where

$$T = [A_b^\rho e_1, \cdots, A_b e_1, I_m]$$
$$A_b = \begin{bmatrix} -b_{m-1}/b_m & \vdots & I_{m-1} & \\ -b_0/b_m & 0 & \cdots & 0 \end{bmatrix}. \tag{99}$$

It is readily verified that

$$Tb = 0, \quad TA = A_b T + TA^\rho b e_1^T. \tag{100}$$

Therefore, from (74) and (78) we obtain

$$\dot{\bar{x}} = A_b \bar{x} + T[A^\rho b y + \varphi(y)] \tag{101}$$
$$\dot{\breve{x}} = A_b \breve{x} + T(A^\rho b - K_0)\tilde{x}_1. \tag{102}$$

These two systems have asymptotically stable homogeneous parts with inputs $y, \tilde{x}_1 \in \mathcal{M}(|X(0)|, \mu)$. This proves that $\bar{x}, \breve{x} \in \mathcal{M}(|X(0)|, \mu)$. So, from (98), we have $x, \tilde{x} \in \mathcal{M}(|X(0)|, \mu)$, and therefore, $\hat{x} \in \mathcal{M}(|X(0)|, \mu)$. Thus, $u \in \mathcal{M}(|X(0)|, \mu)$, and, consequently, $\xi \in \mathcal{M}(|X(0)|, \mu)$. Hence

$$|X(t)| \leq \rho_0(|X(0)|)e^{-dt} + \rho_\mu(\mu) \tag{103}$$

where ρ_0 and ρ_μ are locally Lipschitz class $\mathcal{K}_\infty$ functions. The proof of global asymptotic stability is the same as in Theorem 3.1. We first rewrite the closed-loop system

$$\begin{aligned}
\dot{x} &= Ax + \varphi(x_1) + b\sigma(x_1)u(x_1, \hat{x}, \zeta, m) \\
&\quad + \mu b[c_\Delta \xi + q\sigma(x_1)u(x_1, \hat{x}, \zeta, m)] \\
\dot{\hat{x}} &= A_0\hat{x} + K_0 x_1 + \varphi(x_1) + b\sigma(x_1)u(x_1, \hat{x}, \zeta, m) \\
\dot{\zeta} &= A_0\zeta + K_0 x_1 + \varphi(x_1) \\
\dot{\xi} &= A_\Delta \xi + b_\Delta \sigma(x_1)u(x_1, \hat{x}, \zeta, m) \\
\dot{m} &= -\delta m + |\hat{x}_2 - \zeta_2|_\varepsilon
\end{aligned}$$

in the form (66) and then proceed as in the proof of Theorem 3.1. □

APPENDIX

Definition A.1: The system $\dot{x} = f(t, x, u)$ is said to be input-to-state practically stable (ISpS) if there exist a class $\mathcal{KL}$ function β, a class $\mathcal{K}$ function γ, and a positive real number d such that for any $x(0)$ and for any input $u(\cdot)$ continuous on $[0, \infty)$, the solution exists for all $t \geq 0$ and satisfies $|x(t)| \leq \beta(|x(s)|, t - s) + \gamma(\sup_{s \leq \tau \leq t} |u(\tau)|) + d$ for all $0 \leq s \leq t$.

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Correction to "Adaptive Control of Robot Manipulators with Flexible Joints"

B. Brogliato and R. Lozano

I. INTRODUCTION

The above paper[1] contains two flaws that were recently brought to our attention by Hsu [1]. 1) The first mistake concerns the definition of the signal q_{2d} in (6) and (C.20): At time $t = 0$, the term $q_{2d}(0)$ appears on both sides of the equalities in (6) and (C.20). Hence the initial conditions on the state and on the desired trajectory q_{1d} and its

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[1] R. Lozano and B. Brogliato, *IEEE Trans. Automat. Contr.*, vol. 37, pp. 174–181, Feb. 1992.

first and second derivatives are constrained. 2) The second mistake is that the regressor matrix Y_4 in (C.18) contains integral terms which are not shown to be bounded. Consequently, the stability analysis in Appendix C [(C.42)–(C.47)], based on some signal chasing, is not complete.

