

- [24] —, "Pressure of the lid on the eye, *Arch. Ophthalmol.*, vol. 78, pp. 328-329, 1967.
- [25] S. Mishima and D. M. Maurice, "The oily layer of the tear film and evaporation from the corneal surface," *Exp. Eye Res.*, vol. 1, pp. 39-44, 1961.
- [26] I. S. Nash, P. R. Greene, and C. S. Foster, "Comparison of mechanical properties of keratoconus and normal corneas," *Exp. Eye Res.*, vol. 35, pp. 413-423, 1982.
- [27] D. E. Newland, *Random Vibrations and Spectral Analysis*. New York: Longman, 1981.
- [28] G. Wilson, C. Bell, and S. Chotai, "The effect of lifting the lids on corneal astigmatism," *Amer. J. Opt. Physiol. Opt.*, vol. 59, no. 8, pp. 670-674, 1982.

A Device to Provide Respiratory-Mechanical Unloading

CHI-SANG POON AND SUSAN A. WARD

Abstract—A commercially available and widely used servo-controlled ventilator has been modified to provide controlled inspiratory resistive unloading of the human respiratory system. This is achieved by establishing a positive mouth pressure throughout inspiration in a constant proportion to instantaneous flow, i.e., the ratio of mouth pressure to flow, which defines the "assistance," remaining constant. The performance of this device has been evaluated in four healthy subjects during steady-state, constant-load cycling (20–120 W). It is demonstrated that i) the device can successfully implement controlled degrees of inspiratory assistance on a breath-to-breath basis; ii) the assistance can be sustained over a substantial range of ventilatory drive (i.e., for ventilations up to some 30 l/min) and to an extent which approaches a 100 percent reduction of the normal respiratory resistance. This device should prove useful in experimental and clinical investigations of the respiratory responses to resistive unloading of the respiratory system.

INTRODUCTION

Considerable attention has been directed towards the effects of mechanically loading the human respiratory system, with a view to understanding the etiology of disordered respiratory control in disease states evidencing abnormal lung or chest wall mechanics [1], [2]. In contrast, less is known about the responses to mechanical unloading, as the available techniques suffer, for the most part, from shortcomings of a technical or interpretational character.

For example, by reducing the density of respired air (with substitution of helium for nitrogen or with decompression), it is possible to lower respiratory resistance to some extent through diminished turbulence in the airways, but these approaches are applicable only at relatively high ventilation levels where turbulent dissipation is prominent [3]–[5]. Similarly, in a variety of clinical settings, positive-pressure ventilation is used to provide respiratory assistance [6]. However, as conventionally used, this technique too is not suited to the present purpose.

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C.-S. Poon is with the Department of Electrical and Electronics Engineering, North Dakota State University, Fargo, ND 58105, and with the Pulmonary Division, Veterans Administration Hospital, Fargo, ND 58102.

S. A. Ward is with the Department of Anaesthesiology, School of Medicine, University of California, Los Angeles, CA 90024.

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Firstly, the cycling of commercially available ventilators in synchrony with the subject's intrinsic respiratory rhythm is problematic. While the initiation of an assisted inspiration may be triggered with relative ease by the developing negative airway pressure at inspiratory onset, the termination of the assisted phase based upon the attainment of a preset volume or pressure threshold may not necessarily coincide with the subject's natural phase transition from inspiration to expiration [6]. Hence, the primary ventilatory response to such unloading could be contaminated both by perceptual influences and by mechanoreflexes originating in the lungs and chest wall. In addition, this approach typically does not provide a satisfactory index of unloading; consequently, both the controlled implementation of a particular unloading profile and quantitative assessment of its effects prove to be extremely difficult. For example, while indexes such as peak inspiratory pressure or volume are commonly employed for this purpose [6], they reflect not only the degree of unloading but also factors such as the subject's respiratory mechanics, intrinsic inspiratory drive, and breathing pattern.

A significant advance was provided independently by Harries and Tyler [7] and by Kellogg *et al.* [8], who both described servo-ventilators capable of producing transthoracic pressure changes in proportion to the subject's instantaneous inspiratory flow. As the external pressure waveform remained in synchrony with the subject's intrinsic breathing pattern, undesirable mechanical disturbances were reduced substantially. Moreover, the effect of the external pressure (P_m) could be quantitated readily in terms of the inspiratory flow (\dot{V}_I) as the constant ratio $R = P_m/\dot{V}_I$, which effectively represents a "negative resistance" to airflow. This procedure may therefore be regarded as providing an "assistance" to inspiration which simulates a reduction in the resistive work of breathing.

