The Ancient and Modern History of EM Ground-Wave Propagation[1]

James R. Wait
University of Arizona
Tucson, Arizona USA

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1. Abstract

Radio-wave transmission over the surface of the Earth is a subject of enquiry going back to the beginning of the century. In this review, an attempt is made to describe the ground-wave mechanism that is omni-present. We first call attention to the early analytical contributions of Zenneck and Sommerfeld, based on a flat-Earth model. The subsequent controversies, particularly with regard to the role of the Zenneck surface wave, are outlined. Further developments by other pioneers, such as van der Pol, Fock, Bremmer, Norton, and Millington, are reviewed, and an attempt is made to put these in a modern context. We also show that the trapped surface wave can be a significant contribution to the total ground-wave field, when the earth boundary is sufficiently inductive. Mixed-path theory and confirming model tests by Ray King are described briefly, along with calculated propagation curves for two- and three-section paths. The appended bibliography includes references to related topics, such as tropospheric refraction and topographic influences.

2. Introduction

The propagation of radio waves has been a subject of interest for over a century. Beginning with the original investigations of Heinrich Hertz [1], the influence of intervening obstructions in the path was a question often posed. In particular, Marconi [2] investigated the weakening of the field strength when a hill was located between the transmitting and receiving antennas.

Early conjectures, such as by the prophetic Nikola Tesla [3], were that the signal was guided in some fashion by the air-earth boundary. At that time, the possible existence of the ionosphere was also broached [4]. But the pervading idea at the time (i.e., at the turn of the century) was that somehow the presence of the earth was a key factor to be reckoned with. This was the beginning of the subject that we now call "ground-wave propagation." In fact, such is the topic of this review. Of course, the ionosphere plays a role that may be dominant. Never-the-less, understanding the mechanism of ground-wave transmission is important in many applications.

In what follows, we will attempt to employ a consistent notation, which sometimes may differ drastically from that found in the early literature. Of course, here we shall also employ the SI (or rationalized MKS) units, and a harmonic time factor $\exp(+j\omega t)$ is employed throughout.

3. The Zenneck surface wave

The first analytical concept that whetted the appetite of leading theoretical physicists of the day was the possibility that the air-earth interface supported a surface wave with a low attenuation. Indeed, this was the subject of a seminal paper by Zenneck [5]. His model (in our notation) was a conducting half-space, for $z < 0$, with conductivity σ and permittivity ε. The air or insulator, for $z > 0$, has a permittivity ε_0 equal that for free space. The whole region is assumed to be non-magnetic, in the sense that the permeability is the same as the free-space value μ_0. For transmission in the x direction, following Zenneck, it is assumed that, for $z > 0$, the sole magnetic-field component can be expressed by

$$H_{0y} = b_0 \exp(-u_0 z - jgx), \quad (1)$$

and the corresponding form, for $z < 0$, by

$$H_y = b \exp(+uz - jgx), \quad (2)$$

where b_0 and b are constants, and where

$$u_0 = \left(g^2 + \gamma_0^2\right)^{1/2}, \qquad u = \left(g^2 + \gamma^2\right)^{1/2},$$

$$\gamma_0 = jk = j(\varepsilon_0\mu_0)^{1/2}\omega, \text{ and}$$

$$\gamma = \left[j\mu_0\omega(\sigma + j\varepsilon\omega)\right]^{1/2}.$$

Consistent with Maxwell, the tangential electric fields, corresponding to Equations (1) and (2), are

$$E_{0x} = -K_0 b_0 \exp(-u_0 z - jgx) \quad (3)$$

and

$$E_x = +Kb \exp(+uz - jgx), \quad (4)$$

[1][Editor's note: Jim Wait died October 1, 1998. This article had been reviewed and accepted for publication prior to his death. At Prof. Wait's request and after his death, David Hill responded to the recommendations of the reviewers and made the necessary changes. Jeffrey Young proofed the equations. The *Magazine* is very grateful to David Hill and Jeffrey Young for their efforts, and to Prof. Wait for his many years of active support of and contributions to the *Magazine*. "In Memoria" for Prof. Wait appear elsewhere in this issue. His many contributions to the AP-S and radio science communities, of which this article is but one example, are perhaps his most-fitting memorials. WRS]

where $K_0 = u_0/(j\varepsilon_0\omega)$ and $K = u/(\sigma + j\varepsilon\omega)$ have the nature of wave impedances.

Boundary conditions are that tangential-field components are continuous at the interface $z = 0$. Clearly, this means that $h_0 = h$ and, at the same time, g is to be determined from

$$K_0 + K = 0 \tag{5}$$

Thus, the desired solution for the propagation of the Zenneck wave is

$$jg = jg_s = \gamma_0\gamma\Big/\left(\gamma_0^2+\gamma^2\right)^{1/2}. \tag{6}$$

Now , to be meaningful, the real parts of both u_0 and u should be positive. Then, as a consequence, the imaginary part of u_0 is negative, while the imaginary part of u is positive. Thus, in keeping with the physical concept of a guided surface wave, the field amplitudes are attenuated in the directions away from the interface, but the phase velocities are downwards, both above and below the interface at $z = 0$.

A complementary view is to consider a plane wave, incident from above, where the x variation is fixed according to $\exp(-jgx)$. In this case, the reflection coefficient, at the interface $z = 0$, is found to be $(K_0 - K)/(K_0 + K)$, in terms of the impedances K_0 and K. The pole of the reflection coefficient is determined precisely by Equation (5) in the complex g plane. In either case, the phase velocity of the Zenneck surface wave, in the x direction, is

$$v_x = \omega/(\text{Real part of } g_s), \tag{7}$$

which is *greater* than c, the speed of light in air. Also, the attenuation in the x direction is the real part of jg_s in nepers/m.

Another easily determined property of the Zenneck surface wave is the wave tilt of the electric field vector in air at the interface. It is defined by

$$W = [E_{0x}/E_{0z}]_{z=0}. \tag{8}$$

A simple exercise shows that

$$W = \gamma_0/\gamma = [j\varepsilon\omega/(\sigma + j\varepsilon\omega)]^{1/2}, \tag{9}$$

which is a remarkably simple result. The tip of the electric field vector in the xz plane clearly traces out an ellipse, as shown nicely by Zenneck [1] almost a century ago. The corresponding surface impedance, not described by Zenneck, is given by

$$Z = [-E_x/H_y]_{z=0} = j\mu_0\omega\Big/\left(\gamma^2+\gamma_0^2\right)^{1/2}, \tag{10}$$

which is also remarkably simple, but a result not found in classical texts.

It is useful to note that if the postulated model was a grazing plane wave, jg would be replaced by γ_0 or jk. Then, the wave tilt becomes

$$W = (\gamma_0/\gamma)\left[1-(\gamma_0/\gamma)^2\right]^{1/2}, \tag{11}$$

which reduces to Equation (10) when $|\gamma_0^2/\gamma^2| << 1$. The corresponding surface impedance for the grazing-incidence wave is

$$Z = (j\mu_0\omega/\gamma)\left[1-(\gamma_0/\gamma)^2\right]^{1/2}. \tag{12}$$

The idea that a Zenneck-type surface wave could be employed to investigate the subsurface earth layers was pointed out both by Zenneck himself, and by his colleague, Hack [6]. As we indicated above, the wave tilt and its ellipse of polarization is an indicator of the σ and the ε for the homogeneous half-space. Actually, Hack [6] also extended Zenneck's formalism to a two-layer-earth model. In fact, Grosskopf and Vogt [7] interpreted their measured wave-tilt data in terms of the Zenneck-Hack theory. As we know now, the grazing- or lateral-wave postulate is a more realistic descriptor where, particularly at the higher frequencies, $|\sigma + j\varepsilon\omega|$ is not large compared with $\varepsilon_0\omega$ (i.e., for frequencies above 1 MHz over typical ground conductivities of 10^{-3} S/m).

To put the Zenneck wave in perspective, we need to deal with the excitation mechanism; that is our next task.

4. The Sommerfeld problem

The guiding-wave mechanism of the air-earth interface seemed to be a leading contender to explain the long-distance transmission of radio signals, based on the state of the knowledge at the beginning of the century. Arnold Sommerfeld [8], in particular, took the lead here in an attempt to put the question on a firm mathematical basis. His classical analysis dealt, in a fully rigorous fashion, with the problem of determining the fields of electric and magnetic dipoles located, in the insulating half-space, over a conducting half-space. In fact, he also considered the dipoles to be located in the planar interface, but this formulation seemed to have led to some unneeded mathematical complexities. Here, we just deal with the dipoles being above the interface, but their heights may become as small as desired. The solution is outlined very briefly here.

With reference to Figure 1, a vertical electric dipole or current element, Ids, is located at $z = h$ over the conducting half-space, of conductivity σ and permittivity ε. The geometry, as shown, and the cylindrical coordinates, (r, ϕ, z), will be useful for later reference. Of course, in Figure 1, for the vertical electric dipole excitation, azimuthal symmetry prevails, and thus $\partial/\partial\phi = 0$. Sommerfeld's [8] procedure, recapitulated by many, is to express

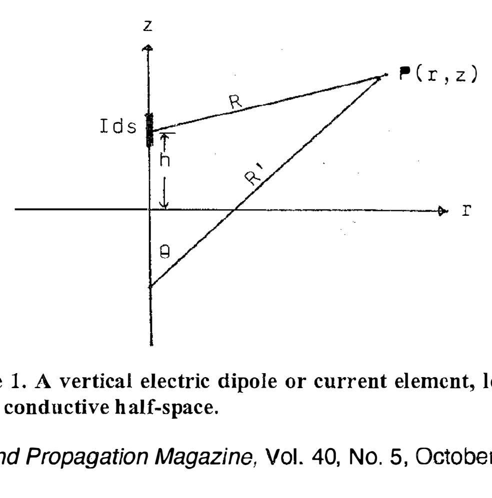

Figure 1. A vertical electric dipole or current element, located over a conductive half-space.

the relevant field components in terms of a Hertz vector, which need have only z components, denoted by U_0 for $z>0$, and by U for $z<0$. Thus, for $z>0$,

$$E_{0r} = \partial^2 U_0 / \partial z \partial r , \tag{13}$$

$$E_{0z} = \left(k^2 + \partial^2/\partial z^2\right) U_0 , \tag{14}$$

$$H_\phi = j\varepsilon_0 \omega \partial U_0 / \partial r , \tag{15}$$

On the other hand, for $z<0$,

$$E_r = \partial^2 U / \partial z \partial r , \tag{16}$$

$$E_z = \left(-\gamma^2 + \partial^2/\partial z^2\right) U , \tag{17}$$

$$H_\phi = (\sigma + j\varepsilon\omega)\partial U / \partial r . \tag{18}$$

Now, we know from basic theory that the primary potential, U_0^p, in the insulating region has the simple form [8, 9]

$$U_0^p = Ids(4\pi j\varepsilon_0\omega)^{-1}\exp(-\gamma_0 R)R^{-1}, \tag{19}$$

where $R = \left[r^2 + (z-h)^2\right]^{1/2}$. The key to the solution is to employ the equivalent form

$$U_0^p = Ids(4\pi j\varepsilon_0\omega)^{-1}\int_0^\infty u_0^{-1}\exp(-u_0|z-h|)\mathrm{J}_0(gr)g\,dg , \tag{20}$$

where $\mathrm{J}_0(gr)$ is the Bessel function of order zero and argument gr. Again, we have $u_0 = \left(g^2 + \gamma_0^2\right)^{1/2} = \left(g^2 - k^2\right)^{1/2}$. Equation (20) is called the Sommerfeld Integral Identity [8], which is an apt descriptor.

