# Corrections to "Optimal Detection Using Bilinear Time-Frequency and Time-Scale Representations"

Akbar M. Sayeed and Douglas L. Jones

*Abstract—* **The authors have made some corrections to a previously published paper.**

## I. Introduction

In [1], the authors develop an optimal detection framework based on quadratic time-frequency and time-scale representations. The expressions for certain test statistics in [1] contain a factor of $\frac{1}{2}$ that is not needed.[1] Moreover, certain "MAP GLRT" detectors are proposed in [1] whose forms should be slightly different than those described in ]1]. The following section describes the appropriate changes that rectify the situation.

## II. Corrections

The factor of $\frac{1}{2}$ should be replaced by 1 in the test statistics $L_O(x)$, $L_{\mathrm{LR}}(x)$, and $L_{\mathrm{LO}}(x)$ in (4), (6) and (9); that is, (4), (6) and (9) in [1] should read

$$L_O(x) = \frac{1}{N_0}\langle(\boldsymbol{R}_s(\boldsymbol{R}_s + N_0\boldsymbol{I})^{-1}x, x\rangle \tag{4}$$

$$\begin{aligned} L_{\mathrm{LR}}(x) &= \langle \boldsymbol{R}_n^{-1}(\boldsymbol{R}_s\boldsymbol{R}_n^{-1} + \boldsymbol{I})^{-1}\boldsymbol{R}_s\boldsymbol{R}_n^{-1}x, x\rangle \\ &\quad - \log(\det(\boldsymbol{R}_s\boldsymbol{R}_n^{-1} + \boldsymbol{I})) \end{aligned} \tag{6}$$

$$\begin{aligned} L_{\mathrm{LO}}(x) &= \langle \boldsymbol{R}_s\boldsymbol{R}_n^{-1}x, \boldsymbol{R}_n^{-1}x\rangle - \mathrm{Trace}(\boldsymbol{R}_n^{-1}\boldsymbol{R}_s) \\ &= L_H(x) - \mathrm{Trace}(\boldsymbol{R}_n^{-1}\boldsymbol{R}_s) \end{aligned} \tag{9}$$

This change should also be reflected in the following equations in [1] by replacing the factors of $\frac{1}{2}$ with 1: (37), (39), (40), (70), (72), (73), (75), (85), and (86). Note that in (70), (73), and (85), the change should be made in both terms on the right-hand side.

Equation (13) in [1], describing the MAP GLRT detectors, should be replaced by (13a), shown on page 762, and the following (13b)

$$(\hat{\alpha}, \hat{\beta}) = \begin{cases} \arg\max_{(\alpha,\beta)} \left[L_O^{(\alpha,\beta)}(x) + \log p(\alpha,\beta)\right] & \text{Case I} \\ \arg\max_{(\alpha,\beta)} \left[\log\left(L_H^{(\alpha,\beta)}(x)\right) + \log p(\alpha,\beta)\right] & \text{Case II} \\ \arg\max_{(\alpha,\beta)} \left[\log\left(L_{\mathrm{LO}}^{(\alpha,\beta)}(x)\right) + \log p(\alpha,\beta)\right] & \text{Case III.} \end{cases} \tag{13b}$$

The above changes should be also be reflected in some parts of Propositions A and B in [1]; we state the whole corrected versions for the sake of completeness.

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A. M. Sayeed is with the Department of Electrical Engineering, Rice University, Houston, TX 77005 USA.

D. L. Jones is with the Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA.

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[1] Although the factor does not matter in the context of the test statistics themselves (it can be absorbed in the threshold), it does affect things in the "MAP GLRT detectors" proposed in [1].