- Concerning point 1), it has been shown in [2] that the positive definite function V in (3) can be modified to

$$V = \frac{1}{2}v_1^T D(q_1)v_1 + \frac{1}{2}v_2^T J_m v_2 + \frac{1}{2}(\tilde{q}_1 - \tilde{q}_2)^T K(\tilde{q}_1 - \tilde{q}_2) \quad (1)$$

so that q_{2d} is now defined as (see [2, (23)])

$$q_{2d} = q_{1d} + K^{-1}(D(q_1)\ddot{q}_{1r} + C(q_1, \dot{q}_1)\dot{q}_{1r} + g(q_1) - B_1 v_1) \quad (2)$$

where $B_1 = B_1^T > 0$ is a constant feedback gain. This allows us to remove the constraint on the initial conditions and to prove exponential Lyapunov stability of the fixed parameters scheme; see [2]. In the adaptive case, the signal q_{2d} in (C.20) can be modified as well to avoid the initial conditions' constraints problem. Indeed the term $K\int_0^t (v_1 - v_2)dt$ in the right-hand side of (C.1) is replaced by $K(\tilde{q}_1 - \tilde{q}_2)$. Then w_5 in (C.2), (C.5), and (C.20) is modified to $w_5 = -q_{1d}$. This does not affect the definition of the other terms in (C.20) since only w_5 has been modified.

Another possible solution was suggested to us [1], presented here in the nonadaptive case (compare with [3, (3) and (6)])

$$V = \frac{1}{2}v_1^T D(q_1)v_1 + \frac{1}{2}v_2^T J_m v_2 + \frac{1}{2}z^T K z \quad (3)$$

where z and q_{2d} are defined as

$$q_{2d} = q_{1d} - \lambda x + K^{-1}(-v_1 + D(q_1)\ddot{q}_{1r} + C(q_1, \dot{q}_1)\dot{q}_{1r} + g(q_1)) \quad (4)$$

$$\dot{q}_{1r} := \dot{q}_{1d} - \lambda\tilde{q}_1 \quad (5)$$

$$\dot{x} := \tilde{q}_1 - \tilde{q}_2 \quad (6)$$

$$z = \lambda x + (\tilde{q}_1 - \tilde{q}_2) \;\; (\dot{z} = v_1 - v_2) \quad (7)$$

$$u = -v_2 - J_m[-\ddot{q}_{2d} + \lambda\dot{\tilde{q}}_2] - K[q_{1d} - q_{2d} - \lambda x]. \quad (8)$$

Proof: Consider $V(v_1, v_2, z)$ as given by (3). Differentiating (3) we obtain

$$\dot{V} = v_1^T[D\dot{v}_1 + Cv_1 + Kz] + v_2^T[J_m v_2 - Kz] \quad (9)$$

where we have used property $P3$ of [3] and the fact that [from (6) and (7)] $\dot{z} = v_1 - v_2$. Now, closely following [3, Appendix B]

$$D\dot{v}_1 + Cv_1 + Kz = \quad (10)$$

$$-g - K(q_1 - q_2) + D[-\ddot{q}_{1d} + \lambda\dot{q}_1] + C[-\dot{q}_{1d} + \lambda\tilde{q}_1] + Kz = \quad (11)$$

$$-g - K(q_1 - q_2) + D[-\ddot{q}_{1d} + \lambda\dot{q}_1] + C[-\dot{q}_{1d} + \lambda\tilde{q}_1] + K[\lambda x + (q_1 - q_2) - (q_{1d} - q_{2d})] = \quad (12)$$

$$Kq_{2d} + \{-g - D\ddot{q}_{1r} - C\dot{q}_{1r} - K[\lambda x - q_{1d}]\}. \quad (13)$$

The last term within braces is the new w_1, using the notation of Appendix B of the original paper. From (4), obtained by requiring the expression in (13) to be equal to $-v_1$, we then have

$$\dot{V} = -v_1^T v_1 + v_2^T[J_m\dot{v}_2 - K(\lambda x + (\tilde{q}_1 - \tilde{q}_2)]. \quad (14)$$