The secondary potential, in the region $z>0$, must be added to U_0^p and the resultant potential, for $z>0$, is then

$$U_0 = Ids(4\pi j\varepsilon_0\omega)^{-1}\int_0^\infty u_0^{-1}\{\exp(-u_0|z-h|) + R_e(g)\exp[-u_0(z+h)]\}\mathrm{J}_0(gr)g\,dg , \tag{21}$$

where $\mathrm{Re}(g)$ is a reflection-coefficient function. For the region $z<0$, a suitable form for the potential is a similar integral, where the integrand here involves only the function $\exp(+uz)\mathrm{J}_0(gr)$. The boundary-value solution, implicitly contained in Sommerfeld's 1909 and 1926 papers [8], leads to the result

$$R_e(g) = (K_0 - K)/(K_0 + K), \tag{22}$$

where, as defined earlier, $K_0 = u_0/(j\varepsilon_0\omega)$ and $K = u/(\sigma + j\varepsilon\omega)$. Clearly, the integrand in Equation (21) has a pole at $g = g_s$, as given by Equation (6). Also, there are branch points at $g = -j\gamma_0 = k$ and at $g = kN$, where N is defined below. Explicit expressions for the field integrals are simply obtained by performing the operations indicated by Equations (13), (14), and (15), which hardly need be written out.

While it is often feasible to employ numerical methods for the infinite integrals discussed here, it is always desirable to examine asymptotic solutions, particularly when kr is a large parameter and/or when z and h are small compared with r. Here, we will just quote some of these results.

In the first case, let us look at the expression for the vertical electric field, in the air, at $z=+0$, and for a source dipole at height $h=+0$. Also, we follow Sommerfeld [8] and Norton [10], and assume, at least initially, that $kr \gg 1$, and also that $|\gamma_0/\gamma|^2 \ll 1$. For convenience here and later on, we define the complex refractive index in the lower half-space by $N = \gamma/\gamma_0$, and thus

$$N^2 = (\sigma + j\varepsilon\omega)/(j\varepsilon_0\omega) = (\varepsilon/\varepsilon_0)(1 - j\tan\delta), \tag{23}$$

where $\tan\delta = \sigma/(\varepsilon\omega)$ is the loss tangent, as conventionally defined. The restriction here, that $|N|^2 \gg 1$, seems to have been invoked by nearly all workers, beginning with Sommerfeld and up until the present era.

With the above provisos,

$$E_{0z} \cong E_0 F(p), \tag{24}$$

where E_0 is a reference field given by

$$E_0 = -j\mu_0\omega Ids(2\pi r)^{-1}\exp(-jkr) \tag{25}$$

being the vertical electric field at a distance r over a perfectly conducting flat earth. With this convenient normalization, the effect of finite ground conductivity is accounted for by the "attenuation function" $F(p)$ as it is commonly described. In accord with Sommerfeld [8] and Norton [10], it is defined by

$$F(p) = 1 - (\pi p)^{1/2} \mathrm{e}^{-p}\,\mathrm{erfc}\left(jp^{1/2}\right), \tag{26}$$

where p, the "numerical distance," is given by

$$p = -jkr/\left(2N^2\right) = |p|\mathrm{e}^{j\arg p} . \tag{27}$$

It is important to note that the error function complement in Equation (26) is defined by

$$\mathrm{erfc}\left(jp^{1/2}\right) = \left(2/\pi^{1/2}\right)\int_{jp^{1/2}}^{\infty}\exp\left(-z^2\right)dz , \tag{28}$$

where the contour, in the complex z plane, is from $jp^{1/2}$ via a straight line to the origin, and then along the real axis to $+\infty$. As we can see, for the Sommerfeld-Norton problem, $\arg p$ ranges from $-\pi/2$ to 0 radians as the phase angle N^{-1} ranges from 0 to $\pi/4$ radians. Within these limits, it follows that for $|p| \gg 1$, the leading asymptotic approximation for Equation (26) is

$$F(p) \cong -1/(2p), \tag{29}$$

where succeeding terms vary as p^{-2}, p^{-3}, p^{-4}, ... In this case,

$$E_{0z} \cong const. \times r^{-2} \exp(-jkr), \qquad (30)$$

which has the expected "lateral wave" behavior to follow contemporary nomenclature [11].

In view of the above, we might ask, "Why doesn't the ZSW (the Zenneck surface wave) emerge in this limit? Mathematically, the answer is that the ZSW pole, g_s, given by Equation (6), is improper and occurs on the lower Riemann sheet of the complex z plane. Physically, the explanation is that the excitation of the ZSW by the highly localized source is weak. But, indeed, at short distances, where $|p|$ is relatively small, the ZSW does play a role, in spite of its being on the "wrong" Riemann sheet. This is evident by the power-series development of Equation (26), given by

$$F(p) = 1 - j(\pi p)^{1/2} + 2p + j\pi^{1/2} p^{3/2} + \ldots . \qquad (31)$$

The second term, when multiplied by E_0, leads to an $r^{-1/2}$ amplitude dependence. But calculations of $|F(p)|$, such as displayed by Norton [10], for the range $0 < |p| < \infty$ and $-\pi/2 < \arg p < 0$, demonstrate that $|F(p)|$ never exceeds one. Thus, in effect, this $r^{-1/2}$ term is being swamped by the other terms, except when $|p|$ is small.

The apparent complexity of the analysis here, even for this highly idealized problem, results from the closeness of the ZSW pole at $g = g_s$, albeit on the "wrong" Riemann sheet, and the branch point at $g = k$. But actually, for smaller values of $|N|$ (e.g., $N = 2$), the integral representation such as Equation (2) can be evaluated by a first-order saddle-point method. In this case, the pole g_s is no longer near the branch point or the associated saddle point [11, 12]. Then the modification of Equation (29) is

$$F(p_e) \cong -1/(2p_e), \qquad (32)$$

where

$$p_e = -j(kr/2)N^{-2}\left(1 - N^{-2}\right), \qquad (33)$$

which is strictly only valid if $|p_e| >> 1$. Nevertheless, this suggests that a useful uniform approximation, in place of Equation (24), is to say that

$$E_{0z} \cong E_0 F(p_e) \qquad (34)$$

can be employed for all values of p_e for the homogeneous half-space of complex refractive index N. In passing, it is also useful to note that Equation (33) is equivalent to $p_e = -j(kr/2)W^2$. W is the complex wave tilt, as given by Equation (11), where it was derived for a plane wave at grazing incidence.

To allow Equation (24) to be employed at smaller ranges, where the condition $kr >> 1$ is violated, we may proceed effectively in two ways. First of all, we note that, for $\sigma = \infty$, for any value of kr,

$$E_{0z} = E_0 = -j\mu_0 \omega I ds (2\pi r)^{-1} \left[1 + (jkr)^{-1} + (jkr)^{-2}\right] e^{-jkr}, \quad (35)$$

where the terms varying as $1/r^{-2}$ and as $1/r^{-3}$ correspond to the induction and static fields, respectively. These terms are important when kr is of the order of one or smaller. But, in this case, the numerical distance defined by Equation (27) will satisfy $|p| << 1$, so that $F(p) \cong 1$. Thus, we might argue that Equations (24) and (34) are uniformly valid for all kr, provided N is not near one.

The second way to deal with the problem is to asymptotically evaluate the integrals in the exact representations for E_{0z} when Equations (14) and (21) are employed. Then, we would find that a working expression, useful for any range, would be

$$E_{0z} \cong -j\mu_0 \omega I ds (2\pi r)^{-1} \left[F(p_e) + (jkr)^{-1} + (jkr)^{-2}\right] e^{-jkr}, \quad (36)$$

where p_e is given by Equation (33). This form differs insignificantly from Equation (34) if E_0, defined by Equation (35), is employed. Actually, Equation (36) is the same as the form proposed by Norton [10].

5. Controversies rage

While we are still talking about the case $z = h = 0$ (i.e. ground-based source and observer), it is appropriate to give a short account of the famous "error in sign" in Sommerfeld's original formulation. It now appears that in his 1909 version of [8], there was an apparent problem on how to define the complex argument $jp^{1/2}$ of the function $\operatorname{erfc}(jp^{1/2})$. [The reader is reminded here that a time factor $\exp(+j\omega t)$ is being employed]. It appears that Sommerfeld placed $jp^{1/2}$ in the wrong quadrant of the complex plane, so, as a consequence $|F(p)|$ actually exceeded one for a range of values of p, even when the argument of p was in the range from 0 to $-\pi/2$ radians. Such was evident in a 1911 paper by Sommerfeld [13], where calculations were carried out using the incorrect sign of $\arg p$. Actually, in the 1926 version of [8], the error function of complex argument was handled properly. But this came after various claims were made that the ZSW played a major role in accounting for long-distance transmission of radio waves.

The key paper to set the record straight was a brilliant analysis by Hermann Weyl [14] who employed a spectral formulation of the problem that indicated that the ZSW was not a dominant contribution to the total field. Unfortunately, the celebrated 1909 paper of Sommerfeld left its mark on the radio-engineering community. For example, a lengthy paper by Bruno Rolf [15] showed exhaustive and detailed plots of the attenuation function $F(p)$, in both amplitude and phase, for a wide range of p. But, alas, the curves are wrong. They exhibit spurious interference patterns that play no role for the homogeneous half-space model. Rolf appears to have used the wrong sign when dealing with $\arg p$. Curiously, his curves apply to the unlikely situation that his half-space model of the homogeneous earth had a negative dielectric constant. An example of such a homogeneous medium is a cold lossy plasma [12], but we will not pursue this question here.

At this time (i.e., in the 1930s), there was a flurry of activity on the question of the validity of the Sommerfeld theory, including the modified 1926 version. An important analysis here was made by van der Pol and Niessen [16], who confirmed that the physical form of the Sommerfeld 1926 version was correct. Further careful analytical studies of the basic integrals were carried out by Rice [17] and by Wise [18]. Both validated the Sommerfeld-Norton form for $F(p)$.

In spite of the apparent closure of the analytical debate, the controversies would not die. The late Kenneth Norton, an eminent radio engineer in the USA, exchanged numerous letters with Sommerfeld. An example [19] from Sommerfeld to Norton was written while the former was on holiday in the Austrian Tyrol. Sommerfeld never agreed an error or miscue had ever been made. Two later letters from Norton [20, 21] indicated his views on the separation of surface waves and space waves. Sommerfeld acknowledged Norton's communications, and suggested he compare his results with those of van der Pol and Niessen [16]. Of course, there is a consistency here, because Norton [10] based his papers on van der Pol and Niessen!

Another leading researcher in the USA, working at the Bell Laboratories, was Charles Burrows. He carefully measured the field strength at 150 MHz, for a distance range from one to 2000 m, over a deep, calm, lake near Seneca, in upper New York state [22]. He showed, using data on the electrical properties of the lake water (independently obtained), that the observed field strength vs. distance was in full conformity with Norton [10], but differed from Rolf [23].

6. The space wave and the Norton surface wave clarified

To simplify the above discussion of physical principles, we set $z = h = 0$. For nearly all practical cases, one or both of the heights z and h are nonzero. There has been a great deal of discussion in the literature, for the past 50 years or more, on the form the theory should take for the general case of raised terminals. The pertinent geometry is again shown in Figure 1. Actually, van der Pol and Niessen [16] had considered this case. But it was the tireless Norton [10] who developed an improved form for the Hertz potential for arbitrary heights (i.e., $(z+h)/r$ need not be small compared with one). This writer [12], and particularly his former colleague, Ray J. King [24], proposed various forms that had the uniform property of reducing exactly to geometrical optics, for any N, at higher elevation angles and under the condition that $kR' >> 1$. Here, $R' = \left[r^2 + (z+h)^2\right]^{1/2}$, as indicated in Figure 1. The form [12] of the solution adopted here, for z and h greater than or equal to zero, is

$$U_0 = (4\pi j\varepsilon_0\omega)^{-1} Ids\left[R^{-1}e^{-jkR} + R'^{-1}e^{-jkR'} - 2P\right], \qquad (37)$$

where

$$P \cong j(\pi p_e)^{1/2} e^{-W'} \operatorname{erfc}\left(jW'^{1/2}\right) e^{-jkR'}, \qquad (38)$$

$$W' = p_e\left[1 + (C/\Delta)\right]^2, \qquad (39)$$

$$C = (z+h)/R',$$

$$p_e = -jkR'\left(\Delta^2/2\right)/S^2, \qquad (40)$$

$$S = r/R',$$

$$\Delta = N^{-1}\left[1 - (S/N)^2\right]^{1/2}. \qquad (41)$$

Of course, in terms of the real angle θ, shown in Figure 1, $C = \cos\theta$ and $S = \sin\theta$. So, for relatively small heights, S is near one. Such is often assumed, for example, in writing Equations (39) and (40), without proper explanation. Also, in such cases, C is, in effect, replaced by $(z+h)/r$.

