Discussion of Time Delay in Reference to Electrical Waves*

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Summary—The time delay of an electrical signal through an ultrasonic delay line may be uniquely expressed by either of the two terms "phase delay" or "group delay." There have been devised numerous ways of determining both of these values for use in the field of ultrasonics. Many of the measuring systems do not determine these constants directly but arrive at a value by using what appears to be, in certain instances, a close approximation to the mathematical definition. The purpose of this paper is to discuss some of these measuring systems as to the degree of the approximation involved. In doing so, the measuring system itself is analyzed together with the type of phase-vs-frequency curve for the delay line under consideration.

Introduction

WITHIN THE past several years there have arisen many new uses for ultrasonic delay lines whose delays are specific functions of frequency. The measurement of such ultrasonic lines presents problems which were not met with lines of constant delay. Therefore, with the introduction of new delay-measuring techniques together with the re-evaluation of present techniques, it was felt useful to review the concept of delay itself.

The problem of defining the delay time of an ultrasonic device may be approached in one of two ways. The first approach, and the one which has found most use up to the present time, attempts to define delay time as the interval between two related signals (input and output) after an arbitrary criteria has been established defining the time position of the two signals.[1] The precision obtained in measurements based on this type of definition can be made to correspond to the stability or repeatability of a particular measuring system by first choosing the equipment and then designing the definition to agree with the results. If a given measuring system can be relied on to determine the acceptability of a delay line for a particular application, then stating precisions in accordance with the stability of the test gear is permissible and exemplifies the use of such a definition. However, in view of the fact that the number obtained is a function of many aspects of the delay line, its precision is of no general use or interest.

With the introduction of ultrasonic lines whose delays vary considerably with frequency, the problems involved in the definition of delay become more complex. Therefore, a second definition of delay, based on linear analysis, which is clear and explicit, is chosen as a more fundamental property of an ultrasonic device. Delay time defined in this manner is given the name "group delay," and pertains solely to the delay line and not to some time relation between two signals. The measurement of "group delay" has presented problems in the evaluation of present delay-measuring techniques. In addition, new or improved systems have been designed to cope with the complex problems presented by delay dispersion. The sole requirement of the analysis to be given here is that the ultrasonic devices be considered linear circuit components within their operating range.

Definitions of Phase Delay and Group Delay

As a foundation on which this analysis is to be built, several terms will be defined. At this point these are pure mathematical concepts. The discussion will be restricted to linear systems, not only because it allows relatively simple analysis by use of the work of Laplace and Fourier, but because it accurately describes a wide variety of ultrasonic devices.

The phase shift of a delay line at a particular frequency may be defined as the angle of the "normal voltage transfer function" evaluated at that frequency (Fig. 1). Another voltage transfer function may be defined by an insertion-type measurement, which in many cases is the more useful of the two. When the input and output impedances of the delay line together with the driving and terminating impedances are equal, the transfer function defined from an insertion measurement is one half that of the "normal voltage transfer function." Each delay line, then, has associated with it a characteristic of phase vs frequency which will be determined, for the purposes of this paper, by an insertion-type measurement. This eliminates the necessity of calculating the effect of any terminating networks which are associated with the line and must be considered as part of the line for impedance-matching purposes. If information is desired on the phase characteristic of the line itself less transducers and/or terminating networks, insertion-type measurements may be made and the necessary calculations performed.

Now that the term "phase shift" has been established, "phase delay" may be defined as

$$T_p = \frac{\Phi(\omega)}{\omega} \tag{1}$$

where Φ is the total phase shift in radians at angular frequency ω. Since no one cycle of a pure sine wave is distinguishable from any other cycle by virtue of its periodicity, the phase shift is ambiguous to any multiple

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[1] D. L. Arenberg, "Measurements of delay in ultrasonic systems," 1958 IRE National Convention Record, pt. 2, pp. 121–133.

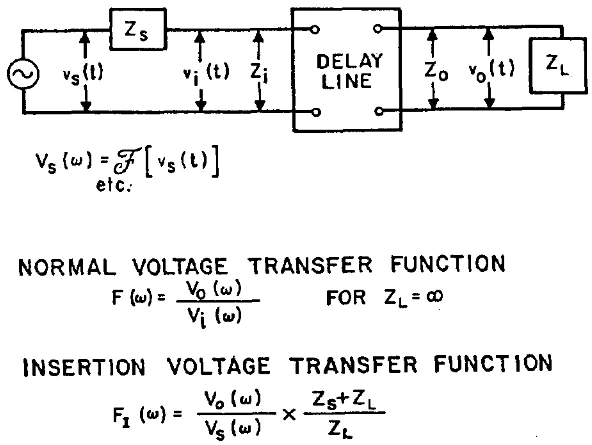

Fig. 1—Definition of transfer function.

of 2π radians. Thus the phase delay itself contains a factor of $2\pi n/\omega$ which is of no interest to the electrical operation of the line. From the physical standpoint this factor may be important and real, but it must be determined from other considerations.

The definition of "group delay" now follows as a simple derivative with respect to angular frequency, *i.e.*,

$$T_g = \frac{+d}{d\omega}\Phi(\omega). \tag{2}$$

The group delay evaluated at a particular frequency then becomes the slope of the phase-vs-frequency characteristic at this frequency. The derivative and its meaning will be further discussed as it takes on physical significance with the application of various signals to specific types of delay lines.

