Theorem 3.2: For the LQG problem (2.1)–(2.3), the linear feedback control given by (3.5) is a.s. optimal, where the Q_i's are determined by (3.7). The pathwise optimal cost is given by (3.8).

Some comments are in order now.

Remark 3.1:

i) The condition $B_i \neq 0$ for each i can be relaxed. If for some i, $B_i = 0$, then the above result will still hold if (A_i, B_i) are stochastically stabilizable in a certain sense [7], [9].

ii) For the multidimensional case, if B_i, C_i, σ_i are positive definite, then all the above results will hold. But the positive definiteness of B_i is a very strong condition, since in many cases the B_i's may not even be square matrices. In such a case if we assume that (A_i, B_i) are stochastically stabilizable, then the above results will again hold. Sufficient conditions for stochastic stabilizability are given in [2], [7], and [9]. If C_i is not positive definite, then the cost does not necessarily penalize the unstable behavior, as discussed in the foregoing. Thus condition (A5) of [5] is not satisfied. In this case under a further detectability condition, the optimality can be obtained in a restricted class of stationary Markov controls [2].

iii) Let ρ be as in (3.8). Then for any admissible policy $u(t)$ it can be shown by the pathwise analysis in [5] that

$$\liminf_{T\to\infty} \frac{1}{T} \int_0^T [C(S(t))X^2(t) + D(S(t))u^2(t)]dt \geq \rho \quad \text{a.s.}$$

This establishes the optimality of the linear feedback control v (3.5) in a much stronger sense, viz., the most "pessimistic" pathwise average cost under v is no worse than the most "optimistic" pathwise average cost under any admissible control.

iv) For $T > 0$, let $V(x, i, T)$ denote the optimal expected cost for the finite horizon $[0, T]$. Then it can be shown as in [2] that

$$\lim_{T\to\infty} \frac{1}{T} V(x, i, T) = \rho$$

where ρ is as in (3.8). Thus the finite horizon value function approaches the optimal pathwise average cost as the length of the horizon increases to infinity. Thus for large T, the linear feedback control (3.5) would be a reasonably good nearly optimal control for the finite horizon case. This would be particularly useful in practical applications since it is computationally more economical to solve the algebraic Riccati system than the Riccati system of differential equations.

v) The condition $\lambda_{ij} > 0$ can be relaxed to the condition that the chain $S(t)$ is irreducible (and hence ergodic). The existence part in [5] can be suitably modified to make the necessary claim here. In the dynamic programming part, the existence of a unique solution in Lemma 3.1 is clearly true under the irreducibility condition.

IV. Conclusion

In this note, we have studied the pathwise optimality of an LQG regulator with Markovian switching parameters. We have assumed that the Markovian parameters are known to the controllers. This is an ideal situation. In practice the controllers may not have a complete knowledge of these parameters. In this case, one usually studies the corresponding minimum variance filter. Unfortunately, this filter is almost always infinite dimensional [9]. A computationally efficient suboptimal filter has been developed in [1] and [8]. We hope that our results will be useful in the dual control problem arising in this situation.

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Corrections to "On the Structure of H^∞ Control Systems and Related Extensions"

Gjerrit Meinsma and Hidenori Kimura

In the above paper[1] Lemma 2.1 on (J, J')-losslessness is not correct. Its implications Lemma 2.2 and Corollary 2.4 are also not fully correct. Corollary 2.5 reappeared in [3] as Lemma 3.1. In [3], Lemma 3.2 is not fully correct for the same reason.

We give a counterexample to these claims and show how they can be fixed by strengthening the assumption. As far as $\mathcal{H}_\infty$ theory is concerned, the additional assumption is satisfied. Throughout, J and J' are signature matrices of the form

$$J := J_{m,r} := \begin{bmatrix} I_m & 0 \\ 0 & -I_r \end{bmatrix}, \qquad J' := J_{\mu,\rho} := \begin{bmatrix} I_\mu & 0 \\ 0 & -I_\rho \end{bmatrix}.$$

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G. Meinsma is with the Department of Electrical and Computer Engineering, The University of Newcastle, University Drive, Callaghan, N.S.W. 2308, Australia.

H. Kimura is with the Department of Systems Engineering, Osaka University, 1-3, Machikaneyama, Toyonaka 560, Japan.

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A proper rational matrix M that has no poles on the imaginary axis is called (J, J')-lossless if $[M(s)]^* J M(s) \leq J'$ for all s in the right-half plane, with equality holding on the imaginary axis. Here * denotes the complex conjugate transpose. The claims are the following.

Claim 1 (Lemma 2.1 of the Above Paper[1]): A proper rational matrix M that has no poles on the imaginary axis is (J, J')-lossless iff there exists a $P > 0$ such that

$$\begin{cases} D^*JD = J'; \\ D^*JC + B^*P = 0; \\ A^*P + PA + C^*JC = 0. \end{cases} \tag{1}$$

Here $[A, B, C, D]$ is any minimal realization of M.