An alternative form of P in Equation (38) is

$$P \cong (p_e/W')^{1/2}\left[1 - F(W')\right]e^{-jkR'}/R', \qquad (42)$$

where

$$F(W') = 1 - j(\pi W')^{1/2} e^{-W'} \operatorname{erfc}\left(jW'^{1/2}\right) \qquad (43)$$

is the same function as $F(p)$ defined by Equation (26).

Using Equation (42), it is now very instructive to rewrite Equation (37) in the decomposed form

$$U_0 = U_0^{(1)} + U_0^{(2)} + De^{-jkR}/R, \qquad (44)$$

where

$$U_0^{(1)} = D\left[1 - 2(p_e/W')^{1/2}\right]e^{-jkR'}/R' \qquad (45)$$

and

$$U_0^{(2)} = D\left[2(p_e/W')^{1/2}F(W')\right]e^{-jkR'}/R', \qquad (46)$$

and where $D = (4\pi j\varepsilon_0\omega)^{-1}$. Then, on using Equation (39), we see that

$$U_0^{(1)} = D\left[(C-\Delta)/(C+\Delta)\right]e^{-jkR'}/R', \qquad (47)$$

$$U_0^{(2)} \cong 2D\left[(C+\Delta)/\Delta\right]F(W')e^{-jkR'}/R'. \qquad (48)$$

In Equation (47), $(C-\Delta)/(C+\Delta)$ can be identified as the exact form of the Fresnel reflection coefficient for a TM plane incident on the half-space with complex refractive index N. Thus, $U_0^{(1)} + De^{-jkR}/R$ can be relabeled U_0^{NSW}, to indicate that it is the Norton Surface Wave. Loosely speaking, it is the correction to geometrical optics to account for the wave dynamics at low elevation angles. It is useful to note that for the grazing limit (i.e., $C \to 0$ or $\theta \to 90°$), the space wave vanishes. Then,

$$U_0 = Ids(2\pi j\varepsilon_0\omega)^{-1}F(p_e), \qquad (49)$$

where

$$p_e = -j(kr/2)\Delta^2, \qquad (50)$$

$$\Delta = N^{-1}\left(1 - N^{-2}\right)^{1/2}. \qquad (51)$$

Then,

$$E_{0z} = -j\mu_0 \omega I ds (2\pi r)^{-1} F(p_e) e^{-jkr}/r, \quad (52)$$

which is in accord with Equation (34).

Sometimes, the fields associated with U_0^{NSW} are identified as lateral waves. Indeed, such is not inappropriate, but the strict equivalence is really only valid for large numerical distances. Our point is that the NSW (Norton Surface Wave) is to be defined as the difference between the total wave field and the easily-computed space wave or geometrical-optical field. [Fortunately, such is consistent with the most recent recommendations of the IEEE Wave Propagation Standards Committee; see the Appendix for definitions of various waves].

At this stage, perhaps it is useful to say something again about the concept of surface impedance in the present context [12]. The assertion (for want of a better word) is to specify that, at the surface of the half-space,

$$\left[E_{0r} = -Z_s H_{0\phi}\right]_{z=0} \quad (53)$$

where, with a certain amount of hindsight, a suitable form for the surface impedance Z_s is selected. It's an interesting exercise to show that the solution for U_0 for the dipole source model, as in Figure 1, can be obtained by applying this impedance condition, provided that we set $Z_s = 120\pi\Delta$, where Δ is given by Equation (41). Actually, Ray King [24] has also employed the surface-impedance model, in an independent integral-equation solution, to obtain essentially the same result as given here by Equations (37) or (44).

7. The relevance to layered models

The beauty (in the eyes of the beholder!) of the surface-impedance technique is that extensions to layered-earth models are readily carried out, without having to deal explicitly with solutions within the layers. As an example, consider the two-layer structure shown in Figure 2. While it is a straight-forward but tedious exercise to solve this particular problem, ab initio, there is a great savings in labor and mental anguish to recognize that the surface impedance in the spectral g plane is [12]

$$Z_s(g) = \frac{K_1\left[K_2 + K_1 \tanh u_1 h_1\right]}{\left[K_1 + K_2 \tanh u_1 h_1\right]}, \quad (54)$$

where $K_1 = \frac{u_1}{(\sigma_1 + j\varepsilon_1\omega)}$, $K_2 = \frac{u_2}{(\sigma_2 + j\varepsilon_2\omega)}$, $u_1 = \left(g^2 + \gamma_1^2\right)^{1/2}$,
$u_2 = \left(g^2 + \gamma_2^2\right)^{1/2}$, $\gamma_1^2 = j\mu_1\omega(\sigma_1 + j\varepsilon_1\omega)$, and
$\gamma_2^2 = j\mu_2\omega(\sigma_2 + j\varepsilon_2\omega)$.

Here, $Z_s(g)$, as given by Equation (54), is exact, but bear in mind it is a function of g. Corresponding expressions for any number of layers are given elsewhere [12, 25]. The equivalent circuit for the present two-layer case is shown in Figure 3. In this case, $Z_s(g)$ is the input impedance, at $z = 0$, of a section of transmission line of length h_1, with propagation constant u_1, and characteristic impedance K_1. The line is terminated by an impedance K_2.

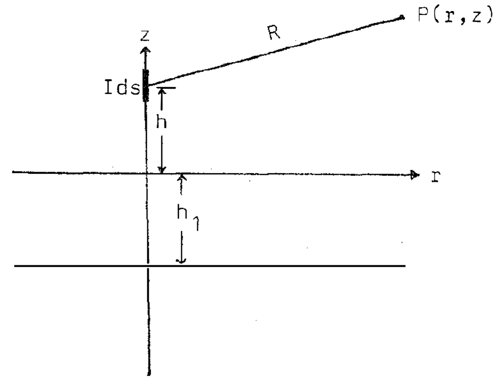

Figure 2. A vertical electric dipole located over a two-layer conductive half-space; otherwise, the geometry is the same as in Figure 1.

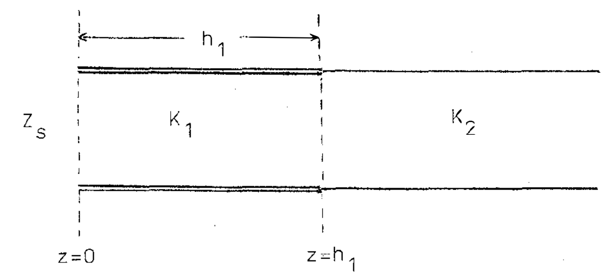

Figure 3. An equivalent transmission-line circuit for a two-layer conductive half-space.

It now follows that the desired solution for U_0, for $z > 0$, is given exactly by Equation (21), but now we replace the expression in Equation (22) for the reflection function by

$$R_e(g) = \frac{(K_0 - Z_s)}{(K + Z_s)}, \quad (55)$$

where Z_s $(= Z_s(g))$ is given precisely by Equation (54). For the problem posed, the solution is exact, as in the case for the homogeneous half-space.

We then argue that the solution given by Equations (37) or (44) is still valid, in an approximate sense, if Δ there is now replaced by Δ_{eff}, the effective normalized surface impedance, given by

$$\Delta_{eff} = \frac{Z_s(kS)}{120\pi}, \quad (56)$$

such that in Equation (54), g is everywhere replaced by kS where, as before,

$$S = \sin\theta = \frac{r}{\left[r^2 + (z+h)^2\right]^{1/2}}.$$

This seemingly ad hoc procedure certainly yields the correct result for the space wave part of U_0 in Equation (45). But the portion $U_0^{(2)}$ should no longer be called the Norton Surface Wave or a lateral wave. As we shall see below, the physics are quite different.

8. The trapped surface wave

To illustrate the form of the layered-earth solution, let us again take $z = h = 0$, which means that the space wave vanishes. The resultant solution is given again by Equation (24), where $F(p)$ is defined by Equation (26), but here, we denote the numerical distance p by

$$p = -j\left(\frac{kr}{2}\right)\Delta_{eff}^2 = |p|e^{j\arg p}. \tag{57}$$

Now it can be determined that $\arg p$ indeed can be positive for various layer parameters. To illustrate this in the context of the model shown in Figure 2, let us assume that the frequency is sufficiently low that $\sigma_1 >> \varepsilon_1\omega$ and $\sigma_2 >> \varepsilon_2\omega$, and let us also neglect any magnetic contrasts (i.e., $\mu_1 = \mu_2 = \mu = \mu_0$). Then, with the common further approximations $u_1 \cong \gamma_1 = (j\sigma_1\mu\omega)^{1/2}$ and $u_2 \cong \gamma_2 = (j\sigma_2\mu\omega)^{1/2}$, it follows readily from Equation (54) that

$$\Delta_{eff} \cong \Delta_1 Q, \tag{58}$$

where

$$\Delta_1 = \frac{jk}{\gamma_1} \cong \left(\frac{\varepsilon_0\omega}{\sigma_1}\right)^{1/2} e^{-j\pi/4}, \tag{59}$$

$$Q = \frac{[\gamma_1 + \gamma_2 \tanh\gamma_1 h_1]}{[\gamma_2 + \gamma_1 \tanh\gamma_1 h_1]}. \tag{60}$$

The formula for the numerical distance is now given by

$$p \cong |p|e^{j2\arg Q} \tag{61}$$

where

$$|p| \cong \left[\frac{\varepsilon_0\omega}{\sigma_1(kr/2)}\right]|Q^2|. \tag{62}$$

As indicated, for this simple model, the complex dimensionless function Q plays an important role. In Figures 4a and 4b, the amplitude and phase, respectively, are shown plotted as a function of the parameter $(\sigma_1\mu\omega)^{1/2}h_1$ for various values of the ratio $\gamma_1/\gamma_2 = (\sigma_1/\sigma_2)^{1/2}$. In particular, it is noted that if this ratio is less than one (i.e., for a relatively higher-conducting substratum), the phase angle of Q, and hence the phase angle of p, can be positive. Such would not occur for any homogeneous earth model.

Another special illustrative case is when the lower layer is perfectly conducting (i.e., $\sigma_2 = \infty$). Then, according to Equation (54), without further approximations,

$$Z_s(g) = K_1 \tanh u_1 h_1. \tag{63}$$

If then one considers that the layer is sufficiently thin, such that $|u_1 h_1| << 1$, we see that

$$Z_s(g) \cong K_1 u_1 h_1 = \frac{u_1^2 h_1}{\sigma_1 + j\varepsilon_1\omega}. \tag{64}$$

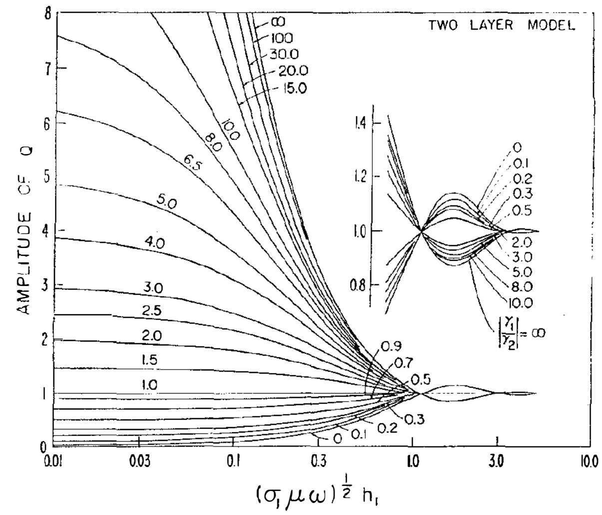

Figure 4a. The amplitude of the correction factor Q for a two-layer conducting half-space, as shown in Figure 2.