Since the construction of these earlier servo-ventilators clearly precludes their widespread application to experimental and clinical situations, we have modified a commercially available and widely used servo-controlled ventilator for the implementation of inspiratory resistive unloading on a breath-to-breath basis. The successful application of such a device in formal investigations of resistive unloading demands that a substantial level of assistance can be generated and maintained over a reasonably wide range of ventilatory drive. Utilizing graded, constant-load cycling to evaluate the performance of the modified servo-ventilator, we have established that its operation is satisfactory up to a ventilation of some 30 l/min and to an extent which approaches 100 percent reduction of the normal respiratory resistance. The application of this device to an experimental investigation of ventilatory control during exercise is described in a separate communication [9].

DESIGN

A Siemens-Elema servo-ventilator (#900B) forms the basic element of the "assistance" device. This ventilator was originally designed to generate either a sinusoidal or a square-wave profile of inspiratory airflow, by means of a servo-mechanism in which the airflow achieved was continuously compared with the airflow required (Fig. 1). This yielded an error signal which drove a step-motor to alter the patency of a servo-valve located in the inspiratory line, thus effecting the required flow.

In order to control the degree of inspiratory "assistance," we have modified the feedback portion of the servo-mechanism, replacing the error-calculating stage with a subsystem which compares the ratio of mouth pressure to inspiratory flow with a constant reference signal proportional to the desired level of "assistance." The design of the modified feedback loop is illustrated in Fig. 2. Input signals to the loop are: inspiratory flow, mouth pressure, and a triggering voltage which changes abruptly from a low level to a high level at inspiratory onset and vice versa at expiratory onset. The output from the loop is an error signal which drives the step-motor and, hence, the servo-valve.

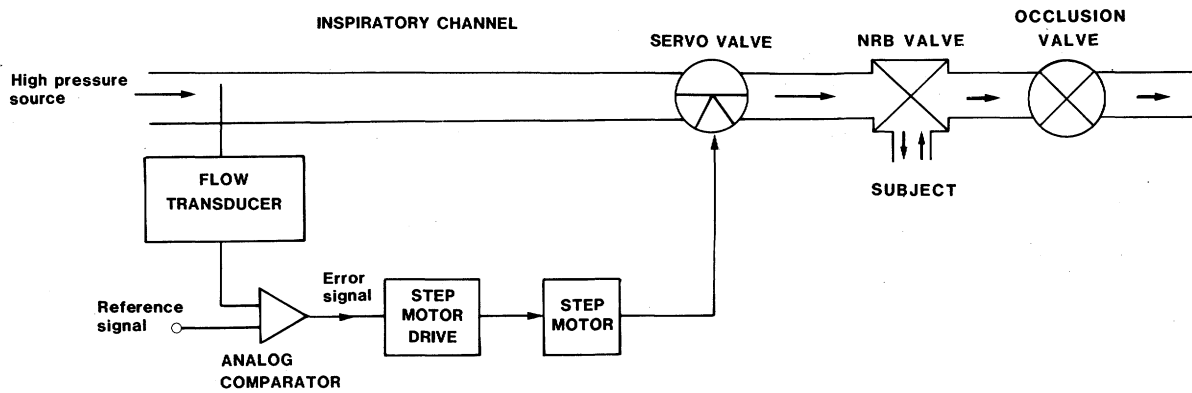


Fig. 1. Block diagram of original servo-mechanism which controls flow output from ventilator.