Types and Characteristics of Lines to be Discussed

In order to discuss, in any analytical sense, specific delay-measuring techniques, certain assumptions must be made as to the phase and attenuation characteristics of the delay lines under test. The lines will contain a pass band, normally assumed flat with frequency, over which it is desired to determine the phase and/or delay characteristic. With lines possessing more than one important mode of transmission, it may be desired to measure the delay of each mode separately. In most cases, however, the delay of the preferred or main mode is all that is required as the device has been carefully designed to suppress unwanted modes. Certain delay-measuring systems will determine only the delay of the main mode without the interfering effects of other modes regardless of their intensity. Mention will be made of these systems as they are discussed. For the present it will be assumed that only one mode of transmission exists.

All delay lines will fall under one of the two headings "dispersive" or "nondispersive." Lines containing a linear phase characteristic over some interval will be termed "nondispersive" over that interval. By definition, then, all other lines will be termed "dispersive" regardless of their phase relation with frequency. The only lines which will be considered in detail are those which possess either a linear or quadratic phase characteristic over the bandwidth of the signal applied.

Nondispersive Ultrasonic Delay Lines

A nondispersive-type delay line may be characterized by the following phase relation with frequency:

$$\Phi(\omega) = K_1\omega + K_0 \tag{3}$$

where K_1 and K_0 are constants. The general linear transfer function of such a device then becomes

$$\begin{aligned} F(\omega) &= A(\omega)e^{-j\Phi} \\ &= A(\omega)e^{-j[K_1\omega + K_0]} \end{aligned} \tag{4}$$

where

$$A(\omega) \text{ is real.}$$

"Phase delay" and "group delay," respectively, may be evaluated directly from the phase characteristic according to their definitions, *i.e.*,

$$T_p = \frac{\Phi(\omega)}{\omega} = K_1 + \frac{K_0}{\omega}, \tag{5}$$

$$T_g = \frac{d\Phi(\omega)}{d\omega} = K_1. \tag{6}$$

Notice that phase delay and group delay are equal only for the condition of $K_0 = 0$.

The presence of a reflected signal will cause the phase characteristic to contain rapid fluctuations about the average curve. In pulse-type applications where the main signal is separated in time from the unwanted signals, these fluctuations are not present. This is mentioned at this point since these fluctuations are troublesome at times in delay-measuring systems using continuous input signals.

Pi-Point Method of Delay Measurement

The pi-point method is a convenient means of determining the phase-vs-frequency relationship of electrical components which possess appreciable amounts of group delay (Fig. 2). Frequencies at which the phase relation is some multiple of pi radians are most easily measured and consequently plotted, from which the system derives its name. If the phase curve is known to be linear, then it is necessary to determine only two points in order to define the characteristic. In general, many pi-points are plotted to confirm the assumption made, or, as in most cases, to evaluate the nonlinearity. The slope of this phase curve at any point, or over the entire band in the case of linearity, is then the group delay of the line

$$\frac{n\pi}{\omega_2 - \omega_1} = \frac{\Delta\Phi}{\Delta\omega} = \frac{d\Phi(\omega)}{d\omega} = T_g \tag{7}$$

where n is the number of pi-points between the frequencies ω_1 and ω_2.

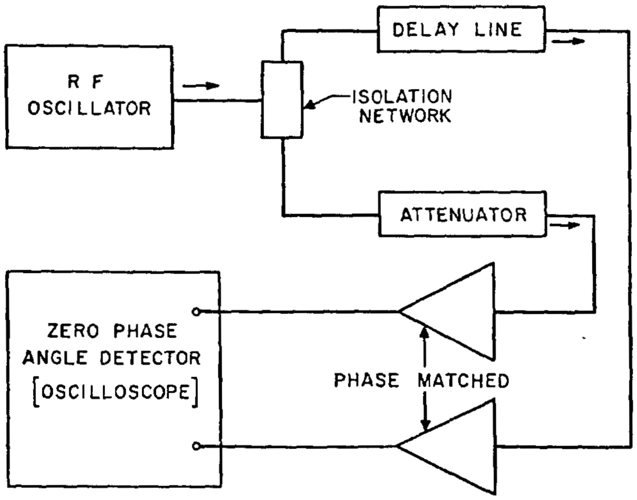

Fig. 2—Pi-point method of delay measurement.

The precision of this measurement depends upon the precision with which the frequency of a single pi-point may be determined, together with the number of pi-points between a set of readings. The accuracy of the measurement is a function of the linearity of the phase characteristic including fluctuations due to reflections.

RF Pulse Method Using Cycle-to-Cycle Alignment

This method has been used a great deal in the measurement of nondispersive delay lines because of its high degree of precision and the elimination of effects due to reflected signals and unwanted modes. Its use in determining the total group delay of a line, as is being discussed here, is highly restricted and must be used with caution. The essence of the system (Fig. 3) is as follows. A burst of RF energy is applied to the delay line and an attenuator simultaneously. The output pulse from the line and the attenuator are applied to the vertical input of an oscilloscope whose horizontal axis is being swept by a separate oscillator. The burst repetition rate is locked to a submultiple of this frequency, which is then carefully adjusted until the two pulses appearing on the oscilloscope are superimposed, with a final adjustment being made by examining the RF phase. The time delay is then stated to be a particular multiple of the period of the horizontal sweep oscillator.[2]

The limitations imposed on this method, when measuring group delay, may be shown by the following analysis of the transmission of an RF burst or pulse. Consider a burst of RF energy which may be represented by a carrier which is amplitude modulated by a pulse. The carrier and the pulse are given respectively as

$$e^{j\omega_c t} \quad \text{(complex)} \tag{8}$$

and

$$s(t) = \frac{1}{2\pi}\int_{-\omega_m}^{+\omega_m} S(\omega)e^{j\omega t}\,d\omega = \mathfrak{F}^{-1}S(\omega) \quad \text{(real)} \tag{9}$$

[2] J. E. May, "Precise measurement of time delay," 1958 IRE National Convention Record, pt. 2, pp. 134-142.