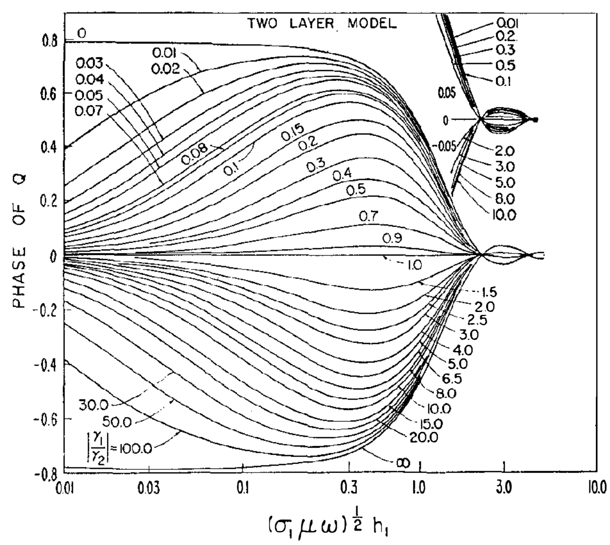

Figure 4b. The phase of the correction factor Q for a two-layer conducting half-space, as shown in Figure 2.

Then, again taking $\mu_1 = \mu_0 = \mu$,

$$Z_s(g) \cong j\mu\omega h_1\left[1 + \frac{g^2}{\gamma_1^2}\right], \tag{65}$$

where $\gamma_1^2 = j\mu\omega(\sigma_1 + j\varepsilon_1\omega)$. Now, we know that if $h_1 = 0$, $Z_s(g) = 0$ and $p = 0$, whence $E_{0z} = E_0$, so the effective value of g, in Equation (65), will be well approximated by k. Furthermore, if $\sigma_1 = 0$, corresponding to a thin dielectric coating layer,

$$Z_s(g) = Z_s \cong j\mu\omega h_1\left(1 - \frac{\varepsilon_0}{\varepsilon_1}\right). \tag{66}$$

This result is identical to Collin's [26] expression for the surface impedance for grazing incidence of a TM plane wave onto such a surface. It is also useful to note that a uniform perfectly conducting surface of small roughness also exhibits a surface impedance which is highly inductive [27]. Accordingly, in such cases, the phase of the numerical distance approaches $+\pi/2$ radians

A relevant exercise is to show how the magnitude $|F(p)|$ varies as a function of $|p|$ for various phase angles of p. To allow for cases of physical interest, $\arg p = b$ can range from $-\pi/2$ to $+\pi/2$ radians, or from -90° to +90°. Here, the explicit expression for the attenuation function is written

$$F(p) = 1 - (\pi p)^{1/2} e^{-p}\left[1 - \mathrm{erf}\left(jp^{1/2}\right)\right], \tag{67}$$

where

$$erf(Z) = \frac{2}{\pi^{1/2}} \int_0^Z \exp\left(-z^2\right) dz . \tag{68}$$

The contour of the error-function integral is from the origin to the complex Z, in the z plane, via a straight line. We note here that $jp^{1/2} = \left|p^{1/2}\right| \exp[j(\pi + \arg p)/2]$ specifies the complex location of the upper limit. The following asymptotic expansions [12], strictly valid for $|p| \to \infty$, are useful:

$$F(p) \cong -\frac{1}{2p} - \frac{1\times 3}{(2p)^3} - \frac{1\times 3\times 5}{(2p)^5} - \cdots \text{ for } b < 0, \tag{69}$$

$$F(p) \cong -2j(\pi p)^{1/2} e^{-p} - \frac{1}{2p} - \frac{1\times 3}{(2p)^3} - \cdots \text{ for } b > 0, \tag{70}$$

where b is the phase angle of p. The apparent discontinuity between Equations (69) and (70), at $b = 0$, is of no consequence, since asymptotically the multiplier e^{-p} vanishes. Actually, the power-series expansion for $F(p)$ given by Equation (31) is valid for all complex values of p for the $\arg p$ from $-\pi/2$ to $\pi/2$, but it is only useful for smaller values of p. In general, numerical integration of Equation (68) provides the needed data in what follows.

The function $|F(p)|$ is shown plotted in Figure 5 for $|p|$ varying from 10^{-3} to 50, for b varying from -90° to +90°. Not surprisingly, there is a considerable enhancement of the field for positive phase angles of p. This is in conformity with Equation (70); the leading term, when multiplied by E_0, can be identified as a trapped surface wave: it has an amplitude variation according to $\left(1/r^{1/2}\right)\exp(-\mathrm{Re}\, p)$, and a phase velocity less than $c = (\mu_0 \varepsilon_0)^{-1/2}$. It is not a Zenneck Surface Wave, which would have a phase velocity greater than c!

The importance of the trapped surface wave (TSW) for ground-wave transmission over stratified or uniformly rough terrain has not been adequately stressed in the past. But further discussion can be found in the references [12, 28-32]. It is interesting to note that in Chapter 15 of [33] and in [34], the authors deal with similar layered models, but the trapped surface waves do not appear.

For the sake of conciseness, we have restricted attention, up to this point, to a flat-earth model for a vertical-electric-dipole

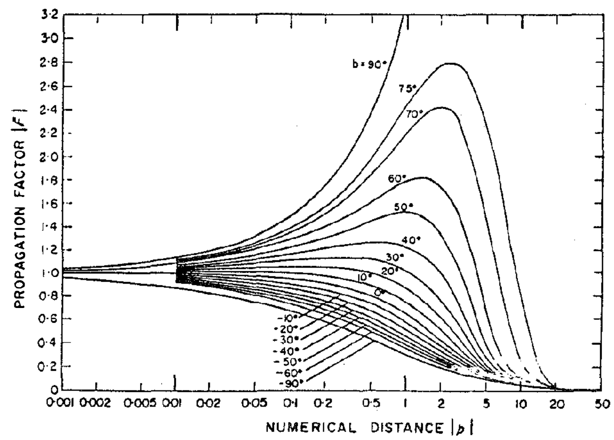

Figure 5. The amplitude of the propagation factor $F(p)$ or the attenuation function as a function of the numerical distance (detail for large p values is not shown).

excitation. As such, we are really talking about vertically polarized ground-wave transmission, to distances where earth curvature is not important. The extensions to magnetic dipole excitation and to arbitrary orientations of both dipole types, again for a flat earth, are described elsewhere [8, 10, 32-41]. Also, the excitation of the Zenneck Surface Wave by an appropriately tapered vertical aperture is considered in [37].

9. Earth sphericity considered

We have deferred the discussion of earth curvature in order to clarify some contentious issues that have arisen in the distant and recent past. But to deal with the actual physical world, we certainly need to adopt a spherical model. Perhaps the key paper here is by G. N. Watson [42]. He began with the classic harmonic-series solution for a dipole, in the presence of a homogeneous sphere of radius a, and with electrical properties given by σ, ε, and μ. The surrounding space had properties ε_0 and μ_0, with zero conductivity. While the solution was mathematically correct, the series representation, involving spherical Bessel functions of integer order, was notoriously poor in convergence, because ka, the sphere radius in wavelengths, was enormous. Watson's first step was to represent the series by a contour integral that enclosed the real axis of the complex wave-number plane. Then, he wrapped the contour around a manageable number of complex poles, which provided a highly convergent residue series in the shadow region. We refer the reader to other places [e.g. 12, 43, 44], or to Watson's original analysis [42] for details.

To implement the residue series solution it was necessary to locate the poles in the complex wave number plane. An essential task here is to deal with the solution of the eigenvalue equation:

$$\frac{\partial}{\partial x}\left[x h_\nu^{(2)}(x)\right]_{x=ka} - j\Delta = 0, \tag{72}$$

where $h_\nu^{(2)}(x)$ is the spherical Hankel function of complex order ν and argument x ($= ka$). In our lexicon, Δ is the surface impedance of the sphere which, strictly speaking, is a function of ν. As usual in our exposition, Δ has been normalized by $(\mu_0/\varepsilon_0)^{1/2}$, or 120π. The discrete pole locations, at $\nu = \nu_s$, determine the angular variation of the propagation. Staying away from the antipode, we can say that the field of an individual mode of order s varies as

$[\sin(d/a)]^{-1/2}\exp(-jv_s d/a)$, where d is the great-circle distance to the observer. Important contributions to determining the complex pole locations v_s were made by Wwedensky [46], van der Pol and Bremmer [47, 48], and Eckersley and Millington [49]. Recognize that for the important modes (i.e., those with low attenuation), both v_s and ka are large parameters, but their difference, $|v_s - ka|$, is of the order of $(ka)^{1/3}$. This suggests that a key variable is t, defined by

$$t = \left(\frac{2}{ka}\right)^{1/3}(v - ka), \quad (73)$$

as employed consistently by Fock [50], Feinberg [51], and other Soviets. The key asymptotic approximation amounts to representing the spherical Hankel function by

$$kah_v^{(2)}(ka) \cong j(ka)^{-1/6}w(t). \quad (74)$$

$w(t)$ is the Airy integral, defined here by

$$w(t) = \pi^{-1/2}\int_C \exp\left[tz - \frac{z^3}{3}\right]dz, \quad (75)$$

where the contour C runs, in the complex z plane, from $\infty e^{j2\pi/3}$ to the origin, and then along the real axis to $+\infty$. Actually, Equation (74), or its equivalent, was employed over a century ago by the Dane, Ludwig Lorenz [52], who employed cylindrical Hankel functions of order 1/3 and 2/3 and complex arguments. van der Pol and Bremmer [47, 48], in their extensive studies, also consistently employed these rather awkward multi-valued functions that are prone to misinterpretation. This writer, in all deference to his mentor Hendricus Bremmer, prefers the more convenient Airy function forms. Then, the mode equation, as given by Equation (72), takes the simpler form

$$w'(t) - qw(t) = 0, \quad (76)$$

where

$$q = -j(ka/2)^{1/3}\Delta. \quad (77)$$

It's also useful to note that $w(t)$ satisfies what is known as Stokes' equation

$$w''(t) - tw(t) = 0, \quad (78)$$

valid for all complex values of t. Primes here in Equations (76) and (77) indicate differentiation with respect to t.

The next non-trivial chore is to determine the roots of Equation (76), which are designated t_s. There are many good references to this particular task, notably [12, 48, 49, 53, 54]. The evaluation of the needed residues of the complex poles is straight forward, where typically one makes use of Stokes' equation or its equivalent.

Leaving aside many details, we will now write down the resulting radial (i.e., vertically oriented) electric field E_{0r} in the air, at a great-circle distance d, for the radial (i.e., vertical) electric dipole source of moment Ids. The result is given by the so-called "residue series:"

$$E_{0r} = E_0 W(x,q), \quad (79)$$

where

$$E_0 = -\frac{j\mu_0\omega Ids}{2\pi d}e^{-jkd} \quad (80)$$

is a reference field, and $W(x,q)$ is the propagation factor. Actually, Equations (79) and (80) are analogous to Equations (24) and (25), respectively, for the flat-earth case. In the present case, $x = (ka/2)^{1/3}(d/a)$ is a normalized range parameter, and q is defined by Equation (77). Then,

$$W(x,q) = \left(\frac{\pi x}{j}\right)^{1/2}\sum_{s=1,2,3,\ldots}^{\infty}\frac{G_s(y_a)G_s(y_b)\exp(-jxt_s)}{\left(t_s - q^2\right)}, \quad (81)$$

where

$$G_s(y) = \frac{w(t_s - y)}{w(t)} \quad (82)$$

is a "height-gain" function. Here, $y_a = [2/(ka)]^{1/3}kh_a$ and $y_b = [2/(ka)]^{1/3}kh_b$, where h_a and h_b are the heights of the source dipole and the observer, respectively, above the surface of the earth. A number of approximations are intrinsic here. These are that h_a and h_b are somewhat less than the range, d, while the latter is less than the earth radius, a. Actually, the approximation $G_s(y) \cong 1 + j\Delta kh$, where $h = h_a$ or h_b, is valid for the dominant modes being the same as for a flat earth.