Claim 2 (Corollary 2.3 of the Above Paper[1] and Lemma 3.1 of [3]): Every (J, J')-lossless matrix M has a J-orthogonal complement N. (That is, such that $[M\ N]T$ is square and (J, J)-lossless for some permutation matrix T.)

As stated these claims are not fully correct. Consider the following rational matrix and minimal realization

$$M(s) = \frac{1}{s+1}\begin{bmatrix} s\sqrt{2} \\ \sqrt{2} \\ 1-s \end{bmatrix} \overset{s}{=} \left[\begin{array}{c|c} -1 & 1 \\ \hline -\sqrt{2} & \sqrt{2} \\ \sqrt{2} & 0 \\ 2 & -1 \end{array}\right]. \tag{2}$$

It is readily checked that $[M(s)]^* J_{2,1} M(s) = 1$ for all s, hence, that M is (J, J')-lossless with $J := J_{2,1}$ and $J' := 1$. The matrix P, however, satisfying (1) is, in this case, $P = 0$, contradicting the first claim. The M in (2) also serves as a counterexample to the second claim. Suppose, to obtain a contradiction, that M defined in (2) does have a J-orthogonal complement N; that is, such that $[M\ N]$ is square and (J, J)-lossless, with $J := J_{2,1}$. Since Claim 1 is correct if $J = J'$ (more of this later), any minimal realization $[A, B, C, D]$ of the square rational matrix $[M\ N]$ satisfies (1) for some $P > 0$. Then M obviously has realization $[A, BV, C, DV]$ where $V := [1\ 0\ 0]^T$. After some manipulation it follows that

$$[M(s)]^* J M(s) = J' - (s + \bar{s}) V^* B^* (\bar{s}I - A^*)^{-1} \times P(sI - A)^{-1} BV. \tag{3}$$

We know that $[M(s)]^* J M(s) = J' = 1$, so the second term on the right in (3) must be identically zero. This cannot be, however, since $P > 0$ and $(sI - A)^{-1}BV$ is nonzero (because $M(s) = C(sI - A)^{-1}BV + DV$ is nonzero, nonconstant). This completes the counterexample to the second claim.

The problems can be fixed by strengthening the assumptions somewhat. In the counterexample the J and J' have a different number of negative eigenvalues (that is, $r \neq \rho$). In $\mathcal{H}_\infty$ control, the type of J-losslessness that is important is when J and J' have the same number of negative eigenvalues.

Lemma 3: Claim 1 and Claim 2 are correct if J and J' have the same number of negative eigenvalues (that is, if $r = \rho$).

This result is known, although it is not stated explicitly in the form of Claims 1 and 2 (see [4]). In the above paper[1] it is shown that Claim 1 implies Claim 2, so we only need to prove Claim 1 for the case that $r = \rho$. If $P > 0$ satisfies (1) then

$$[M(s)]^* J M(s) = J' - (s + \bar{s}) B^* (\bar{s}I - A^*)^{-1} P (sI - A)^{-1} B$$

and, hence, in that case M is (J, J')-lossless. Conversely, suppose the $(m + r) \times (\mu + \rho)$ rational matrix M is (J, J')-lossless and partition M compatibly as

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}.$$

Since $r = \rho$ we have that M_{22} is square. It then follows (see [2]) that U defined as

$$U := \begin{bmatrix} 0 & I_r \\ M_{11} & M_{12} \end{bmatrix} \begin{bmatrix} I_\mu & 0 \\ M_{21} & M_{22} \end{bmatrix}^{-1} \tag{4}$$

is inner, hence, in particular that U is stable. As in [1], a minimal realization of M easily gives a minimal realization of U. By minimality and stability of U, the observability gramian of U is positive definite and this gramian, call it P, can be shown to satisfy the three conditions (1). Note that we used here that the M_{22} is square, which is the reason we need that $r = \rho$.

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Sampled-Data Controller Reduction Procedure

Anton G. Madievski and Brian D. O. Anderson

***Abstract*—The problem of controller order reduction aimed at preserving the closed-loop performance of a sampled-data closed-loop system is investigated. Fast sampling of the system at a multiple of the sampling frequency followed by lifting allows capturing of the system's intersample behavior and yields a time-invariant single-rate system; this then permits standard order-reduction ideas to be applied. Special weighting functions aimed at preserving the closed-loop transfer function are obtained, and weighted balanced truncation is used to reduce the controller. An example shows that without the use of fast-sampling, an unstable closed loop can result from the reduction.**

I. Introduction

The great importance and usefulness of controller reduction is now widely recognized, and much attention has been paid to the subject over the past years. The main reason is that the linear quadratic Gaussian (LQG) and H_∞ design procedures lead to controllers which have order equal to, or roughly equal to, the order of the plant ([2] for LQG). Often, controllers of a lower order will result in acceptable performance and will be desired for their greater simplicity.

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The authors are with the Department of Systems Engineering, Australian National University, Canberra, ACT, 0200, Australia.

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