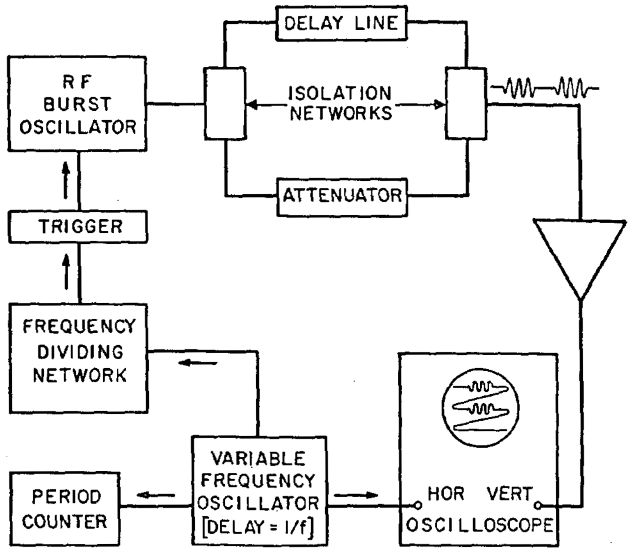

Fig. 3—RF-pulse method of delay measurement.

where

$$2\omega_m \leq \text{bandwidth of delay line}$$

and

$$\omega_m < \omega_c.$$

The Fourier transform representation will be used to express the signal. For simplicity the amplitude of the carrier has been taken as twice the positive frequency content of the real signal cos $\omega_c t$. The real part of the complex output signal would be the signal obtained if cos $\omega_c t$ had been used instead of $e^{j\omega_c t}$. The frequency spectrum of the pulse has been limited in such a way that the entire spectrum after modulation will be contained within the pass band of the delay line. If this were not the case, amplitude distortion would take place due to bandlimiting caused by the delay line and a more complicated analysis would be necessary. The conclusions reached in regard to the limitations imposed on the system under discussion are quite general and will not be affected by the assumption made.

The input signal is obtained by multiplication in the time domain of the carrier and the pulse envelope. Thus the input signal becomes

$$e(t) = s(t)e^{j\omega_c t} \quad \text{(complex)}$$
$$= \frac{1}{2\pi}\int_{-\omega_m}^{+\omega_m} S(\omega)e^{j(\omega+\omega_c)t}\,d\omega. \tag{10}$$

A new variable is defined, $w = \omega + \omega_c$, and substituting this into (10) yields the input signal

$$e(t) = \frac{1}{2\pi}\int_{\omega_c-\omega_m}^{\omega_c+\omega_m} S(w-\omega_c)e^{jwt}\,dw. \tag{11}$$

This signal is contained entirely within the pass band of the delay line so the appropriate delay line transfer function may be applied to (11) giving the output signal.

The delay line transfer function may be written as

$$F(\omega) = e^{-j(K_1\omega + K_0)}, \tag{12}$$

and the output signal becomes

$$v(t) = \frac{1}{2\pi}\int_{\omega_c-\omega_m}^{\omega_c+\omega_m} S(w-\omega_c)e^{jw(t-K_1)-jK_0}\,dw. \tag{13}$$

Now in (13) make the inverse substitution $\omega = w - \omega_c$, yielding

$$v(t) = \frac{1}{2\pi}e^{j\omega_c(t-K_1)-jK_0}\int_{-\omega_m}^{+\omega_m} S(\omega)e^{j\omega(t-K_1)}\,d\omega. \tag{14}$$

The integral which represents the envelope is back at base-band frequency and the signal may again be considered as a carrier modulated by a particular pulse envelope.

There are two cases to consider depending on the value of K_0. In the first case for $K_0 = 0$ or $2\pi n$ (n being any positive or negative integer) the output signal is identical to the input signal except for a new variable in time indicating a time delay equal to K_1, which in turn is the group delay of the line. For all other values of K_0 the second condition prevails. The carrier now has an additional phase shift of K_0 radians relative to the envelope. This means that any group-delay-measuring system depending on the alignment of the carrier wave rather than merely the envelope itself, is susceptible to an error of τ sec in the measurement of group delay, where

$$-\frac{\pi}{\omega_c} < \tau < \frac{\pi}{\omega_c}.$$

Dispersive-Type Ultrasonic Delay Lines

Ultrasonic delay lines having a phase characteristic which is not a linear function with frequency fall under the general category of "dispersive"-type delay lines. This dispersion, or nonconstant group delay, is often an undesired effect which is present to some extent in lines which are designed to be nondispersive. On the other hand, many important applications have been found for delay lines having very specific nonlinearities in their phase characteristics. In most instances these applications involve replacing a linear electrical network which could in theory perform the same function, but from the practical standpoint has serious disadvantages as compared to the ultrasonic line. The problems involved in determining the type and amount of nonlinearity can become quite complex when stringent requirements are to be met. Some of these problems will be discussed in the following paragraphs referring to several specific measuring systems which have been used.