The most extensive presentation of numerical results for E_{0r} is by Rothcram [55], who also accounted for the modification caused by normal atmospheric refraction. The well-known and still very useful paper by Norton [56] is based essentially on the leading term of Equation (81), which is then augmented by the complementary flat-earth results. The transition between the spherical and the planar predictions is "engineered" by a graphical procedure, which is surprisingly accurate. Another early and also very useful development was made by Burrows and Gray [57]. Their "shadow factors," in a sense, were a concept later known as "Fock Theory." In this connection, there exists a beautiful exposition in a Lockheed report by Nelson Logan [58]. A follow-on paper by Logan and Yee [59] dealt specifically with the attenuation and phase characteristics of the creeping waves on convex cylinders.

An interesting question can be posed. What happens to the residue-series representation, such as given here by Equation (81), when the earth curvature vanishes, that is, when $a \to \infty$? The convergence of the series would seem to be increasingly bad. Earlier numerical studies showed that the spherical-earth propagation factor did tend to the flat-earth case, if 100 or more terms in the residue series were taken. But the analytical connection was not at all obvious. Here, the genius of Bremmer [60] came to the fore. Using his approach, this writer [45, 61] found that the following series, in inverse powers of ka, was valid:

$$W(x,q) \cong F(p) - \left(\frac{\delta^3}{2}\right)\left[1 - j(\pi p)^{1/2} - (1+2p)F(p)\right]$$
$$+\delta^6\left\{1 - j(\pi p)^{1/2}(1-p) - 2p + \frac{5}{6}p^2 + \left[\frac{p^2}{2} - 1\right]F(p)\right\} \quad (83)$$
$$+ \text{ terms in } \delta^9,\ \delta^{12},\ \text{etc.},$$

where $\delta^3 = -1/(2q^3) = j/(ka\Delta^3)$. This type of expansion was useful in a region where the flat-earth attenuation $F(p)$ was becoming inadequate to predict the phase of the ground wave. Of course, at larger distances, the residue series has a manageable convergence. Furthermore, in the overlap region at intermediate ranges, the consistency of the flat-earth-curvature-corrected form, as given by Equation (83), with the residue series, was an excellent check on the numerical work.

There is another theoretical point of view which helps one (i.e., this writer) reconcile the apparent disparity of the solution forms between the flat-earth results and the residue series at short ranges over the spherical earth. The original Sommerfeld solution for the dipole over the homogeneous half-space involved an integral with a branch cut at $g = k$. The corresponding branch cut, in the lower half-space of the complex g plane, led to a continuous spectrum of waves. But this was really equivalent to the totality of the residue series, when the infinite string of these was considered in the limit of zero curvature.

A related and highly desirable property of the spherical-earth residue-series solution is that the radial wave functions $h_{\nu_s}^{(2)}(kr)$ are orthogonal over the range $a < r < \infty$, *when* the physical problem is formulated with an impedance boundary condition at $r = a$. [The joker, of course, is that the surface impedance must not depend on the mode number. In this case, for a finite earth radius a, the mode spectrum is entirely discrete, much to the surprise of some of my esteemed colleagues].

10. Mixed-path theory

The needed extension of the spherical earth, as formulated above, to allow for lateral non-uniformity is now considered in a rather limited context. The classic example is the situation when the path, between source and observer, passes from land to sea or from sea to land. Apart from some early empirical approaches to predict the field-strength-versus-distance variation, the first analytical proposal was put forth by George Millington [62]. He argued that the transmission up to the coastline, from a land-based transmitting antenna, behaved in the manner as for a fully uniform or homogeneous path. Beyond the coastline, the propagation was assumed to behave as it should for an all-sea path, but with an adjustment to allow continuity of the vertical electric field at the coastline. For transmission in the reverse direction, he employed the same argument. But, lo and behold, he found that the resultant fields, for the same source-dipole current moment, were different. Undeterred, he then took the geometric mean of the two predictions and asserted that such was a "good" estimate of the actual field. Indeed, reciprocity is now satisfied. George's keen intuition did not fail him, because later analytical developments, as discussed below, showed that such a "lashed-together formulation" had validity, in a limited sense.

Again it was Bremmer [63] who was inspired then to provide a firm analytical basis for this mixed-path problem. This writer received an early personal account of this work in 1955, from Bremmer, following his presentation at the 1954 URSI General Assembly in The Hague. His multiple-scattering approach, while rather complicated, did, in an asymptotic limit, reproduce Millington's geometric-mean formula.

A somewhat more general technique to deal with mixed-path geometries is based on the compensation theorem, as formulated by G. D. Monteath [64, 65]. I had the good fortune to visit "Monty" at the BBC Research Labs in Tadworth, Surrey, in May, 1956. At the time, we discussed how his formulation could be employed to set up an integral equation for propagation over laterally inhomogeneous paths. But, to introduce the topic here, let me outline the basic concepts.

Monteath's compensation theorem is actually related to Lorentz's reciprocal relationship for vector $\vec{E}$ and $\vec{H}$ fields [66]. In the present context, it is applied to a smooth surface or interface of surface impedance Z', which may be any function of the surface coordinates. Then, an identical reference surface is taken, which has a laterally uniform surface impedance Z. In what follows, Z' and Z are referred to as the modified and unmodified surface impedances, respectively. Now, we locate two short antennas, denoted a and b, over the surface. Then, the mutual impedance between the terminals of these antennas is denoted z'_{ab} for the modified surface, and z_{ab} for the unmodified surface. Following Monteath [64, 65], we have the elegant-but-deceptively-simple-appearing result:

$$z'_{ab} - z_{ab} = (I_a I_b)^{-1}\iint_S \left[\vec{E}_a \times \vec{H}'_b - \vec{E}'_b \times \vec{H}_a\right]_n dS, \qquad (84)$$

where $\vec{E}'_b$ and $\vec{H}'_b$ are the vector electric and magnetic fields over the modified surface for a current I_b injected into the terminals of antenna b, and $\vec{E}_a$ and $\vec{H}_a$ are the vector fields over the unmodified surface for a current I_a injected into the terminals of antenna a. As indicated by the subscript n in Equation (84), the normal components of the cross products are taken, and the integration is over the surface S where Z' differs from Z. In our discussion, the tangential fields satisfy the appropriate surface-impedance boundary conditions, but actually, Equation (84) is valid in a more general sense.

In Monteath's engineering research, Equation (84) was applied to antenna ground systems at MF, where the modified and unmodified tangential magnetic fields did not differ significantly, so that the modified tangential field $\vec{H}'_{bt}$ could be replaced by the unmodified field $\vec{H}_{bt}$. The modified tangential electric field $\vec{E}'_{bt}$ is then replaced by $-Z'\left[\vec{n} \times \vec{H}_{bt}\right]$. Such an approach can be very useful for a number of problems, in order to gain an initial assessment. But, here we will go a step further, and show how one accounts for the change of the modified tangential fields before doing the double integral in Equation (84).

To simplify the discussion, we consider an idealized two-section mixed path, as indicated in Figure 6, where the great-circle distance between antennas a and b is d. The surface impedance is a constant Z, except for the modified portion of constant surface impedance Z_1 of length d_1, measured from antenna b. Now, according to Equation (84),

$$z'_{ab} - z_{ab} = (I_a I_b)^{-1}(Z_1 - Z)\iint_{S_1} \vec{H}_{at} \cdot \vec{H}'_{bt}\, dS,$$

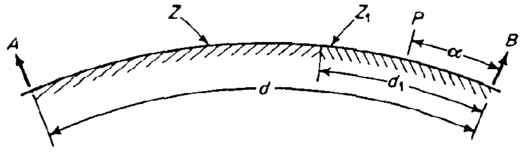

Figure 6. The geometry for propagation over a two-section spherical earth, where great-circle distances d and d_1 are shown.

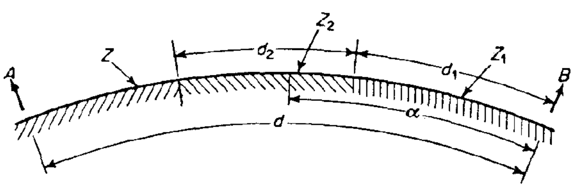

Figure 7. The geometry for a three-section spherical-earth path.

where $\vec{H}_{at}$ and $\vec{H}'_{bt}$ are the tangential magnetic fields over the surface S_1 to the right of the junction between the two regions. The double integral can be simplified if we apply a stationary-phase argument, to reduce it to a line integral of length d_1. This is justified by noting that contributions from off the great-circle path tend to be self-canceling. This aspect of the problem is discussed in some detail elsewhere [31, 36, 67]. Also, in Equation (85), it is useful to note that

$$z'_{ab} = \frac{\ell_a \ell_b j \mu_0 \omega}{2\pi d} e^{-jkd} W'(x,q,q_1), \tag{86}$$

and

$$z_{ab} = \frac{\ell_a \ell_b j \mu_0 \omega}{2\pi d} e^{-jkd} W(x,q), \tag{87}$$

where W' and W are the propagation factors for the modified and the unmodified cases, respectively. Here, ℓ_a and ℓ_b are the effective lengths of antennas a and b, respectively. We can also express $\vec{H}_{at}$ and $\vec{H}'_{bt}$ in terms of these propagation factors. Leaving aside further details, we arrive at the following one-dimensional integral equation [68]:

$$W'(x,q,q_1) = (x/\pi)^{1/2} e^{-j\pi/4}(q_1 - q) \times \int_0^{x_1} W(x-x'',q) W'(x'',q_1,q) \left[x''(x-x'')\right]^{-1/2} dx'', \tag{88}$$

where, to be consistent with earlier notation,

$$x = \left(\frac{ka}{2}\right)^{1/3} \frac{d}{a}, \quad x_1 = \left(\frac{ka}{2}\right)^{1/3} \frac{d_1}{a}, \quad x'' = \left(\frac{ka}{2}\right)^{1/3} \frac{\infty}{a},$$

$$jq = \left(\frac{ka}{2}\right)^{1/3} \Delta, \quad jq_1 = \left(\frac{ka}{2}\right)^{1/3} \Delta_1,$$

$$\Delta = \frac{Z}{120\pi}, \quad \Delta_1 = \frac{Z_1}{120\pi}.$$

Here, the unknown attenuation function or propagation factor is to be determined. Various numerical methods can be brought to bear without making any further approximations. But in the case of radiowave propagation over the earth's surface, across junctions such as flat-lying coastlines, the reflections are very weak. This means that when x_1 is negative or when $d_1 < 0$ in Figure 6, $W'(x,q,q_1)$ can be replaced by $W(x,q)$. For the same reason, $W'(x'',q_1,q)$ in the integrand of Equation (88) can be replaced, for the case x_1 positive or for $d_1 > 0$, by $W(x'',q_1)$. Thus, in effect, we have reduced the integral equation to an integral formula to calculate the field variation as the observer crosses the coastline. It is somewhat of an improvement over the case where we replace W' in the integrand of Equation (88) by $W(x'',q_1)$, which would be a first-order perturbation.

