

legitimate to assume that identical currents exist in all antennas that are well removed from the ends of the array. A great simplification in the problem is thus realized.

An obvious way to remove array "end effects" altogether is to locate the antennas at the vertices of regular polygons. For identical antennas carrying currents of equal amplitude, only one arbitrary constant appears in the analysis of any such radiating system. Furthermore, an appropriate progressive phase shift from one antenna to the next is easily taken into account.<sup>2</sup>

#### APPENDIX

The integrals (24) to (35) have the following values:

$$F_1(0) \approx \overline{\text{Ci}} 2\beta h + j \text{Si } 2\beta h \quad (42)$$

$$F_1(h) \approx \frac{1}{2} \cos \beta h \{ \overline{\text{Ci}} 4\beta h + j \text{Si } 4\beta h \} \\ - \frac{1}{2} \sin \beta h \{ \text{Si } 4\beta h - j \overline{\text{Ci}} 4\beta h \} \quad (43)$$

$$G_1(0) \approx - \text{Si } 2\beta h + j \overline{\text{Ci}} 2\beta h \quad (44)$$

$$G_1(h) \approx - \frac{1}{2} \cos \beta h \{ \text{Si } 4\beta h - 2 \text{Si } 2\beta h \\ - j \overline{\text{Ci}} 4\beta h + j 2 \overline{\text{Ci}} 2\beta h \} \\ - \frac{1}{2} \sin \beta h \{ \overline{\text{Ci}} 4\beta h + j \text{Si } 4\beta h - j 2 \text{Si } 2\beta h \\ - 2 \overline{\text{Ci}} 2\beta h - 4 \ln 2 \} \quad (45)$$

$$P_1(0) = - \text{Ci } \beta(x+h) + \text{Ci } \beta(x-h) \\ - j \text{Si } \beta(x-h) + j \text{Si } \beta(x+h) \quad (46)$$

$$P_1(h) = - \frac{1}{2} \cos \beta h \{ \text{Ci } \beta(y+2h) - \text{Ci } \beta(y-2h) \} \\ - j \text{Si } \beta(y+2h) + j \text{Si } \beta(y-2h) \} \\ + j \frac{1}{2} \sin \beta h \{ 2 \text{Ci } \beta b - j 2 \text{Si } \beta b \\ - \text{Ci } \beta(y-2h) - \text{Ci } \beta(y+2h) \\ + j \text{Si } \beta(y-2h) + j \text{Si } \beta(y+2h) \} \quad (47)$$

$$Q_1(0) = j \{ 2 \text{Ci } \beta b - j 2 \text{Si } \beta b - \text{Ci } \beta(x+h) \\ - \text{Ci } \beta(x-h) + j \text{Si } \beta(x+h) \\ + j \text{Si } \beta(x-h) \} \quad (48)$$

$$Q_1(h) = \frac{j}{2} \{ e^{-i\beta h} (2 \text{Ci } \beta(x-h) - j 2 \text{Si } \beta(x-h) \\ - \text{Ci } \beta(y-2h) + j \text{Si } \beta(y-2h) \\ - \text{Ci } \beta b + j \text{Si } \beta b) + e^{i\beta h} (2 \text{Ci } \beta(x+h) \\ - j 2 \text{Si } \beta(x+h) - \text{Ci } \beta(y+2h) \\ + j \text{Si } \beta(y+2h) - \text{Ci } \beta b + j \text{Si } \beta b) \}. \quad (49)$$

In the above

$$\text{Ci } z = \int_{\infty}^z \frac{\cos u}{u} du \quad (50)$$

$$\text{Si } z = \int_0^z \frac{\sin u}{u} du \quad (51)$$

$$\overline{\text{Ci}} z = \int_0^z \frac{1 - \cos u}{u} du = 0.5772 + \ln z - \text{Ci } z \quad (52)$$

$$x = \sqrt{(h)^2 + (b)^2} \quad (53)$$

$$y = \sqrt{(2h)^2 + (b)^2}. \quad (54)$$

Observe that  $J_1(0)$  is similar to  $P_1(0)$ ,  $J_1(h)$  is similar to  $P_1(h)$ ,  $K_1(0)$  is similar to  $Q_1(0)$ , and  $K_1(h)$  is similar to  $Q_1(h)$ . For example to obtain  $K_1(h)$  from  $Q_1(h)$ , replace  $x$  by

$$\bar{x} = \sqrt{(h)^2 + (2b)^2} \quad (55)$$

and replace  $y$  by

$$\bar{y} = \sqrt{(2h)^2 + (2b)^2}. \quad (56)$$

Also write  $\text{Ci } \beta b$  and  $\text{Si } \beta b$  as  $\text{Ci } 2\beta b$  and  $\text{Si } 2\beta b$ , respectively.

#### ACKNOWLEDGMENT

This paper has been reviewed by Professor Ronald King, Cruft Laboratory, Graduate School of Engineering, Harvard University. The writer expects to use the material in a thesis on antennas at some future date.

#### Correction

Edward Leonard Ginzton and Arthur E. Harrison have brought to the attention of the Editor an error which appeared in their paper "Reflex-Klystron Oscillators," PROCEEDINGS of the I.R.E., vol. 34, pp. 97-114,

March, 1946. Fig. 5, which is shown on page 101P, should be Fig. 6; Fig. 6, which appears on page 102P, should be Fig. 5.