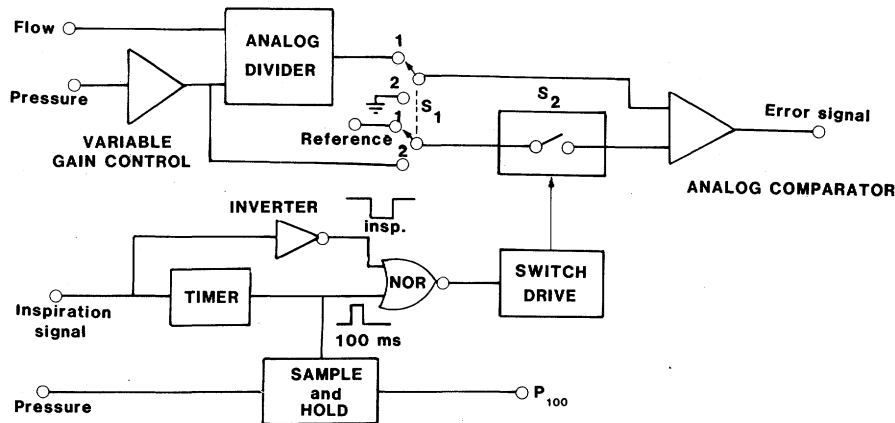


Fig. 2. Block diagram of electrical circuit for feedback loop of ventilator servo-mechanism.

Transducers built in to the ventilator provide signals proportional to i) inspiratory flow by way of a pneumotachograph connected to a strain-gauge manometer upstream of the servo-valve, and ii) mouth pressure by a second strain-gauge manometer which monitors the air pressure at the outlet of the servo-valve (Fig. 1). The ratio between these two signals is calculated continuously by an analog divider circuit (Fig. 2), and reflects the "assistance" provided by the ventilator. Appropriate adjustment of the pressure signal gain allows the required degree of "assistance" to be generated (Fig. 2).

The feedback loop can operate in one of two modes: assisted or unassisted. Inspiratory "assistance" is provided when the DPDT switch S_1 is set in position 1, permitting the pressure-flow ratio signal to be compared with the selected reference level (Fig. 2). Any difference between these two signals will yield an error signal, generated by the analog comparator (Fig. 2), which drives the step-motor to adjust the patency of the servo-valve (Fig. 1) so that the pressure/flow ratio can be restored to the desired level. The unassisted mode is designed to simulate normal inspiration. With the switch S_1 set in position 2, the pressure signal is referenced to ground level by the analog comparator (Fig. 2), thereby maintaining the mouth pressure at zero.

To prevent loss of the inspirate through the expiratory port of the breathing valve (Rudolph, #2700), the expiratory line is actively closed during inspiration by means by an occlusion valve (Fig. 1). As the original valve was found to have an unacceptably high resistance to flow when patent ($2.2 \text{ cm H}_2\text{O/l/s}$ at a flow of 1 l/s), it was replaced by a pneumatically-driven occlusion valve [10] having a substantially lower airflow resistance ($0.6 \text{ cm H}_2\text{O/l/s}$).

During expiration, the inspiratory servo-valve is maintained in a closed position so that inspiratory gas does not leak across the

breathing valve and contaminate the expirate. Furthermore, if breath-to-breath monitoring of inspiratory occlusion pressure (P_{100}) [10] is required, the period of closure can be extended beyond inspiratory onset by 100 ms or so. This is achieved by opening an analog switch S_2 at the analog comparator input for the required interval (Fig. 2). The switch drive is deactivated by a NOR logic circuit when at least one of the two inputs to the latter has a high level. Thus, inversion of the trigger voltage (see above) provides the logic circuit with a high-level input throughout expiration; the trigger voltage itself provides a second high-level input during inspiration which, after passage through a timer, is abruptly reset to a low level 100 ms following inspiratory onset. As both inputs to the logic circuit are now at low level (and remain so until the onset of the next expiration), the logic circuit is activated so that switch S_2 closes. This allows the inspiratory servo-valve to reopen and, some 20 ms later, inspiratory airflow to be resumed (the delay reflecting the mechanical inertia of the servo-valve).

EVALUATION AND DISCUSSION

The performance of the device was evaluated in four healthy adults during the steady state (i.e., in the fifth minute) of constant-load exercise on a cycle ergometer (Collins), with work loads which ranged from 20 to 120 W on different occasions. To minimize the likelihood of audible cues from the servo-ventilator influencing their respiration, the subjects listened to music of their choice through headphones. The following assistance modes were utilized: a) control mode (C), where the servo-ventilator outlet was disconnected from the inspiratory line; b) nonassist mode (A_0), where the ventilator was in place but P_m was actively held at zero throughout inspiration; c) low-level assist mode (A_1), where P_m and \dot{v}_I were controlled to approximate a negative resistance ($-R$) of 1.5 cm