The extension of the above procedure can be carried out for a multi-section path. For example, in the case of a three-section path, such as shown in Figure 7, the resultant form for the propagation factor is given by

$$\begin{aligned} W'(x,q,q_2,q_1) &= W(x,q) \\ &+ \left[x/(j\pi)\right]^{1/2}(q_1 - q) \int_0^{x_1} W(x-x'',q) W(x'',q_1) \left[x''(x-x'')\right]^{-1/2} dx'' \\ &+ \left[x/(j\pi)\right]^{1/2}(q_2 - q) \int_{x_1}^{x_1+x_2} W(x-x'',q) W'(x'',q_1,q_2) \left[x''(x-x'')\right]^{-1/2} dx'' \end{aligned} \tag{89}$$

where

$$jq_2 = \left(\frac{ka}{2}\right)^{1/3} \Delta_2, \qquad x_2 = \left(\frac{ka}{2}\right)^{1/3} \frac{d_2}{a}.$$

Using Equation (88), we see that in Equation (89)

$$W'(x'',q_1,q_2) = W(x'',q_2) + \left[x''/(j\pi)\right]^{1/2}(q_2 - q_1) \int_0^{x''-x_1} W(x''-x',q_1) W(x',q_2) \left[(x''-x')x'\right]^{-1/2} dx' \tag{90}$$

Thus, Equation (89) is an explicit formula to calculate transmission over the three-section path. Actually, the integrations can be done employing the residue-series representations for the W functions,

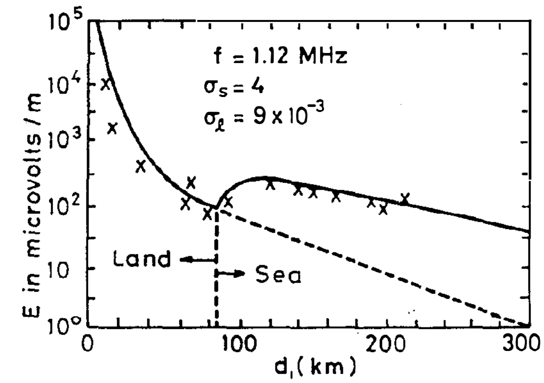

Figure 8. Millington's data for transmission from land to sea, where the coastline is 80 km from the transmitter (the solid curve is calculated).

but these lead to doubly and triply infinite series. Usually, it is simpler to employ numerical integration of these convolution integrals.

As indicated earlier, these mixed-path formulations consistently neglect reflections at the junctions between different homogeneous regions. Also, near-field effects are ignored, in that static and induction fields are not incorporated into formulations. Further discussion of the methodologies, such as reviewed here, and extensions of the analytical techniques, are given in the references [29, 36, 61, 69-72, 74, 75-90].

11. Experimental studies

The experimental confirmation of mixed-path propagation theories is available in a few limited places. The classic example is shown in Figure 8, where the transmission from a BBC transmitter in England at 1.12 MHz is recorded as a function of distance, for propagation across a coastline, 80 km from the source, out over the North Sea to a range of 200 km. The data points are taken from Millington [62], while the calculations are based on Equation (88) for a sea conductivity of 4 mhos/m and a land conductivity of 9×10^{-3} and a dielectric constant of 10. The dashed curve is the calculated extension, assuming the path was all land. The celebrated "recovery effect" is certainly in evidence here. One should

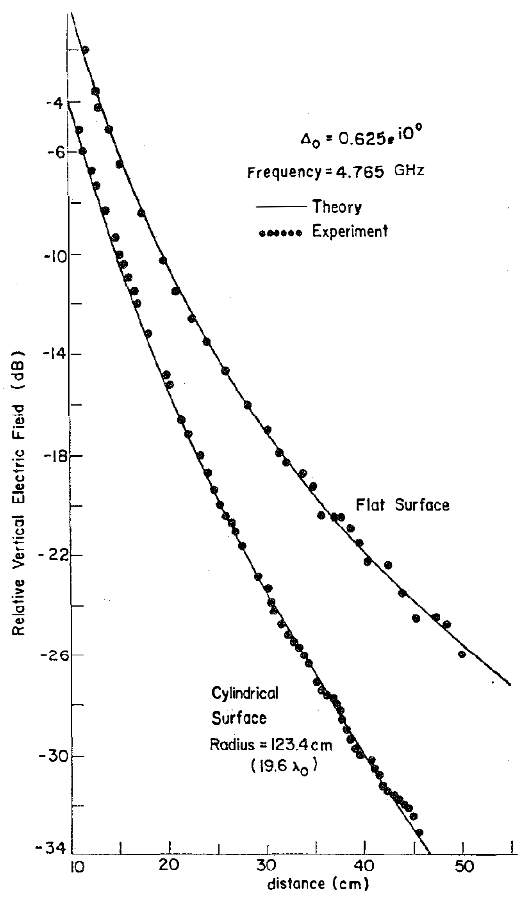

Figure 9. The field strength as a function of distance for both planar and cylindrical surfaces, showing a comparison between theory and experiment (courtesy of Ray J. King).

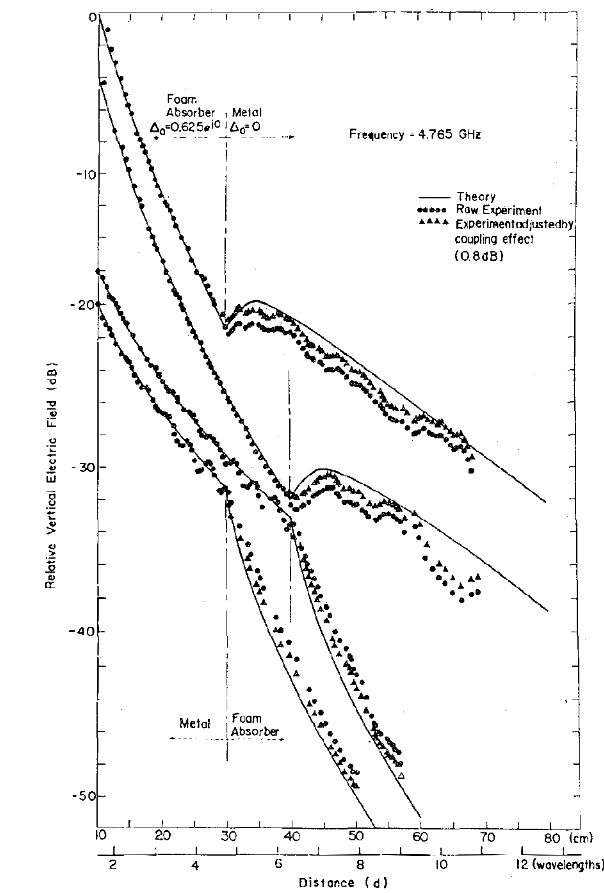

Figure 10. The field strength as a function of distance over a two-section path, showing a comparison between theory and experiment (courtesy of Ray J. King).

bear in mind that the observations of the vertical electric field were effectively at zero elevation, as were the calculations. For receiving antenna heights of 300 m or more, the "recovery effect" is much diminished.

While experiments in-situ are useful, the conditions upon which they are based are not usually known in sufficient detail to allow for a fully satisfactory confirmation of the theory. In such cases, there is much to be said for scale-model tests. When dealing with low or medium radio frequencies (e.g., 10 kHz to 3 MHz), the microwave range is particularly convenient for scaling the essential frequencies. This writer was involved in facilitating a program at the University of Colorado, in Boulder, which was carried forth by Professors Sam Maley, Ray King, David Chang, Ezzy Bahar, and others, including graduate students. The time period was roughly from 1957 to 1977, but the activity was not continuous. Certainly, the pinnacle of achievement during this era was the PhD research by Ray King. His thesis [67] was completed in 1965. He continued his efforts at the University of Wisconsin for the next decade.

An early example of the scale-model work was to confirm the ground-wave propagation theory over a smooth homogeneous spherical earth. Actually, in the laboratory model, the surface employed was cylindrical, rather than spherical. A 2.5 cm foam coating on the metal cylinder exhibited an effective (normalized) surface impedance $\Delta_0 = 0.625 e^{j0}$ at 4.765 GHz. In Figure 9, the results of Ray King [36, 67] are shown when the relative fields are plotted for the distance d from 10 to 60 cm, both for a flat surface and for a cylindrical surface with a radius of 123.4 cm. (i.e., 19.6 free-space wavelengths). The agreement between theory and experiment is rather good. While this particular test was for a cylindrical surface, the diffraction process is closely related to that for a spherical coated metal sheet, which is a bit difficult to fabricate. The main difference is that the reference field for the cylindrical model is slightly different than for the spherical model.

A rather convincing confirmation of the two-section mixed-path theory is shown in Figure 10. Here, propagation takes place from the simulated land of (normalized) surface impedance $\Delta_0 = 0.625$ up to a distance of 40 cm, and then beyond the simulated coastline, out to a distance of 70 cm over the simulated sea, where the effective conductivity is effectively infinite. Also shown in Figure 10 are the results for transmission in the reverse direction, where the junction (simulated coastline) is at a distance $d = 30$ cm from the source over the "sea." As Ray King [67] points out, the experimental data were "adjusted" by 0.8 dB, to account for the changed sensitivity of the receiving probe as it crossed the junction. This change was not empirical, but determined by independent means [36]. Also, as may be seen in Figure 10, Millington's "recovery effect" was quite noticeable when transmission was from land to sea, while for transmission in the reverse direction, the field strength was weakened beyond the junction. Many related microwave scale-model tests, by Ray King and his colleagues, are described in the references [36, 67, 76, 91].

12. Sample terrestrial mixed-path calculations

It's fair to say that one should have considerable confidence in the mixed-path ground-wave theory, as expounded above. Thus, it is appropriate to show some samples of calculated field-strength curves, for terrestrial paths at low radio frequencies. Following the well-founded practice [21, 56, 75], the effective earth's radius, a, was taken as 4/3 the actual radius, to account for normal atmospheric refraction. Thus, in Figures 11a and 11b, we show the amplitude and phase, respectively, of the propagation factor, W, as a function of distance for 1 to 1000 km, for a land-to-sea trans-path where the junction (i.e., the coastline) is at various distances d_2 from the source [88]. The two special cases shown are for an all-sea path and for an all-land path. The land conductivity is 10^{-2} mhos/m, while the sea conductivity is 4 mhos/m. These calculations where carried out for a frequency of 1 MHz. Clearly, here

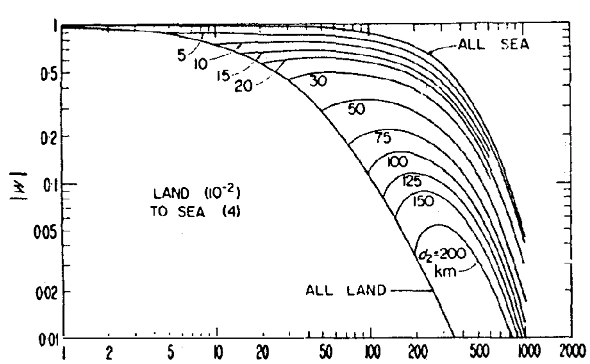

Figure 11a. The calculated amplitude of the attenuation function for a two-section path on a spherical earth, at a frequency of 1 MHz (note that $d_2 = d - d_1$).

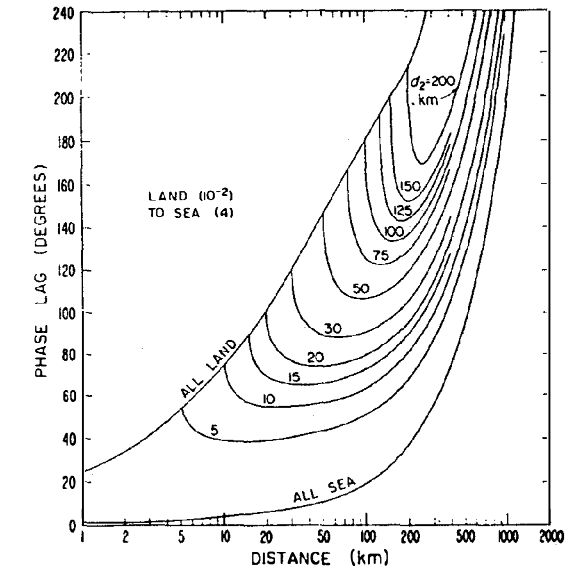

Figure 11b. The calculated phase of the attenuation function for a two-section path on a spherical earth, at a frequency of 1 MHz (note that $d_2 = d - d_1$).