The last term within brackets is the new w_2, following our original notation. After some manipulations using the robot motor equations, one gets

$$w_2 = u + J_m[-\ddot{q}_{2d} + \lambda\dot{\tilde{q}}_2] + K[q_{1d} - q_{2d} - \lambda x]. \quad (15)$$

Then, as in [3], setting $w_2 = -v_2$, one obtains (8). Consequently, the proposed control law yields (as in [3])

$$\dot{V} = -v_1^T v_1 - v_2^T v_2 \quad (16)$$

which is semi-definite negative as required.

- Concerning point 2): Fundamentally, the flaw comes from the fact that to be able to define signals q_{2d}, $\dot{q}_{2d}$ which are measurable and $\ddot{q}_{2d}$ that does not depend on $\ddot{q}_2$ (so that the control input in (14) is well defined), we need to filter the first dynamical equation (1) to suitably parameterize Dv_1 in (C.18). In the above paper we filtered (1) with an integrator, hence the possible unboundedness of the regressor Y_4 in (C.18). We now propose to use a first-order filter with transfer function $\frac{1}{s+1}$ instead of the integrator.

Applying the filter to (1) one gets (we drop the arguments for convenience)

$$\frac{1}{s+1}\{D\ddot{q}_1 + C\dot{q}_1 + g + K(q_1 - q_2)\} = 0. \quad (17)$$

Now we have

$$\frac{1}{s+1}\{D\ddot{q}_1\} = D\dot{q}_1 - D(0)\dot{q}_1(0) - \frac{1}{s+1} \times \{D\dot{q}_1 - D(0)\dot{q}_1(0)\} - \frac{1}{s+1}\{\dot{D}\dot{q}_1\}. \quad (18)$$

Indeed (18) can be obtained by first noting that $D\ddot{q}_1 = \frac{d}{dt}(D\dot{q}_1) - \dot{D}\dot{q}_1$. Now

$$\frac{1}{s+1}\{D\ddot{q}_1\} = \int_0^t e^{-(t-\tau)}D\ddot{q}_1 d\tau. \quad (19)$$

Then using integration by parts one gets

$$\frac{1}{s+1}\{D\ddot{q}_1\} = e^{-t}\left[\left[e^{\tau}\left(D\dot{q}_1 - D(0)\dot{q}_1(0) - \int_0^{\tau}\dot{D}\dot{q}_1 dy\right)\right]_0^t - \int_0^t e^{\tau}\left(D\dot{q}_1 - D(0)\dot{q}_1(0) - \int_0^{\tau}\dot{D}\dot{q}_1 dy\right)d\tau\right] \quad (20)$$

which finally yields

$$\frac{1}{s+1}\{D\ddot{q}_1\} = D\dot{q}_1 - D(0)\dot{q}_1(0) - \int_0^t \dot{D}\dot{q}_1 d\tau - \int_0^t e^{-(t-\tau)} \times \left(D\dot{q}_1 - D(0)\dot{q}_1(0) - \int_0^{\tau}\dot{D}\dot{q}_1 dy\right)d\tau. \quad (21)$$

Using the fact that (still integrating by parts)

$$\int_0^t e^{-(t-\tau)}\dot{D}\dot{q}_1 d\tau = \int_0^t \dot{D}\dot{q}_1 d\tau - \int_0^t e^{-(t-\tau)}\left(\int_0^{\tau}\dot{D}\dot{q}_1 dy\right)d\tau \quad (22)$$

we then get (18) by combining (21) and (22).

Now from (17) and (18) we have

$$D\dot{q}_1 = D(0)\dot{q}_1(0) + \frac{1}{s+1}\{D\dot{q}_1 - D(0)\dot{q}_1(0)\} + \frac{1}{s+1}\{\dot{D}\dot{q}_1\} - \frac{1}{s+1}\{C\dot{q}_1 + g + Kq_1\} + \frac{1}{s+1}\{Kq_2\}. \quad (23)$$

The terms between brackets can be written as $Y_i(q_1, \dot{q}_1)\theta_i$ for some constant vector θ_i. Hence $\frac{1}{s+1}$ $\{Y_i(q_1, \dot{q}_1)\theta_i\} = \frac{1}{s+1}\{Y_i(q_1, \dot{q}_1)\}\theta_i \triangleq Y_{if}\theta_i$ with $\dot{Y}_{if} + Y_{if} = Y_i(q_1, \dot{q}_1)$.