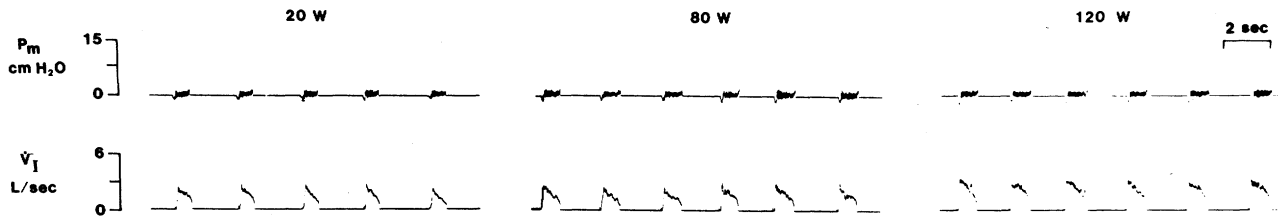


Fig. 3. Breath-by-breath recordings of mouth pressure (P_m) and inspiratory flow (\dot{v}_I) from one subject in the nonassist breathing mode (A_0), at three levels of cycle ergometer exercise.

TABLE I
INFLUENCE OF SERVO-VENTILATOR OPERATING IN NONASSIST MODE (A_0) ON STEADY-STATE VENTILATION (\dot{V}_E) AND END-TIDAL PCO_2 ($P_{ET}CO_2$) DURING CYCLE ERGOMETER EXERCISE

Subject	Control		Unassisted		Control		Unassisted	
	\dot{V}_E l/min	$P_{ET}CO_2$ mmHg	\dot{V}_E l/min	$P_{ET}CO_2$ mmHg	\dot{V}_E l/min	$P_{ET}CO_2$ mmHg	\dot{V}_E l/min	$P_{ET}CO_2$ mmHg
	20 W				80 W			
1	12.9 ± 0.6	35.3 ± 1.0	13.0 ± 0.7	35.9 ± 2.0	22.2 ± 0.5	38.9 ± 1.0	21.7 ± 0.3	38.6 ± 1.0
2	12.8 ± 0.6	35.5 ± 0.6	13.2 ± 0.3	34.9 ± 0.5	20.7 ± 0.6	36.9 ± 0.5	20.8 ± 0.3	36.8 ± 0.3
3	13.7 ± 0.4	40.0 ± 1.3	13.5 ± 0.9	39.3 ± 0.1	25.7 ± 0.4	42.1 ± 0.8	24.8 ± 0.5	41.3 ± 0.4
4	11.9 ± 0.2	39.5 ± 0.4	10.3 ± 0.6	39.6 ± 0.6	19.7 ± 0.7	44.6 ± 0.7	17.5 ± 0.4	45.4 ± 0.6

Values represent means ± S.E. ($n = 3$).
Individual "control" and "unassisted" values are not significantly different ($p > 0.05$).

$H_2O/l/s$; and d) high-level assist mode (A_2), where $-R = 3.0$ cm $H_2O/l/s$. These assistance levels correspond, respectively, to reductions of approximately 50 percent and 100 percent of the normal respiratory resistance [11]. The PCO_2 in respired gas was monitored with an infrared CO_2 analyzer (Beckman, LB-2). Expiratory tidal volume was obtained by electronic integration of the instantaneous flow signal recorded by a pneumotachograph (Fleisch, #3) and a pressure transducer (Validyne, MP45). All respiratory variables were continuously displayed on a chart recorder (Beckman, RM Dynograph). Subsequent off-line computations yielded breath-to-breath values of ventilation (\dot{V}_E) and end-tidal PCO_2 ($P_{ET}CO_2$).

The ability of the device to simulate unassisted (i.e., normal) breathing conditions at three different levels of exercise is demonstrated in Fig. 3. Mouth pressure is effectively maintained at zero by the ventilator throughout each inspiration, except for a brief period of occlusion immediately following inspiratory onset which is required for triggering the ventilator into the inspiratory phase. Thus, although the ventilator is in position, it is operating with zero assistance (mode A_0).