Pi-Point Method of Delay Measurement

One possible approach to the problem is merely to plot the phase characteristic of the delay line and from this information determine the average slope or group delay over any region. The pi-point method, though it may be used to determine group delay directly, is in essence only a convenient way of plotting phase. To determine group delay directly two pi-points are chosen at ω_1 and ω_2 in such a way that they enclose the frequency of interest ω_c, *i.e.*,

$$\omega_1 < \omega_c < \omega_2.$$

The following argument is then performed:

$$\frac{m\pi}{\omega_2-\omega_1} = \frac{\Delta\Phi}{\Delta\omega} = \frac{d\Phi}{d\omega}\bigg|_{\omega=\omega_0} = T_g(\omega_0) \tag{15}$$

for $\omega_1 < \omega_0 < \omega_2$ where m is the number of pi-points between ω_1 and ω_2. The necessity for a compromise in the value of m chosen becomes at once evident. As the frequency interval is widened, the precision in the evaluation of $\Delta\Phi/\Delta\omega$ increases. However, to offset this effect, the uncertainty between the value of ω_c and that of ω_0 also increases. Thus, if the delay characteristic contains rapid fluctuations which are to be detected in their proper position and magnitude, it is necessary that $\Delta\omega$ be much less than the minimum period of these variations. For delay characteristics which do not exhibit rapid fluctuations, large frequency intervals may be taken since the difference of ω_c and ω_0 can be taken into account.

The pi-point method possesses several distinct disadvantages as compared with other delay-measuring techniques when used on highly dispersive lines. This method is relatively time consuming, not only because of the point-by-point measurements, but also due to the calculations which are required to arrive at group delay. The next two disadvantages are a result of the close frequency spacing required between consecutive readings in order to plot a highly nonlinear phase curve. It becomes difficult to establish a single pi-point with sufficient precision in both angle and frequency so that an accurate determination of the slope between two consecutive pi-points may be made. The third disadvantage is common to all systems using continuous input signals, as was mentioned previously. A reflected signal causes ripples in the phase characteristic of a delay line with a periodicity in frequency given to a first approximation by $1/2T_g$. In order to avoid this problem in the phase plot, one would like to take phase readings with the same periodicity as the ripples. At first glance this is seen to be accomplished by the pi-point method. However, the period of the phase ripples does not correspond exactly to that of the pi-points due to the location of the boundaries at which reflections take place, thus allowing for the possibility of errors.

Envelope Delay

The term "envelope delay" is frequently used synonymously with the term "group delay." For the purposes of this paper it will be helpful to clarify the usage of this term. Any real signal in the time domain $x(t)$ may be represented by a complex notation in terms of envelope and angle. A complex time function $z(t)$ corresponding to the real time function $x(t)$ may be defined by the following equations where $X(\omega)$, $Y(\omega)$, and $Z(\omega)$ are the Fourier transforms of $x(t)$, $y(t)$, and $z(t)$, respectively:

$$Z(\omega) = X(\omega) + jY(\omega), \tag{16}$$

$$Y(\omega) = \begin{matrix} -jX(\omega) & \text{for} & \omega > 0 \\ +jX(\omega) & \text{for} & \omega < 0. \end{matrix} \tag{17}$$

It may be shown that the following time relations will exist:

$$z(t) = x(t) + jy(t) = r(t)e^{j\Phi(t)}, \tag{18}$$

$$y(t) = \frac{1}{\pi} x(t) \circledast \frac{1}{t} = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t - \tau} d\tau \quad \text{[Hilbert Transform]}. \tag{19}$$

The envelope of $x(t)$ is then defined as the magnitude of the complex function $z(t)$,[3,4]

$$r(t) = [x(t)^2 + y(t)^2]^{1/2}. \tag{20}$$

The reason for defining the envelope in this way is that narrow-band waveforms (such as double- or single-sideband *AM*) appear to be a sinusoidal function with a slowly varying amplitude which conforms with this definition. It is only under such conditions that the envelope has any physical significance. The problem of whether or not particular types of envelope detectors and demodulators will give an output equivalent to the mathematical envelope must be considered on the basis of this definition.

It is entirely possible that the output envelope of a given device will be a completely different time function than the input envelope. A time reference must then be arbitrarily chosen on the output as well as the input waveform if envelope delay is to be considered. Thus, while group delay is unambiguously defined at each and every frequency, envelope delay is not only arbitrary at times, but also requires the presence of at least two frequencies. In spite of the problems just mentioned, the envelope of a real signal may be used in a variety of systems to determine group delay quite accurately.

Sine Wave Modulation Delay-Measuring Technique

The first system which would most logically fall under the category of envelope-type measurements is one using a cosine wave amplitude-modulated carrier of the form shown below. The input signal is

$$e(t) = E(1 + m_a \cos \omega_m t) \cos \omega_c t, \tag{21}$$

where m_a is the index of modulation, ω_m is the modulating frequency and ω_c is the carrier frequency. For convenience the following are defined:

$$E = 1,$$

$$\omega_1 = \omega_c - \omega_m, \quad \text{the lower sideband, and}$$

$$\omega_2 = \omega_c + \omega_m, \quad \text{the upper sideband.}$$

The input signal is then

$$e(t) = \cos \omega_c t + \tfrac{1}{2} m_a \cos \omega_1 t + \tfrac{1}{2} m_a \cos \omega_2 t. \tag{22}$$

This signal is composed of three distinct frequencies, each of which will be altered in phase and amplitude as defined by the transfer function of the delay line. Any attenuation of the carrier relative to the sidebands may be taken into account by a corresponding change in the modulation index and thus will not be considered. The result may be analyzed in three separate cases.