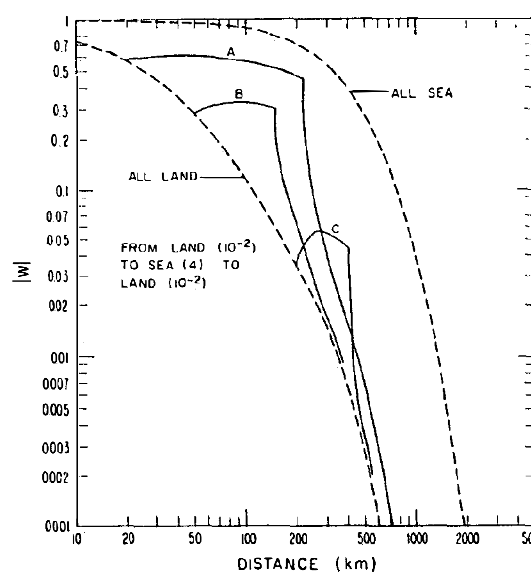

Figure 12. The calculated amplitude of the attenuation function for a three-section path on a spherical earth, at a frequency of 1 MHz.

the recovery effect is very striking. Also, one should notice here that there is a corresponding drastic drop in the phase lag as the observer crosses the junction. The case for a three-section path is shown in Figure 12, where the amplitude of W is plotted as a function of distance, going from land to sea, and then from sea to land. Again, the same electrical properties are adopted as in Figure 11a, and the frequency is also 1 MHz. Other such examples are available [88, 92].

13. Concluding remarks

As indicated in the foregoing text, we have restricted attention to smooth, flat, and curved surfaces, with piece-wise uniform sections. Extensions to irregular and topographic obstacles are covered elsewhere [79, 93-97]. Further details on the refraction effects in the lower neutral atmosphere, and the validity of the effective-earth-radius concept, are found in references [12, 49, 55, 56, 98, 99]. But most importantly, we should stress again that the influence of the ionosphere has not been considered at all. By definition, "ground-wave propagation" refers to that mechanism of transmission that does not include ionospherically reflected signals. Such an assumption is meaningful, in a physical sense, out to a range of 400 km in the daytime, and for shorter distances at night. Of course, one can treat the combined effect of ground waves and the reflected sky waves using a modified ray theory, which is feasible out to 2000 km or more [12, 53]. Also, transient or pulse transmission has not been considered; relevant references are [100-104]. Other related references, covering material not cited, are [105-122].

14. Appendix: Definitions of Various Waves

Over the years, the terminology for the various waves involved in ground-wave propagation has not been totally consistent in the literature. The terminology in this article follows that of the recently revised IEEE Standard 211, "IEEE Standard Definitions of Terms for Radio Wave Propagation." The most relevant definitions are reproduced below [123].

Ground wave. From a source in the vicinity of the surface of the Earth, a wave that would exist in the vicinity of the surface in the absence of an ionosphere. Note: The ground wave can be decomposed into the Norton surface wave and a space wave consisting of the vector sum of a direct wave and a ground-reflected wave.

Improper mode. Refers to a mode of propagation which cannot be excited by a physical source in the absence of other modes, e.g., Zenneck surface wave.

Lateral wave. A wave, not predicted by geometrical optics, excited at and propagated along the interface of two (possibly lossy) dielectric media. For sufficiently large distances from the source, the magnitude of the field varies as the inverse square of the distance measured along the interface. Note: The lateral wave is similar to the component of the radio ground wave when the geometrical-optical component is separated out.

Norton surface wave. The propagating electromagnetic wave produced by a source over or on the ground. The Norton wave consists of the total field minus the geometrical-optics field.

Surface wave. A wave guided by a boundary with a surface impedance whose reactive part exceeds the resistive part. A surface wave is generally characterized as a slow wave having a magnitude which exponentially decreases with distance from the interface but may be modified by curvature. [This definition applies equally well to the trapped surface wave analyzed in this paper.]

15. References

1. H. Hertz, *Electric Waves*, London, Macmillan & Co., 1893.

2. G. Marconi, "Wireless Telegraphy," Jour. IEE (UK), **28**, 1899, pp. 273-315.

3. N. Tesla, "Art of Transmitting Electrical Energy Through Natural Mediums," US Patent 787 412, April 18, 1905.

4. O. Heaviside, "The Theory of Telegraphy," in *Encyclopedia Britannica*, London, 1902.

5. J. Zenneck, "Propagation of Plane EM Waves Along a Plane Conducting Surface," *Ann. Phys.* (Leipzig), **23**, 1907, pp. 846-866.

6. F. Hack, "Propagation of EM Waves Over a Plane Conductor," *Ann. Phys.* (Leipzig), **27**, 1908, pp. 27-37.

7. J. Grosskopf and K. Vogt, "The Measurement of Electrical Conductivity for a Stratified Ground," *Hochfrequenztech. u. Elettroakust.*, **58**, 1941, pp. 52-57.

8. A. N. Sommerfeld, "Propagation of Waves in Wireless Telegraphy," *Ann. Phys.* (Leipzig), **28**, 1909, pp. 665-737; **81**, 1926, pp. 1135-1153.

9. J. R. Wait, "Radiation From a Vertical Electric Dipole Over a Stratified Ground," *IRE Transactions on Antennas and Propagation*, **AP-1**, 1953, pp. 9-12; **AP-2**, 1954, pp. 144-146.

10. K. A. Norton, "The Propagation of Radio Waves Over the Surface of the Earth," *Proceedings of the IRE*, **24**, 1936, pp. 1367-1387; **25**, 1937, pp. 1203-1236.

11. L. Brekhovskikh, *Waves in Layered Media*, New York, Academic Press, 1970.

12. J. R. Wait, *Electromagnetic Waves in Stratified Media*, New York, Pergamon Press, first edition 1962; second enlarged edition 1970; first edition reprinted by IEEE Press 1996.

13. A. N. Sommerfeld, "Propagation of Waves in Wireless Telegraphy–Influence of the Nature of the Soil," *Jahrb. d. Drahtl. Tel.*, **4**, 1910, pp. 157-176.

14. H. Weyl, "The Propagation of Electromagnetic Waves Over a Plane Conductor," *Ann. Phys.*, **60**, 1919, pp. 481-500.

15. B. Rolf, "Graphs to Prof. Sommerfeld's Attenuation Formula for Radio Waves," *Proceedings of the IRE*, **18**, 1930, pp. 391-402.

16. B. van der Pol and K. F. Niessen, "The Propagation, of Electromagnetic Waves Over a Plane Earth," *Ann. Phys.*, **6**, 1930, pp. 273-294.

17. S. O. Rice, "Series for the Wave Function of a Radiating Dipole at the Earth's Surface," *Bell System Technical Journal*, **16**, 1937, pp. 101-109.

18. W. H. Wise, "The Physical Reality of the Zenneck Surface Wave," *Bell System Technical Journal*, **16**, 1937, pp. 35-44.

19. A. N. Sommerfeld, letter to K. A. Norton from the Zirmerhof, Tyrol, September 5, 1937.

20. K. A. Norton, letters to Prof. Sommerfeld, both dated September 20, 1937.

21. K. A. Norton, "The Sommerfeld Error in Sign," Annex to Lecture X.2, EM Propagation Course, Boulder Colorado, July 3, 1968.

22. C. R. Burrows, "The Surface Wave in Radio Propagation Over Plane Earth," *Proceedings of the IRE*, **25**, 1937, pp. 219-229.

23. B. Rolf, *Numerical Discussion of Prof. Sommerfeld's Attenuation Formula for Radio Waves*, Stockholm, Ingeniors Vetenskaps Akademien, 1929.

24. R. J. King, "EM Wave Propagation Over a Constant Impedance Plane," *Radio Science*, **4**, 1969, pp. 225-268.

25. J. R. Wait, "Propagation of Radio Waves Over a Stratified Ground," *Geophysics*, **18**, 1953, pp. 416-422.

26. R. E. Collin, *Field Theory of Guided Waves*, New York, McGraw Hill, 1960.

27. J. R. Wait, "Guiding of EM Waves by Uniformly Rough Surfaces," *IEEE Transactions on Antennas and Propagation*, **AP-7**, 1959, pp. S154-S168 (supplement).

28. J. R. Wait, "Note on the Theory of Radio Propagation Over an Ice Covered Sea," Radio Phys. Lab. Report 18-07, Department of National Defence, Ottawa, 1954.

29. J. R. Wait, "Electromagnetic Surface Waves," in *Advances in Radio Research, Volume 1*, New York, Academic Press, 1964, pp. 157-217.

30. L. E. Vogler, "A Note on the Attenuation Function for Propagation Over a Flat Layered Ground," *IEEE Transactions on Antennas and Propagation*, **AP-12**, 1964, pp. 240-242.

31. J. R. Wait, *Electromagnetic Wave Theory*, 1985 (notably Chapter 7).

32. T. S. M. Maclean and Z. Wu, Radiowave Propagation Over Ground, London, Chapman & Hall, 1993.

33. R. W. P. King, M. Owens, and T. T. Wu, *Lateral Em Waves: Theory and Applications to Communications, Geophysical Exploration, and Remote Sensing*, New York, Springer-Verlag, 1992.

34. R. W. P. King and S. S. Sandler, "The EM Field of a Vertical Electric Dipole in the Presence of a Three Layered Region," *Radio Science*, **29**, 1994, pp. 97-113; J. R. Wait, "Comment," and reply by authors, *Radio Science*, **33**, 1998, pp. 251-253.

35. H. von Hoerschelmann, "On the Mode of Operation of the Marconi Bent Antenna in Wireless Telegraphy," *Jahrb. d. Drahtl. Tel.*, **5**, 1911, pp. 14-34.

36. R. J. King and J. R. Wait, "Electromagnetic Groundwave Theory and Experiment," in *Symposia Mathematica, Volume XVIII*, New York, Academic Press, 1976.

37. D. A. Hill and J. R. Wait, "Excitation of the 'Zenneck Surface Wave' by a Vertical Aperture," *Radio Science*, **13**, 1978, pp. 969-977.

38. J. R. Wait, "Low Frequency Behaviour of the Beverage Aerial," *Electronics Letters*, **13**, 1977, pp. 272-273.

39. G. K. Karawas, *Theoretical and Numerical Investigation of Dipole Radiation Over a Flat Earth*, PhD thesis (Advisor: R. E. Collin), Case Western Reserve University, 1985.

40. B. van der Pol, "Theory of the Reflection of Light from a Point Source by a Finitely Conducting Flat Mirror: With Application to Radiotelegraphy," *Physics*, **2**, 1935, pp. 843-853.

41. J. R. Wait, "Fields of a Line Current Source Over a Stratified Conductor," *Appl. Sci. Research* (The Hague), **B3**, 1953, pp. 279-292.

42. G. N. Watson, "The Diffraction of Radio Waves by the Earth," *Proceedings of the Royal Society*, **A95**, 1918, pp. 83-99.

43. L. B. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves*, Englewood Cliffs, New Jersey, Prentice Hall, 1973.

44. D. G. Dudley, *Mathematical Foundations for Electromagnetic Theory*, Piscataway, New Jersey, IEEE Press, 1994.

45. J. R. Wait, "Radiation (and Propagation) from a Vertical Antenna Over a Spherical Earth," *Journal of Research of the National Bureau of Standards*, **56**, 1956, pp. 237-244.

46. B. Wwedensky, "The Diffractive Propagation of Radio Waves, Pt. II: Elevated Transmitter and Receiver," *Tech. Phys. USSR*, **3**, 1936, pp. 915-925.

47. B. van der Pol and H. Bremmer, "The Diffraction of EM Waves from an Electrical Point Source Round a Finitely Conducting Sphere with Applications to Radiotelegraphy and the Theory of the Rainbow," *Phil. Mag.*, 1937, pp. 141-176 (Pt. I); 1937, pp. 825-864 (Pt. II).

48. B. van der Pol and H. Bremmer, "The Propagation of Radio Waves over a Finitely Conducting Spherical Earth," *Phil. Mag.*, **25**, 1938, pp. 817-834; Further note on above, **27**, 1939, pp. 261-275.

49. T. L. Eckersley and G. Millington, "Application of the Phase Integral Method to the Analysis of the Diffraction and Refraction of Wireless Waves Around the Earth," *Phil. Trans. Roy. Soc.*, **237**, 1938, pp. 273-309.