To examine whether this procedure, per se, might elicit secondary ventilatory responses (consequent, in some way, to the cycling of the ventilator) other than that which might result directly from a change in mouth pressure, we compared the respiratory responses to exercise under control and nonassist conditions (modes C and A_0). As is evident from Table I, in none of our subjects could we discern any significant difference between the control and nonassist modes in terms of \dot{V}_E or $P_{ET}CO_2$ at either of the work rates investigated. These results demonstrate that the mere presence of the assisting device is without effect on the normal ventilatory response to moderate exercise, thereby ensuring that the device may adequately simulate normal breathing conditions in this mode of operation. It is thus possible for an experimenter to readily impose transitions between normal and assisted breathing using the servo-ventilator, with minimal modification of the apparatus being required.

The typical inspiratory flow and mouth pressure characteristics during assisted breathing (modes A_1 and A_2) are shown in Fig. 4 at three different levels of exercise. In each of these circumstances P_m is elevated throughout inspiration and is controlled to continuously

track the airflow signal. Consequently, the instantaneous pressure-flow ratio (P_m/\dot{v}_I) remains almost constant at the chosen assistance level, except for some occasional overshoots and undershoots (± 20 percent) and the more rapid and small fluctuations which reflect the response characteristics of the servo-mechanism (the momentary overshoot occasionally seen in the assistance signal at the end of inspiration is of little significance as the instantaneous flow is effectively zero then). The high-frequency fluctuations are caused by the added transport delay of inspired gas between the ventilator and the subject, and may be easily eliminated by shortening the connecting tubing. The pressure-flow ratio is typically attained within 50 ms of inspiratory onset, the actual rise time being somewhat longer for higher peak pressures. Thus, the initial response speed is influenced by the assistance level as well as the rate of rise of inspiratory activity. The control of the P_m/\dot{v}_I ratio is generally satisfactory, except at the highest airflow level (i.e., A_2 assistance at 120 W) where the mouth pressure is sometimes seen to drop abruptly during inspiration, owing to premature emptying of the ventilator bellows which supply the pressure.

Thus, we have found the airflow output of the ventilator to be adequate for those studies in which the ventilatory demands are not excessively high (e.g., moderate exercise and CO_2 stimulation). If required, the operating range of the ventilator may easily be extended by increasing the capacity of the bellows or their rate of filling (e.g., by raising the pressure of the compressed air source).

It should be noted that the relatively low external pressures required for assisted breathing (cf. Fig. 4) are unlikely to significantly impair cardiopulmonary functions. Thus, any influence of the assistance on intrathoracic pressure is probably small; the resistance of the intervening airways serving to diminish the pressure downstream as the peripheral air spaces are approached. This is in contrast to conventional methods of mechanical ventilation in which the peak mouth pressure (which typically occurs at end inspiration where airflow is minimal) is completely transmitted to the alveoli, thus increasing the likelihoods of barotrauma and decreased pulmonary blood flow. We have shown that with the present ventilation technique, no alterations of the normal heart rate and systemic blood pressure could be discerned even at the highest assistance level (mode A_2) [9]. Based upon these considerations, therefore,