Case 1 (No Distortion): This condition is obtained only with a nondispersive delay line that attenuates both sidebands equally. The output signal, as derived in the Appendix, is shown to be

$$v(t) = \left[1 + m_a \cos\left(\omega_m t + \frac{\Phi_2 - \Phi_1}{2}\right)\right] \cdot \cos\left(\omega_c t + \frac{\Phi_2 + \Phi_1}{2}\right). \tag{23}$$

Notice that the phase delay of the envelope $[-\Phi_2 + \Phi_1/2\omega_m]$ is equal to the group delay $T_g(\omega)$ for $\omega_1 < \omega_c < \omega_2$. Thus the group delay in the vicinity of the carrier frequency, and hence everywhere, may be evaluated directly by measuring the phase delay of the envelope.

Case 2 (Phase Distortion): In essence this is the condition which exists when a dispersive-type line is used. It will be shown that in many cases what may be considered the phase delay of the envelope is a very good approximation to the group delay at the carrier frequency, thus justifying the usefulness of this method for dispersive line measurements. The mathematical envelope has been derived in the Appendix and is seen to have a distortion factor dependent on the phase shift of the carrier relative to the average phase shift of the sidebands ($\delta\Phi$). The envelope may be represented as

$$\left| \cos \delta\Phi + m_a \cos\left(\omega_m t + \frac{\Phi_2 - \Phi_1}{2}\right) + j \sin \delta\Phi \right| \tag{24}$$

where $\delta\Phi$ is the relative phase shift just mentioned. This distortion is of such a nature that the side-to-side symmetry of the envelope wave shape is maintained. It is possible then to define the time position of the output envelope merely by neglecting this distortion term. Envelope delay, as in Case 1, remains defined as $-\Phi_2 + \Phi_1/2\omega_m$, which in turn is equal to $T_g(\omega)$ for $\omega_1 < \omega < \omega_2$. Notice that ω is in general not equal to ω_c.

[3] M. F. Gardner and J. L. Barnes, "Transients in Linear Systems," John Wiley & Sons, Inc., New York, N. Y., vol. 1; 1957.
[4] E. Weber, "Linear Transient Analysis," John Wiley & Sons, Inc., New York, N. Y., vol. 1; 1954.

Case 3 (Amplitude Distortion): In all measurements involving a signal with a finite bandwidth some amplitude distortion is inevitably present. This effect is treated briefly in the Appendix where the mathematical envelope is derived showing the type and amount of distortion involved. It turns out that the distortion is again of the symmetrical type and may be treated similarly to that caused by phase distortion.

To sum up the conclusions which have been reached, it may be said that with a nondispersive line the phase delay of the envelope is the group delay of the line. However, the real advantage of the sine wave modulation system is in the measurement of dispersive lines. The delay of the envelope now becomes the group delay evaluated at a frequency which is confined somewhere between the upper and lower sidebands. Another way to state the result is to say that the phase delay of the envelope is the average group delay over the frequency interval ω_1 to ω_2. This is seen by writing the average group delay, *i.e.*,

$$\frac{1}{2\omega_m}\int_{\omega_1}^{\omega_2} T_g(\omega)\,d\omega = \frac{-\Phi_2 + \Phi_1}{2\omega_m}. \tag{25}$$

Analysis of Pulse Delay in a Dispersive System

The signal which will be used is a fairly general burst of RF energy. Several qualifications will be put on the waveform which are justified in practical systems and which will simplify the analysis. The signal may be considered as a carrier which is amplitude modulated by a pulse. The carrier and the pulse are given respectively as

$$e^{j\omega_c t} \qquad \text{(complex)} \tag{26}$$

and

$$s(t) = \mathfrak{F}^{-1}[S(\omega)]$$
$$= \frac{1}{2\pi}\int_{-\omega_m}^{+\omega_m} S(\omega)e^{j\omega t}\,d\omega \quad \text{(real)} \tag{27}$$

where

$$s(t) > 0 \text{ for all } t$$

and

$$\omega_m \ll \omega_c.$$

The pulse is bandlimited in such a way that after linear modulation the entire signal will be in the positive frequency domain. In most practical applications using pulse techniques $2\omega_m \ll$ total bandwidth of the delay line. The pulse is further assumed to be a real function in time symmetric about the point $t = 0$.

For $s(t)$ real:

$$S(\omega) = \int_{-\infty}^{+\infty} s(t)e^{-j\omega t}\,dt$$
$$S(-\omega) = \int_{-\infty}^{+\infty} s(t)e^{+j\omega t}\,dt = S^*(\omega). \tag{28}$$

For $s(t)$ symmetric about $t = 0$: $s(t) = s(-t)$

$$\int_{-\omega_m}^{+\omega_m} S(\omega)e^{j\omega t}\,d\omega = -\int_{+\omega_m}^{-\omega_m} S(-\omega)e^{j\omega t}\,d\omega$$
$$= \int_{-\omega_m}^{+\omega_m} S(-\omega)e^{j\omega t}\,d\omega,$$
$$S(\omega) = S(-\omega). \tag{29}$$

Hence $S(\omega)$ is also real and symmetrical about $\omega = 0$. The input signal is the product of the carrier and the pulse, *i.e.*,

$$e(t) = s(t)e^{j\omega_c t}$$
$$= \frac{1}{2\pi}\int_{-\omega_m}^{+\omega_m} S(\omega)e^{j(\omega+\omega_c)t}\,d\omega. \tag{30}$$

Since this signal is merely twice the positive frequency content of the real signal formed by modulating $\cos \omega_c t$ rather than $e^{j\omega_c t}$, the envelope may be given as

$$|\, s(t)e^{j\omega_c t} \,| = s(t). \tag{31}$$

A new variable is defined, $w = \omega + \omega_c$, and substituting this in (30), yields the input

$$e(t) = \frac{1}{2\pi}\int_{\omega_c-\omega_m}^{\omega_c+\omega_m} S(w - \omega_c)e^{jwt}\,dw. \tag{32}$$

