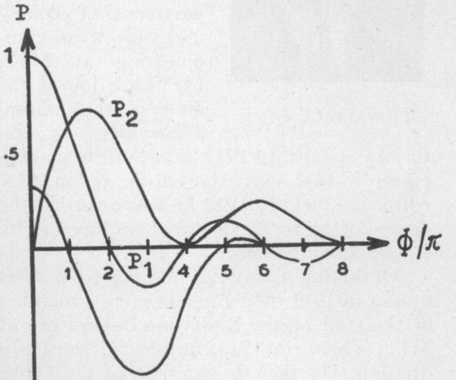

Fig. 2—The dynamic sensitivities of the three systems shown in Fig. 1 versus transit-time angle.

The inversion spectrograph[2,7,8] produces the inversion spectra shown in Fig. 3. The stray fields,[6] not included in the multiphase analysis, assure only a qualitative agreement between formulas and experiment. The fact that the maxima of an N-phase system remain below one and do not appear accurately at $N\pi$ is caused by the transit-time effects of the first kind, i.e., by the transit time elapsing in each individual field as well as by the phase-jumps.

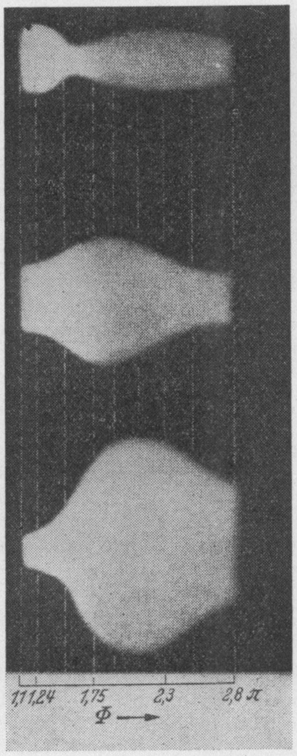

Fig. 3—Experimental inversion spectra of the multiphase systems.

The disadvantage of the earlier multiphase systems with equal and adjacent fields can be overcome by various means. The simplest method is to diminish the axial length of the partial fields so that they operate quasi-statically with sufficient interspace in-between; however, this does not eliminate the stray field effects. Another method was applied by Pierce[9] in his multiphase or traveling-wave oscilloscope, wherein the former phase opposition is reduced by means of lumped-constant circuits, each feeding an individual pair of plates. From this device, only a short step leads to the traveling-wave oscilloscope described in Heaff's patent[10] and by the Japanese authors.

The ultradynamic Lissajous figures shown in the Japanese paper are the same as the writer's figures taken as far back as ten years ago.[2,8,11,12] The writer's method of a graphical analysis may well be applied to the Japanese figures. This may easily be understood because the traveling-wave system eliminates only the transit-time effect of the first kind but does not affect that of the second kind, namely, the transit time between both perpendicular deflecting fields.

All in all, the analogy between the step from the multiphase plates to the traveling-wave oscilloscope on the one hand and the step from the linear electron decelerator to the traveling-wave tube on the other hand is quite obvious.

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[4] H. E. Hollmann, "Die Braunsche Röhre bei sehr hohen Frequenzen" (The cathode-ray tube at vhf), *Zeit. für Hochfrequenz.*, vol. 40, p. 97; 1932.

[5] H. E. Hollmann, "The use of the cathode-ray oscilloscope at ultra-high frequencies," *Wireless Eng.*, vol. 10, pp. 430 and 484; 1933.

[6] H. E. Hollmann, "The dynamic sensitivity and calibration of cathode-ray oscilloscopes at very high frequencies," Proc. I.R.E., vol. 38, p. 32; January, 1950.

[7] H. E. Hollmann, "Das Inversionsspektrum einer Braunschen Röhre," (The inversion spectrum of a cathode-ray tube), *Zeit. für. Tech. Phys.*, vol. 19, p. 259; 1938.

[8] H. E. Hollmann, "Ultra-high frequency oscillography," Proc. I.R.E., vol. 28, p. 213; 1940.

[9] J. R. Pierce, "Traveling-wave oscilloscope," *Electronics*, vol. 22, p. 97; November, 1949.

[10] A. Heaff, U. S. Patent No. 2,064, 469.

[11] H. E. Hollmann, "Ultradynamische Lissajous-Figuren," (Ultradynamic Lissajous figures), *Zeit. für Hochfrequenz.*, vol. 54, p. 19; 1939.

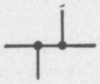
[12] H. E. Hollmann, "Mikrowellen-Oszillographie" (Microwave oscillography), *Zeit. für Hochfrequenz.*, vol. 54, p. 188; 1939.

Representation on Circuit Diagrams of Conductors in Contact*

In Fig. 14 of M. A. Schultz's paper[1] on "Linear Amplifiers," the lead to C26 and J4 (discriminator output) is drawn as making contact with the "ground" line. This is an obvious mistake, caused by putting a spot at the intersection of two conductors.

This occurrence prompts me to draw attention to the recommendation which has appeared for the last sixteen years in the British Standard Specification No. 530:[2]

"Of wires meeting at a connecting point, not more than two should be shown collinear."

They should be shown thus: 

and not: .

In my experience, neglect of this recommendation has been responsible for many mistakes, and I am careful never to draw a cross with a spot at the point of intersection.

I suggest that this recommendation might with advantage be adopted in an American Standard.

It is suggested that readers having comments on this subject address them to the chairman, IRE Symbols Committee, 1 East 79 Street, New York 21, N. Y.—*The Editor.*

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* Received by the Institute, June 22, 1950.

[1] M. A. Schultz, "Linear amplifiers," Proc. I.R.E., vol. 38, pp. 475–485; May, 1950.

[2] For further details, see L. Bainbridge-Bell, "Drawing circuit diagrams," *Wireless World*, vol. 55, pp. 179–180; May, 1949.