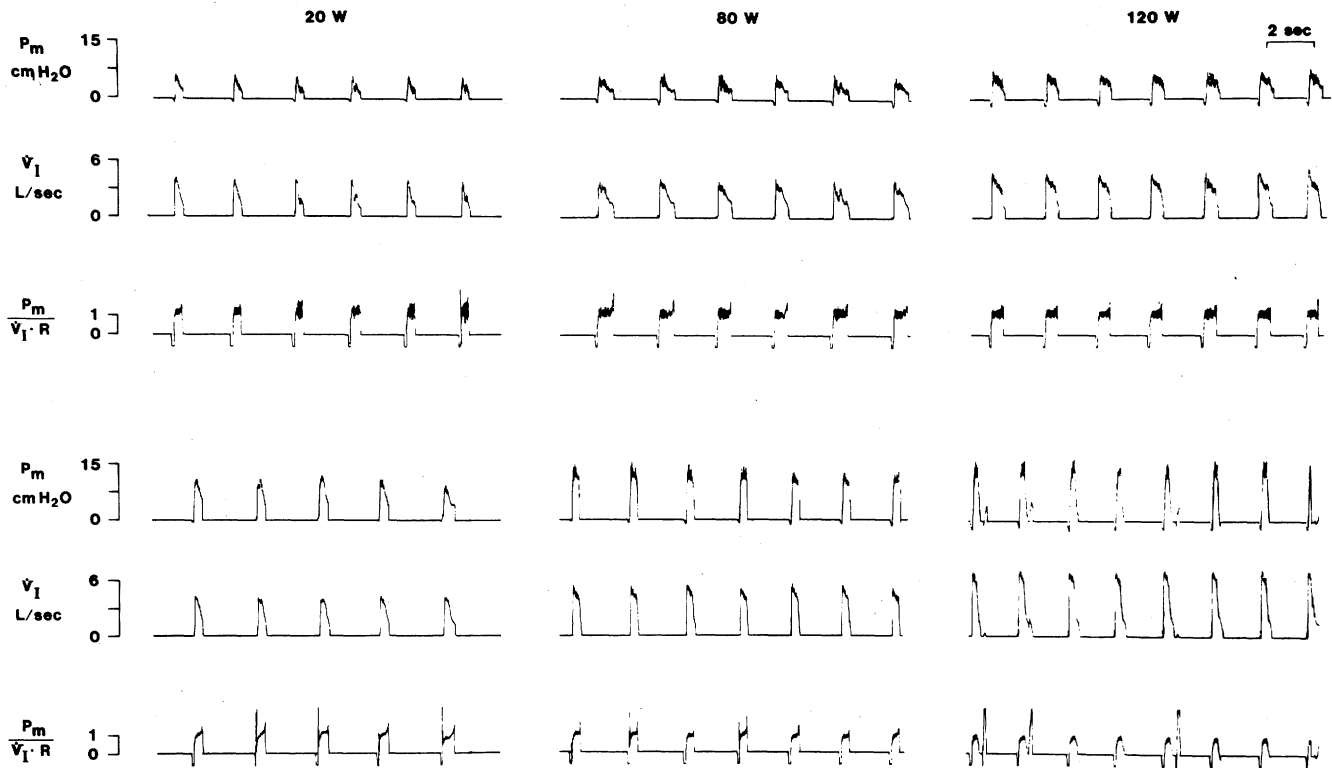


Fig. 4. Breath-by-breath recordings of mouth pressure (P_m), inspiratory flow (\dot{v}_I), and the normalized ratio ($P_m/(\dot{v}_I \cdot R)$, where R is the desired ratio) from one subject in breathing modes A_1 (upper panel) and A_2 (lower panel), at three levels of cycle ergometer exercise.

the performance of the assistance device is evidently appropriate for routine laboratory use.

In addition to its physiological use, the technique of negative-resistance ventilation could well find applications in the clinical area. In such considerations it is important to recognize that conventional positive-pressure ventilators provide assistance according to a preset mechanical pattern and, hence, rarely operate in synchrony with the patient's intrinsic respiratory efforts. It is not uncommon for patients to "fight" the ventilator, so actually increasing the intrinsic work of breathing. In order for the assistance to be successfully implemented, therefore, it is often necessary to administer sedatives and muscle relaxants if the patient cannot be adequately motivated and coached.

These shortcomings were acknowledged by Harries and Tyler [7] who developed a patient-controlled servo-ventilator from a body tank which could provide transthoracic pressure changes in proportion to respiratory airflow. Using this mode of assisted ventilation in a group of severely emphysematous patients, these authors observed a significant reduction in the oxygen cost of breathing and, importantly, an increase in ventilation sufficient to lower arterial PCO_2 (although the obligatory enclosure of the lower body limits the experimental and clinical application of this particular device).

The demonstrated efficacy of negative-resistance ventilation may be attributed to the improved patient-machine interface, which is designed to be responsive to the patient's intrinsic respiratory demands rather than to a preset mechanical pattern. Thus, the ventilator allows the patient to "set" the rate and depth of his breathing; the external pressure serving only to lower the resistive work of breathing required of the respiratory muscles. Furthermore, consequent to the improved synchronization of patient and ventilator, the degree of physical and psychological distress which often attends artificial ventilation [12] may be greatly reduced. This mode of assisted ventilation may be of considerable benefit in the management of neonates and patients with severe respiratory disorders

of neuromuscular origin, where patient-ventilator coordination is difficult or impossible.