The transfer function of the delay line which will be considered may be given as

$$F(\omega) = e^{-j[K_2\omega^2+K_1\omega+K_0]}. \tag{33}$$

The output signal is obtained by applying the transfer function to the input function, *i.e.*,

$$v(t) = \frac{1}{2\pi}\int_{\omega_c-\omega_m}^{\omega_c+\omega_m} S(w - \omega_c)e^{j(wt-K_2w^2-K_1w-K_0)}\,dw. \tag{34}$$

Making the inverse substitution $\omega = w - \omega_c$ in (34) yields the output

$$v(t) = \frac{1}{2\pi}e^{+j\omega_c(t-K_2\omega_c-K_1)}e^{-jK_0}$$
$$\cdot\int_{-\omega_m}^{+\omega_m} S(\omega)e^{-jK_2\omega^2}e^{j\omega(t-2K_2\omega_c-K_1)}\,d\omega. \tag{35}$$

The output envelope according to the definition set forth may be expressed as

$$s(t) = \frac{1}{2\pi}\left|\int_{-\omega_m}^{+\omega_m} S(\omega)e^{-jK_2\omega^2}e^{j\omega(t-2K_2\omega_c-K_1)}\,d\omega\right|$$
$$= \frac{1}{2\pi}\left|\int_{-\omega_m}^{+\omega_m} S(\omega)\cos(K_2\omega^2)e^{j\omega(t-\tau)}\,d\omega - j\int_{-\omega_m}^{+\omega_m} S(\omega)\sin(K_2\omega^2)e^{j\omega(t-\tau)}\,d\omega\right|$$
$$= \frac{1}{2\pi}\,|\, x(t-\tau) - jy(t-\tau) \,| \tag{36}$$

where

$$\tau = 2K_2\omega_c + K_1 = T_g(\omega_c).$$

Since $S(\omega)$ cos $(K_2\omega^2)$ is a real function symmetric about $\omega = 0$, it is implied that $x(t - \tau)$ is real and symmetric about $t - \tau = 0$. The same argument will hold for $y(t - \tau)$. Therefore, the output envelope though changed in shape still contains symmetry about $t = 2K_2\omega_c + K_1$, and the time delay between pulses may be measured taking advantage of this symmetry.

Group Delay Measurements by Narrow-Band RF Pulse Techniques

The particular envelope which has been found most advantageous for group delay measurements on dispersive delay lines using pulse techniques is the Gaussian or Normal Distribution curve. This particular function has the well-known property of possessing the same shape in the frequency and time domains.[5] The Gaussian envelope contour was chosen because it is mathematically smooth and well confined in both domains. The bandwidth possessed by the pulse may be taken as six times the standard deviation in the frequency domain. Two delay-measuring systems will be discussed which, according to this definition, use pulses whose bandwidths range from 4 per cent to 12 per cent of the total line bandwidth. These systems were designed to plot the group delay vs frequency characteristics of delay lines containing a large amount of linear dispersion (quadratic phase) together with second-order effects of higher degree dispersion. The correlation in readings between the two systems is in the range of 0.01 per cent or better of the total delay at any frequency within the band.

The first technique measures insertion group delay directly by determining the envelope delay of the line when the signal is a Gaussian-shaped burst of RF frequency (Fig. 4). The input signal is applied simultaneously to the delay line and an attenuator through an isolation network. The envelopes of the output signals from the line and the attenuator are detected and applied to the vertical axis of an oscilloscope. The horizontal axis is swept by an audio oscillator whose frequency is then adjusted for perfect envelope alignment. The period of the audio oscillator is the group delay of the line at the particular carrier frequency used. The pulse repetition rate is locked to a submultiple of the audio frequency to eliminate interference caused by reflected signals and unwanted modes of transmission. As some pulse spreading in the time domain will take place as a result of delay dispersion, readings are made on the leading and trailing portions of the envelope and then averaged. Because of the visual limitations involved in aligning the envelopes, it is only possible to repeat a single measurement to several

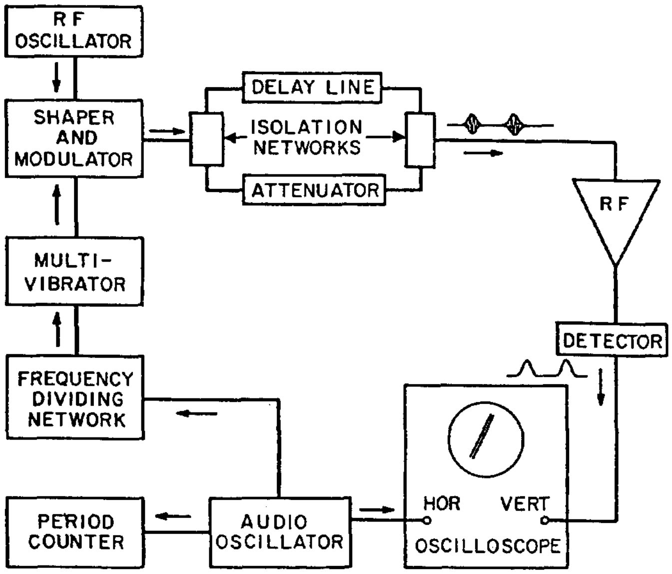

Fig. 4—Group delay measurement by envelope alignment.