*Proposition A:* In the composite hypothesis testing problem (10), if the dependence of the Gaussian signal on the parameters $(\alpha, \beta) = (\tau, \nu)$ is characterized by the family of correlation functions $\{R_s^{(\tau,\nu)}\}$ defined in (24), then the test statistics for both the ML and MAP GLRT detectors identified in Section III can be implemented using bilinear TFR's as in (41), shown on page 762, where

$$y = \begin{cases} x & \text{Case I} \\ \boldsymbol{R}_n^{-1}x & \text{Cases II and III} \end{cases} \tag{42}$$

$$\begin{aligned} F_A(\tau,\nu) &= P_{R_n^{-1}}(\tau,\nu; \Phi = \mathrm{WS}_{R_{TF}}) \\ &= \iint \mathrm{WS}_{R_n^{-1}}(t,f)\mathrm{WS}_{R_{TF}}(t-\tau, f-\nu)\,dt\,df \end{aligned} \tag{43}$$

$(\hat{\tau}, \hat{\nu})$ are defined in and (44), shown on page 762, and the kernel $\Phi$ characterizing the TFR $P_y(\Phi)$ can be expressed as

$$\Phi(t,f) = \begin{cases} \mathrm{WS}_{\hat{R}_{TF}}(t,f) & \text{Case I} \\ \mathrm{WS}_{R_{TF}}(t,f) & \text{Cases II and III} \end{cases} \tag{45}$$

where

$$\mathrm{WS}_{\hat{R}_{TF}}(t,f) = \frac{1}{N_0}\sum_k \frac{\lambda_k}{\lambda_k + N_0} W_{u_k}(t,f) \tag{46}$$

and

$$\begin{aligned} \mathrm{WS}_{R_{TF}}(t,f) &= \int R_{TF}(t+\tau/2, t-\tau/2)e^{-j2\pi f\tau}\,d\tau \\ &= \sum_k \lambda_k W_{u_k}(t,f). \end{aligned} \tag{47}$$

□

*Proposition B:* In the composite hypothesis testing problem (10), if the dependence of the Gaussian signal on the parameters $(\alpha, \beta) = (\tau, c)$ is characterized by the family of correlation functions $\{R_s^{(\tau,c)}\}$ defined in (25), then the test statistics for both the ML and MAP GLRT detectors identified in Section III can be implemented using bilinear TSR's as in (48), shown on page 762, where

$$y = \begin{cases} x & \text{Case I} \\ \boldsymbol{R}_n^{-1}x & \text{Cases II and III} \end{cases} \tag{49}$$

$$\begin{aligned} F_B(\tau,c) &= C_{R_n^{-1}}(\tau, 1/c; \Pi = \mathrm{WS}_{R_{TS}}) \\ &= \iint \mathrm{WS}_{R_n^{-1}}(t,f)\mathrm{WS}_{R_{TS}}((t-\tau)c, f/c)\,dt\,df \end{aligned} \tag{50}$$

$(\hat{\tau}, \hat{c})$ are defined in (51), shown on page 762, and the kernel $\Pi$ characterizing the TSR $C_y(\Pi)$ can be expressed as

$$\Pi(t,f) = \begin{cases} \mathrm{WS}_{\hat{R}_{TS}}(t,f) & \text{Case I} \\ \mathrm{WS}_{R_{TS}}(t,f) & \text{Cases II and III} \end{cases} \tag{52}$$

where

$$\mathrm{WS}_{\hat{R}_{TS}}(t,f) = \frac{1}{N_0}\sum_k \frac{\mu_k}{\mu_k + N_0} W_{v_k}(t,f) \tag{53}$$

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$$L_{\mathrm{MAP}}(x) = \begin{cases} \max_{(\alpha,\beta)} \left[ L_O^{(\alpha,\beta)}(x) + \log p(\alpha,\beta) \right] - \log p(\hat{\alpha},\hat{\beta}) & \text{Case I} \\ \max_{(\alpha,\beta)} \left[ \log\left( L_H^{(\alpha,\beta)}(x) \right) + \log p(\alpha,\beta) \right] - \log p(\hat{\alpha},\hat{\beta}) & \text{Case II (deflection optimal)} \\ \max_{(\alpha,\beta)} \left[ \log\left( L_{\mathrm{LO}}^{(\alpha,\beta)}(x) \right) + \log p(\alpha,\beta) \right] - \log p(\hat{\alpha},\hat{\beta}) & \text{Case III (locally optimal)} \end{cases} \tag{13a}$$