In conclusion, we propose that the modified servo-ventilator can successfully implement controlled degrees of inspiratory assistance on a breath-to-breath basis. The assistance can be sustained over a substantial range of ventilatory drive, and to an extent which approaches a 100 percent reduction of the normal respiratory resistance. This device should prove valuable in future investigations of the ventilatory responses to resistive unloading of the respiratory system in humans and experimental animals, and in the clinical management of chronic or acute respiratory failure secondary to various respiratory mechanical or neuromuscular disorders.

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REFERENCES

- [1] N. S. Cherniack and M. D. Altose, "Respiratory responses to ventilatory loading," in *Regulation of Breathing*, part II, T. F. Hornbein, Ed. New York: Dekker, 1981, pp. 905-964.
- [2] L. D. Pengelly, A. S. Rebeck, and E. J. M. Campbell, Eds., *Loaded Breathing*. Toronto, Canada: Longman, 1974.
- [3] T. M. Murphy, W. H. Clark, I. P. B. Buchingham, and W. A. Young, "Respiratory gas exchange in exercise during helium-oxygen breathing," *J. Appl. Physiol.*, vol. 26, pp. 303-307, 1969.
- [4] E. E. Nattie and S. M. Tenney, "The ventilatory response to resistance unloading during muscular exercise," *Resp. Physiol.*, vol. 10, pp. 249-262, 1970.
- [5] S. A. Ward, B. J. Whipp, and C. S. Poon, "Density dependent airflow and ventilatory control during exercise," *Resp. Physiol.*, vol. 4, pp. 267-277, 1982.
- [6] S. P. McPherson, *Respiratory Therapy Equipment*. St. Louis, MO: Mosby, 1977.
- [7] J. R. Harries, and J. M. Tyler, "Mechanical assistance to respiration in emphysema," *Am. J. Med.*, vol. 36, pp. 68-78, 1964.

- [8] R. H. Kellogg, D. E. Leith, and K. Konno, "Ventilatory response to CO₂ when respiratory resistance and elastance are reduced in normal subjects," *Fed. Proc.*, vol. 23, no. 2, p. 259, 1964.
- [9] C. S. Poon, S. A. Ward, and B. J. Whipp, "Influence of inspiratory assistance on ventilatory control during moderate exercise," *J. Appl. Physiol.*, to be published.
- [10] S. A. Ward, K. Aqleh, and C. S. Poon, "Breath-to-breath monitoring of inspiratory occlusion pressures in man," *J. Appl. Physiol.*, vol. 51, pp. 520-523, 1981.
- [11] A. B. DuBois, "Resistance to breathing," in *Handbook of Physiology, Respiration*, vol. 1, W. O. Fenn and H. Rahn, Eds. Washington, DC: Am. Physiol. Soc., 1964, pp. 451-462.
- [12] J. L. Gibbons, and D. C. Myler, Jr., "Human care of the ventilator patient," in *Clinical Use of Mechanical Ventilation*, C. C. Rattenborg, Ed. Chicago, IL: Year Book Medical, 1981, pp. 328-338.

Roundoff Errors in Signal Averaging Systems

ROGER P. GAUMOND

Abstract—In biomedical signal averaging applications where a small repetitive signal is to be extracted from a very noisy waveform (noise variance σ_n^2), the A/D converter range is set at $\pm A\sigma_n$ where A typically has a value of 3 or 4. In this case, A/D roundoff noise using a $(b + 1)$ -bit A/D converter degrades the SNR of the resulting signal estimate by an amount slightly greater than $10 \log(1 + A^2 2^{-2b}/12)$. Application of this result to the design of a system for digital acquisition and averaging of the acoustic brainstem response indicates that an increase of A/D resolution beyond the first few bits yields little improvement in the SNR of the measurement.

INTRODUCTION

The reduction of roundoff errors is often an important consideration in specifying the number of bits required of an A/D converter in a given application. The SNR of a quantized signal for a perfect $(b + 1)$ -bit A/D converter which yields roundoff errors, such as a successive-approximation A/D converter, is given in [1] as

$$\text{SNR} = 6.02b + 10.79 + 10 \log_{10}(\sigma_s^2) \quad (1)$$

where σ_s^2 is the variance of the (stochastic) sampled signal. In deriving (1), the sequence of roundoff errors is assumed to be a stationary random process whose elements are uniformly distributed on the quantization interval and are uncorrelated with the sequence of signal samples and with each other. No noise sources other than roundoff errors are considered. In this case, (1) suggests SNR increases by roughly 6 dB per converter bit.