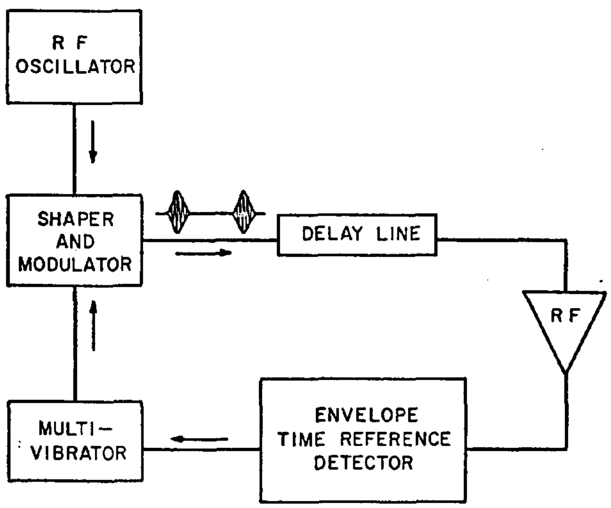

FIG. 5

Fig. 5—Group delay measurement by closed-loop pulse repetition rate.

hundredths of a per cent. A group of readings is therefore taken at each frequency and averaged to increase the precision.

The second system using a narrow-band RF pulse was adapted primarily for its speed and convenience combined with a high degree of precision in the comparison of two similar delay lines (Fig. 5). The system uses a regenerative-type closed loop, whereby the peak of the output envelope is used as a time reference to generate a subsequent pulse. The repetition rate within the closed loop is measured to determine the total loop delay, of which the delay line itself is the major portion. The remaining delay may be associated with the regenerating and amplifying equipment and is carefully designed to be independent of as many variables as possible. The system is best adapted to measuring the difference of delay be-

[5] A. A. Kharkevich, "Spectra and Analysis," Consultants Bureau, New York, N. Y.; 1960.

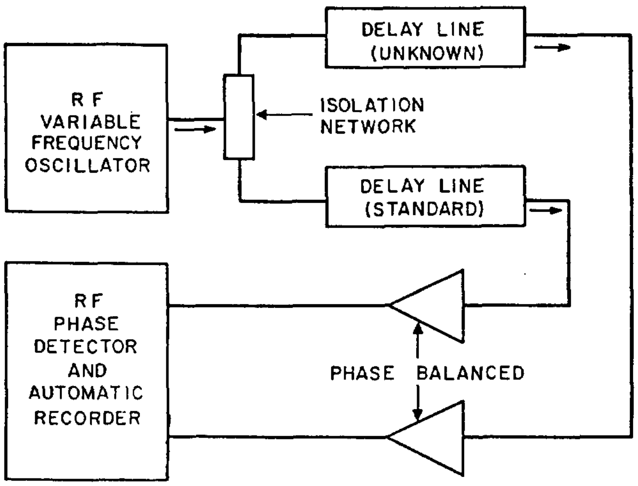

Fig. 6—Phase-difference method of delay line comparison.

tween two similar lines. It is capable of repeating measurements to 0.001 per cent or better. Under these conditions the limiting factor in its accuracy appears to be pulse distortion. The delay added by the system components is sufficient to eliminate the effects of the major reflected signal known as "third time around".

Group Delay Measurements by Comparison

In the process of manufacturing lines it becomes important to be able to accurately reproduce a given phase or group delay characteristic. As was just mentioned, the regenerative pulse system offers excellent precision for comparison-type measurements. Another way of comparing lines is to plot the phase-difference curve (Fig. 6). Using a continuous sine wave signal, the output of a standard line is compared in phase to that of an unknown line. For accuracy and convenience it is possible to automatically record this phase-difference characteristic as a function of frequency across the entire bandwidth of the delay lines. The slope of the curve at any frequency represents the difference in group delay of the two lines at that point. This method offers a fairly simple and quick measurement of the average delay difference between two similar but highly dispersive lines.

Conclusions

An ultrasonic delay line which fulfills the requirements of a linear circuit element may be completely specified by a transfer function. This function contains amplitude and angle. The angle plotted as a function of frequency is known as the phase characteristic of the line. When this phase characteristic is differentiated with respect to frequency a new function is formed having units of time and given the name "group delay." It is this property of a delay line which is most useful in electrical circuits.

Under certain conditions the group delay of a line manifests itself in the form of what may be called an envelope delay. The conditions for defining an envelope have been discussed so that a clear correlation may be made between group delay and envelope delay. Several systems have been mentioned which, by using envelope methods, give accurate measurements of group delay for nondispersive delay lines. One of the most serious problems arising in the measurement of dispersive lines is the distortion of the signal envelope, thus making it difficult to determine time positions of the input and output wave which will give results corresponding to group delay. The combination of phase distortion due to dispersion and attenuation distortion sets very definite limits on the accuracy of any particular type of envelope measurement. Some of the problems have been minimized by selecting narrow-band mathematically smooth time functions for the envelopes of the input signals. These methods have proved quite successful as fairly rapid and accurate means of plotting group delay vs frequency.