$$L_A(x) = \begin{cases} \max_{(\tau,\nu)} [P_y(\tau,\nu;\Phi)] & \text{ML detectors; Cases I and II} \\ \max_{(\tau,\nu)} [P_y(\tau,\nu;\Phi) - F_A(\tau,\nu)] & \text{ML detectors; Case III} \\ \max_{(\tau,\nu)} [P_y(\tau,\nu;\Phi) + \log p(\tau,\nu)] - \log p(\hat{\tau},\hat{\nu}) & \text{MAP detectors; Case I} \\ \max_{(\tau,\nu)} [\log\{P_y(\tau,\nu;\Phi)\} + \log p(\tau,\nu)] - \log p(\hat{\tau},\hat{\nu}) & \text{MAP detectors; Case II} \\ \max_{(\tau,\nu)} [\log\{P_y(\tau,\nu;\Phi) - F_A(\tau,\nu)\} + \log p(\tau,\nu)] - \log p(\hat{\tau},\hat{\nu}) & \text{MAP detectors; Cases III} \end{cases} \tag{41}$$

$$(\hat{\tau},\hat{\nu}) = \begin{cases} \arg\max_{(\tau,\nu)} [P_y(\tau,\nu;\Phi) + \log p(\tau,\nu)] & \text{MAP detectors; Case I} \\ \arg\max_{(\tau,\nu)} [\log\{P_y(\tau,\nu;\Phi)\} + \log p(\tau,\nu)] & \text{MAP detectors; Case II} \\ \arg\max_{(\tau,\nu)} [\log\{P_y(\tau,\nu;\Phi) - F_A(\tau,\nu)\} + \log p(\tau,\nu)] & \text{MAP detectors; Case III} \end{cases} \tag{44}$$

$$L_B(x) = \begin{cases} \max_{(\tau,c)} [C_y(\tau,1/c;\Pi)] & \text{ML detectors; Cases I and II} \\ \max_{(\tau,c)} [C_y(\tau,1/c;\Pi) - F_B(\tau,c)] & \text{ML detectors; Case III} \\ \max_{(\tau,c)} [C_y(\tau,1/c;\Pi) + \log p(\tau,c)] - \log p(\hat{\tau},\hat{c}) & \text{MAP detectors; Case I} \\ \max_{(\tau,c)} [\log\{C_y(\tau,1/c;\Pi)\} + \log p(\tau,c)] - \log p(\hat{\tau},\hat{c}) & \text{MAP detectors; Case II} \\ \max_{(\tau,c)} [\log\{C_y(\tau,1/c;\Pi) - F_B(\tau,c)\} + \log p(\tau,c)] - \log p(\hat{\tau},\hat{c}) & \text{MAP detectors; Case III} \end{cases} \tag{48}$$

$$(\hat{\tau},\hat{c}) = \begin{cases} \arg\max_{(\tau,c)} [C_y(\tau,1/c;\Pi) + \log p(\tau,c)] & \text{MAP detectors; Case I} \\ \arg\max_{(\tau,\nu)} [\log\{C_y(\tau,1/c;\Pi)\} + \log p(\tau,c)] & \text{MAP detectors; Case II} \\ \arg\max_{(\tau,\nu)} [\log\{C_y(\tau,1/c;\Pi) - F_B(\tau,c)\} + \log p(\tau,c)] & \text{MAP detectors; Case III} \end{cases} \tag{51}$$

and

$$\begin{aligned} \mathrm{WS}_{R_{TS}}(t,f) &= \int R_{TS}(t+\tau/2, t-\tau/2) e^{-j2\pi f\tau} d\tau \\ &= \sum_k \mu_k W_{v_k}(t,f). \end{aligned} \tag{54}$$

□

## References

[1] A. M. Sayeed and D. L. Jones, "Optimal detection using bilinear time-frequency and time-scale representations," *IEEE Trans. Signal Processing*, vol. 43, pp. 2872–2883, Dec. 1995.