Equation (1) does not take into account reductions in SNR due to additive "inherent" noise processes corrupting the signal prior to sampling. When there is substantial "inherent" noise, its presence limits SNR. When the signal is repetitive, signal averaging can be used to reduce the effect of inherent noise [2]. We shall show that signal averaging also reduces the effect of roundoff noise, and that under conditions in which signal averaging is appropriate, any additional precision in A/D conversion beyond the first few bits yields little improvement in overall SNR of the signal estimate.

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The author is with the Bioengineering Program, Pennsylvania State University, University Park, PA 16802.
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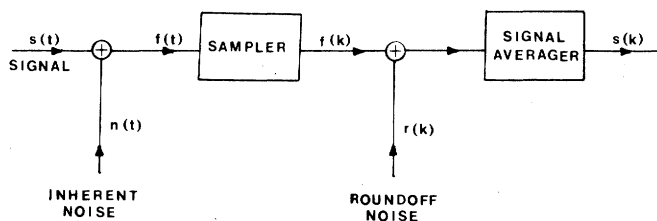


Fig. 1. Signal model for the averaging process. A/D conversion is represented by a sampling operation followed by the addition of roundoff noise.

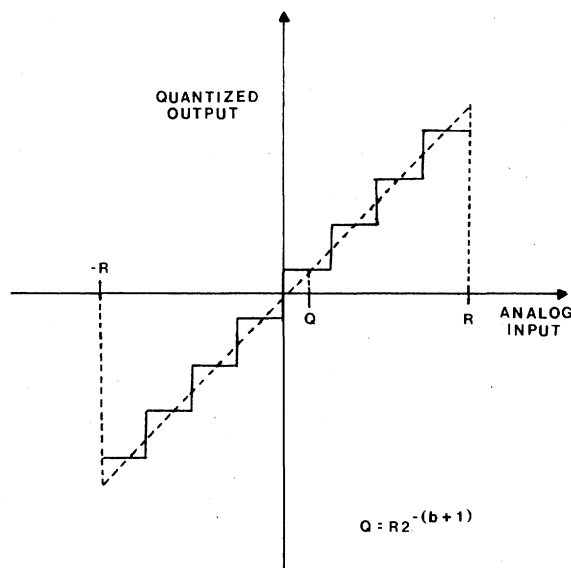


Fig. 2. Quantization using an ideal 3-bit A/D converter ($b = 2$) set to produce roundoff errors for analog input data in the range $-R$ to $+R$.

SNR CALCULATION

We proceed as in [2] to establish a signal model for the sampling process of Fig. 1. Waveform $f(t)$ consists of a signal $s(t)$ and additive noise $n(t)$. The A/D conversion process is represented as signal sampling followed by the addition of error term $r(k)$ to account for roundoff errors of finite bit representation. We assume that the $(b + 1)$ -bit A/D converter has been adjusted to yield roundoff errors as shown in Fig. 2 for a 3-bit A/D converter. Over the range of input analog data values from $-R$ to $+R$, such a $(b + 1)$ -bit converter yields quantization errors $r(k)$ which range from $-R2^{-(b+1)}$ to $+R2^{-(b+1)}$.

We assumed that segments of $s(t)$ are repeated identically during a succession of J signal epochs. A signal averager can then be used to produce an estimate of $s(t)$ by summing appropriate samples during each epoch and dividing by J . Thus, if j indexes successive epochs,

$$\hat{s}(k) = \frac{1}{J} \sum_j [f_j(k) + r_j(k)] \quad (2)$$

$$= s(k) + \frac{1}{J} \sum_j n_j(k) + \frac{1}{J} \sum_j r_j(k). \quad (3)$$

The error in signal estimate $e(k) = \hat{s}(k) - s(k)$ is thus equal to the average value of $2J$ inherent and roundoff noise terms. We make the following assumptions about these noise terms.

- 1) $\{n_j(k)\}, j = 1, 2, \dots, J$ is a sample sequence of a stationary random process whose elements are identically distributed with zero mean and with variance σ_n^2 .
- 2) $\{r_j(k)\}, j = 1, 2, \dots, J$ is a sample sequence of a stationary