Appendix

Determination of the Mathematical Envelope of an Amplitude-Modulated Signal in the Presence of Phase Distortion

Delay line input signal:

$$e(t) = (1 + m_a \cos \omega_m t) \cos \omega_c t, \qquad m_a < 1.$$

Delay line transfer function:

$$F(\omega) = e^{j\Phi_1} \quad \text{at} \quad \omega_1 = \omega_c - \omega_m,$$

$$F(\omega) = e^{j\Phi_c} \quad \text{at} \quad \omega_c$$

$$F(\omega) = e^{j\Phi_2} \quad \text{at} \quad \omega_2 = \omega_c + \omega_m,$$

$$2\,\delta\Phi = 2\Phi_c - \Phi_1 - \Phi_2.$$

By applying this transfer function to the input signal, the output signal becomes

$$v(t) = \cos\left(\omega_c t + \frac{\Phi_2 + \Phi_1}{2} + \delta\Phi\right)$$

$$+ m_a \cos\left(\omega_c t + \frac{\Phi_2 + \Phi_1}{2}\right) \cos\left(\omega_m t + \frac{\Phi_2 - \Phi_1}{2}\right).$$

The envelope of the output signal is given by twice the magnitude of the positive frequency components.

$$s(t) = \Bigg| e^{j(\omega_c t + [\Phi_2 + \Phi_1/2])} e^{j\delta\Phi}$$

$$+ m_a \cos\left(\omega_m t + \frac{\Phi_2 - \Phi_1}{2}\right) e^{j(\omega_c t + [\Phi_2 + \Phi_1/2])} \Bigg|$$

$$= \left| \cos \delta\Phi + m_a \cos\left(\omega_m t + \frac{\Phi_2 - \Phi_1}{2}\right) + j \sin \delta\Phi \right|.$$

Notice that the envelope is symmetric in time about the point $t = [-\Phi_2 - \Phi_1]/2\omega_m$ which represents the group delay at some frequency ω located between ω_2 and ω_1.

For the condition of no distortion, merely let $\delta\Phi = 0$.

Then the output envelope, as given below, is identical to the input envelope, except for a displacement of $[\Phi_2 - \Phi_1]/2\omega_m$ in time,

$$s(t) = 1 + m_a \cos\left(\omega_m t + \frac{\Phi_2 - \Phi_1}{2}\right).$$

Determination of the Mathematical Envelope of an Amplitude-Modulated Signal in the Presence of Attenuation Distortion

Delay line input signal:

$$e(t) = (1 + m_a \cos \omega_m t) \cos \omega_c t, \qquad m_a < 1.$$

Delay line transfer function:

$$F(\omega) = \begin{cases} (1 - K)e^{j\Phi_1} & \text{at } \omega_1 = \omega_c - \omega_m \text{ for } K < 1 \\ e^{j\Phi_c} & \text{at } \omega_c, \\ (1 + K)e^{j\Phi_2} & \text{at } \omega_2 = \omega_c + \omega_m, \end{cases}$$

$$2\Phi_c = \Phi_2 + \Phi_1.$$

By applying this transfer function to the input signal, the output signal becomes

$$v(t) = \cos(\omega_c t + \Phi_c) + \frac{(1 + K)}{2} m_a \cos(\omega_2 t + \Phi_2) + \frac{(1 - K)}{2} m_a \cos(\omega_1 t + \Phi_1)$$

$$v(t) = \left[1 + m_a \cos\left(\omega_m t + \frac{\Phi_2 - \Phi_1}{2}\right)\right] \cdot \cos\left(\omega_c t + \frac{\Phi_2 + \Phi_1}{2}\right) - K m_a \cdot \sin\left(\omega_c t + \frac{\Phi_2 + \Phi_1}{2}\right) \sin\left(\omega_m t + \frac{\Phi_2 - \Phi_1}{2}\right).$$

The envelope of the output signal is given as

$$s(t) = \left| 1 + m_a \cos\left(\omega_m t + \frac{\Phi_2 - \Phi_1}{2}\right) + jK m_a \sin\left(\omega_m t + \frac{\Phi_2 - \Phi_1}{2}\right) \right|.$$

Notice that the envelope is symmetric in time about the point $t = [-\Phi_2 - \Phi_1]/2\omega_m$ which represents the group delay at some frequency ω located between ω_2 and ω_1.

The undistorted envelope may be obtained by letting $K = 0$.

The Depletion Layer Transducer*

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Summary—**The depletion layer transducer is an ultrasonic transducer for use at UHF and microwave frequencies. Its potential advantages are high efficiency, large bandwidth, and comparative simplicity in fabrication. The region which generates or detects the ultrasonic waves is a thin flat high-resistance depletion layer, such as a *p-n* junction or a rectifying metal to semiconductor contact, in an extrinsic piezoelectric semiconductor. When an ac voltage is applied to the material, the depletion layer behaves in a manner similar to an extremely thin piezoelectric crystal bonded to the conducting substrate. Since the depletion layer can be generated on the surface of the semiconductor, the problems of handling extremely thin piezoelectric plates are avoided. Depletion layer transducers have worked at frequencies as high as 1000 Mc. It is anticipated that improvements in circuit and fabrication techniques will greatly extend their frequency range and efficiency.**

I. Introduction

THE ULTRASONIC depletion layer transducer is a new transducer for converting electrical energy into ultrasonic energy and vice versa. Its frequency range extends from near 300 Mc to perhaps higher than 10,000 Mc.

The electromechanical coupling is due to the piezoelectric effect in a material which is simultaneously an extrinsic semiconductor. Examples of such materials are gallium arsenide and cadmium sulfide. The novel feature of this transducer is that the electromechanically active portion is a high-resistance depletion layer such as a *p-n* junction or rectifying metal-semiconductor contact instead of the high resistance bulk material of ordinary piezoelectric transducer material such as quartz. As will be shown in the text, a flat depletion layer formed on the surface of a low-resistivity piezoelectric semiconductor behaves very similarly to a plate of high-resistivity piezoelectric material bonded to a metal substrate.

The two unique advantages of a depletion layer used as a piezoelectric transducer are:

1) Since the layer is so thin, it has its greatest efficiency at very high frequencies;
2) The thickness of the layer, hence the "resonant" frequency, can be controlled by a dc bias voltage.

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