It follows that (C.18) can be written as

$$Dv_1 = Y_{4f}\theta_4 \quad (24)$$

with $\dot{Y}_{4f} + Y_{4f} = Y_4$ for some $Y_4(q_1, \dot{q}_1, q_2)$. Note also that q_{2d}, $\dot{q}_{2d}$ in (C.20) can be computed from position and velocities measurements only which is crucial for the algorithm to be implementable. From the new parameterization, it follows that if q_1, $\dot{q}_1$ and q_2 are bounded, so is Y_4. This was not the case when integrators were used.

The stability analysis is much simplified. Note first that the modification of $Y_4\theta_4$ leaves the rest of the analysis and algorithm unchanged. Hence from (C.41), q_1 and $\dot{q}_1$ are bounded. From (23) it follows that $q_{2f} = \frac{1}{s+1}\{q_2\}$ is bounded. Hence from (C.20) q_{2d} is bounded, and consequently q_2 and $\ddot{q}_1$ are bounded. Boundedness of $\dot{q}_2$ follows by differentiating (C.20) which proves that $\dot{q}_{2d}$ is bounded. Hence from (C.41) $\dot{q}_2$ is bounded. Boundedness of u can be inferred by inspecting (C.27) and (C.32). Consequently $\ddot{q}_2$ is bounded also.

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Comments on "Control of Vector Discrete-Event Systems II—Controller Synthesis"

Zhengyi Zhao

***Abstract*—This paper notes that the proof of Appendix 7 in the above paper[1] is wrong. A conterexample is given.**

I. Introduction

In the proof of Appendix 7 of the above paper,[1] the following conclusion was used: To maximize $v_{1,1} - \Sigma_{(i,j)\in D} v_{i,j}$, $v_{i,j}$ for $(i, j) \in D$ should be set to be as large as possible. This conclusion is wrong.

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II. A Counterexample

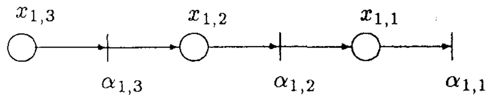

Fig. 1. A tree structure of TS2.

Fig. 1 is a TS2

$$E = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \quad B = [1, -1, -1].$$

The following special type of LIP:

$$\max BV \quad \text{s.t.} \quad \begin{cases} EV \geq -X \\ V \geq 0 \end{cases}$$

has a solution

$$V^* = [x_{1,1}, 0, 0]^T$$

and

$$BV^* = [1, -1, -1][x_{1,1}, 0, 0]^T = x_{1,1}.$$

According to the conclusion mentioned above,[1] $v_{1,1}, v_{1,2}, v_{1,3}$ should be set to be as large as possible, that is

$$\hat{V} = [x_{1,1} + x_{1,2} + x_{1,3}, x_{1,2} + x_{1,3}, x_{1,3}]^T.$$

But

$$B\hat{V} = x_{1,1} - x_{1,3} \leq BV^*.$$

So it is shown that the proof of Appendix 7[1] is incorrect.

Authors' Reply by Y. Li and W. M. Wonham

The result of Appendix 7[1] is correct, and its proof[1] is intentionally sketchy. However, without further clarification, the statement of the "basic idea" in the proof is indeed unclear and can lead to an incorrect conclusion. We thank Zhao for noting this. A complete proof is now provided in the following.

As in Fig. 4,[1] we arrange the events and state variables of a TS2 G into levels as shown in Fig. 1. Denote the set of events in the TS2 as Σ and the set of state variables as Ω. Let Z be the set of integers. Define the index set

$$D = \{(i, j) \mid B(\alpha_{i,j}) < 0\}.$$

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