50. V. A. Fock, "Diffraction of Radio Waves Around the Earth's Surface," *Jour. Phys. (Moscow)*, 9, 1945, pp. 256-266.

51. E. Feinberg, *Radio Propagation Along the Earth's Surface*, Academy of Sciences of USSR, 1961, Chapter 7.

52. L. V. Lorenz, "Lysbevoegelsen i. og uden for en plane Lysbølger belyst kugle," *Vid. Selsk. Skr.*, **6**, 1890, pp. 403-420.

53. H. Bremmer, *Terrestrial Radio Waves*, Amsterdam, Elsevier, 1949.

54. V. A. Fock, *Electromagnetic Diffraction Problems*, Oxford, Pergamon Press, 1966 [Volume 1 in The Series on EM Waves, edited by A. L. Cullen and J. R. Wait].

55. S. Rotherham, "Ground Wave Propagation," *Marconi Review*, **XLV**, 1982, pp. 18-48; also, a longer version is in *Proceedings of the IEE (UK), Pt. F*, **128**, 1981, pp. 275-295.

56. K. A. Norton, The Calculation of Ground-Wave Field Intensity Over a Finitely Conducting Spherical Earth, *Proceedings of the IRE*, **29**, 1941, pp. 623-639.

57. C. R. Burrows and M. C. Gray, Thc Effcct of the Earth's Curvature on Ground Wave Propagation, *Proceedings of the IRE*, **29**, 1941.

58. N. A. Logan, "General Research in Diffraction Theory, Volume 1," Lockheed Aircraft Corporation, Sunnyvale, California, (NTIS #241228), 1959.

59. N. A. Logan and K. S. Yee, "A Mathematical Model for Diffraction by Convex Surfaces," in R. E. Langer (ed.), *Electromagnetic Waves*, University of Wisconsin Press, 1962, pp. 139-180.

60. H. Bremmer, "Applications of Operational Calculus to Groundwave Propagation, Particularly for Long Waves," *IRE Transactions on Antennas and Propagation*, **AP-6**, 1958, pp. 267-274.

61. J. R. Wait, "The Propagation of EM Waves Along the Earth's Surface, in R. E. Langer (ed.), *Electromagnetic Waves*, University of Wisconsin Press, 1962, pp. 243-290.

62. G. Millington, "Groundwave Propagation Over an Inhomogeneous Smooth Earth," *Proceedings of the IEE (UK), Pt. III*, 1949, pp. 53-64.

63. H. Brcmmcr, "The Extension of Sommerfeld's Formula for the Propagation of Radio Waves Over a Flat Earth to Different Conductivities of the Soil," *Physica*, **20**, 1954, pp. 441-460.

64. J. R. Wait, "Coupled Mode Analysis for Ground Wave Propagation Over Inhomogeneous Paths," *Electromagnetics*, **1**, 1981, pp. 91-100.

65. G. D. Monteath, *Applications of the EM Reciprocity Principle*, Tarrytown, New York, Press, 1973.

66. J. A. Stratton, *Electromagnetic Theory*, New York, McGraw Hill, 1941.

67. R. J. King, *Analytic and Experimental Studies of Propagation of EM Surface Waves Across Mixed Paths*, PhD Thesis, University of Colorado, 1965.

68. J. R. Wait, "Mixed-Path Ground Wave Propagation on a Spherical Earth," *Journal of Research of the National Bureau of Standards, Part D: Radio Propagation*, **65**, 1961, pp. 401-410.

69. P. C. Clemmow, "Radio Propagation Over a Flat Earth Across a Boundary Separating Two Different Media," *Phil. Trans. Royal Soc., Series A*, **246**, 1953, pp. 1-55.

70. S. Christiansen and T. Larsen, "Numcrical Application of the Compensation Theorem to Mixed-Path Propagation Problems," *Radio Science*, **2**, 1967, pp. 1471-1480.

71. K. Furutsu, "Propagation of EM Waves Over a Spherical Earth Across Boundaries Separating Earth Media," *Jour. Radio Res. Labs. (Japan)*, **2**, 1955, pp. 345-388.

72. Z. Godzinski, "The use of Equivalent Secondary Sources in the Theory of Ground-Wavc Propagation Over an Inhomogeneous Earth, *Proceedings of the IEE, Part C (UK)*, **105**, 1958, pp. 448-464.

73. Yu K. Kalinin and E. Feinberg, "Groundwave Propagation Over the Surface of an Inhomogeneous Spherical Earth," *Radiotechnika i Electronika*, **3**, 1985, pp. 1122-1132.

74. G. Grunberg, "Theory of Coastal Refraction of EM Waves," *Jour. Physics (Moscow)*, **6**, 1942, pp. 185-209.

75. J. R. Wait and J. Householder, "Mixed-Path Ground Wave Propagation, Pt. 2, Larger Distances (for Spherical Earth)," *Journal of Research of the National Bureau of Standards*, **59**, 1957, pp. 19-26.

76. R. J. King, S. W. Maley, and J. R. Wait, "Experimental and Theoretical Studies of Propagation Across Mixed Paths," in J. Brown (ed.), *Electromagnetic Wave Theory (URSI Symposium)*, New York, Pergamon Press, 1967, pp. 217-224.

77. G. A. Hufford, "An Integral Equation Approach to the Problem of Wave Propagation Over an Irregular Surface," *Quarterly Journal of Applied Mathematics*, **9**, 1952, pp. 391-404.

78. J. R. Wait and K. P. Spies, "Integral Equation Approach to the Radiation (and Propagation) from a Vertical Antenna Over an Inhomogeneous Ground Plane," *Radio Science*, **5**, 1970, pp. 73-79.

79. R. H. Ott, "An Alternative Equation for Propagation Over Irregular Terrain," *Radio Science*, **6**, 1971.

80. O. V. Soloviev and V. V. Agapov, "An Asymptotic 3D Technique to Study Radio Wave Propagation in the Presence of a Localized Perturbation of Environment," *Radio Science*, **32**, 1997, pp. 515-524.

81. J. R. Wait and K. P. Spies, "Propagation of Radio Waves Past a Coastline with a Gradual Change of Surface Impedance," *IEEE Transactions [on Antennas and Propagation?]*, **12**, 1964, pp. 570-575.

82. K. Furutsu, "Propagation of EM Waves Over the Spherical Earth Across Boundaries Separating Different Earth Media," *Jour. Radio Res. Lab. (Tokyo)*, **2**, 1955, pp. 345-398.

83. D. A. Hill and J. R. Wait, "HF Radio Transmission Over Sea Ice and Remote Sensing Possibilities," *IEEE Transactions on Geoscience and Remote Sensing*, **GRS-19**, 1981, pp. 204-209.

84. J. R. Wait, "Oblique Propagation of Ground Waves Across a Coastline," *Journal of Research of the National Bureau of Standards, Part D: Radio Propagation*, 67, 1963, pp. 617-630; **68**, 1964, pp. 291-296.

85. D. A. Hill and J. R. Wait, "HF Ground Wave Propagation Over Mixed Land, Sea, and Sea-Ice Paths," *IEEE Transactions on Geoscience and Remote Sensing*, **GRS-19**, 1981, pp. 210-216.

86. S. H. Cho and J. R. Wait, "Transmission of Creeping-Wave Modes Around a Periodically Modulated Impedance Cylinder," *Canadian Journal of Physics*, **54**, 1976, pp. 1839-1849.

87. A. Buyukaksoy and G. Uzgoren, "High Frequency Scattering from the Impedance Discontinuity on a Cylindrical Curved Surface," *IEEE Transactions on Antennas and Propagation*, **AP-35**, 1987, pp. 234-236.

88. J. R. Wait, "Theory of Ground Wave Propagation," in J. R. Wait (ed.), *Electromagnetic Probing in Geophysics*, Boulder, Golem Press, 1971, pp. 163-207; revised version of article in *Advances in Radio Research, Volume 1*, New York, Academic Press, 1964, pp. 157-217.

89. J. R. Wait, "Recent Analytical Investigations of EM Ground Wave Propagation Over Inhomogeneous Earth Models," *IEEE Proceedings*, **62**, 1974, pp. 1061-1072.

90. J. R. Wait, "Propagation of Electromagnetic Waves Over a Smooth Multi-Section Curved Earth–An Exact Theory," *Jour. Math. Phys.*, **11**, 1970, pp. 2851-2860.

91. R. J. King, S. H. Cho, D. L. Jaggard, and V. Sokolov, "Height-Gain Experimental (Model) Data for Groundwave Propagation," *Radio Science*, **8**, 1973, pp. 7-22.

92. J. R. Wait and L. C. Walters, "Curves for Ground-Wave Propagation Over Mixed Land and Sea Paths," *IEEE Transactions on Antennas and Propagation*, **AP-11**, 1963, pp. 38-45.

93. J. C. Schelleng, C. R. Burrows, and E. B. Ferrell, "Ultra Shortwave Propagation," *Proceedings of the IRE*, **21**, 1933, pp. 427-463.

94. J. R. Wait, "Diffraction and Scattering of EM Groundwave by Terrain Features," *Radio Science*, **3**, 1968, pp. 995-1003.

95. Z. Wu, T. S. M. Maclean, D. Bagwell, and M. J. Mehler, "Propagation Over an Inhomogeneous Irregular Surface," *Radio Science*, **23**, 1988, pp. 33-40.

96. J. R. Wait and A. Murphy, "Further Studies of the Influence of a Ridge on the Low Frequency Ground Wave," *Journal of Research of the National Bureau of Standards*, **61**, 1958, pp. 57-60.

97. D. A. Hill and J. R. Wait, "Ground Wave Propagation Over a Mixed Path with Elevation Change," *IEEE Transactions on Antennas and Propagation*, **AP-30**, 1982, pp. 139-141.

98. M. C. Gray, "Horizontally Polarized EM Waves Over a Spherical Earth, *Phil. Mag.*, 1939, pp. 421-436.

99. J. R. Wait and H. H. Howe, "Amplitude and Phase Curves for Ground Wave Propagation in the Band 200 c/s(Hz) to 500 kc/s (kHz)," US National Bureau of Standards Circular 574, 1956 [also NTIS No. PR272292/AS].

100. J. R. Wait, *(Lectures on) Wave Propagation Theory*, New York, Pergamon, 1981.

101. J. R. Wait, "Ground-Wave Attenuation Function for a Gaussian Modulated Carrier Signal, *Electronics Letters*, **6**, 1970, pp. 106-107.

102. J. R. Wait, "The Transient Behavior of the EM Ground Wave on a Spherical Earth," *IRE Transactions on Antennas and Propagation*, **AP-5**, pp. 198-202.

103. J. R. Wait, "A Note on the Propagation of the Transient Ground Wave," *Canadian Journal of Physics*, **35**, 1957, pp. 1146-1151.

104. J. R. Wait and M. Assis, "Ground Wave Pulse Propagation," *Revista Brasileria*, **2**, 1971, pp. 193-199.

105. A. N. Sommerfeld, *Partial Differential Equations in Physics-Lectures on Theoretical Physics, Vol. VI*, New York, Academic Press, 1964, Chapter 6.

106. H. G. Booker and P. C. Clemmow, "A Relation Between the Sommerfeld Theory of Radio Propagation Over a Flat Earth and the Theory of Diffraction at a Straight Edge," *Proceedings of the IEE (UK)*, **97, Pt. III**, 1950, pp. 18-27.

107. J. R. Wait, "Low Frequency Radiation from a Horizontal Earth," *Canadian Journal of Physics*, **34**, 1956, pp. 586-595.

108. J. R. Wait, "On the Excitation of EM Surface Waves on a Curved Surface, *IRE Transactions on Antennas and Propagation*, **AP-8**, 1960, pp. 445-449.

109. T. B. A. Senior, "Radio Propagation Over a Discontinuity in the Earth's Electrical Properties, *Proceedings of the IEE (UK), Pt. C*, **104**, 1947, pp. 43-53 and 139-147.

110. J. R. Wait and K. P. Spies, "On the Calculation of the Groundwave Attenuation Factor at Low Frequencies, *IEEE Transactions on Antennas and Propagation*, **AP-14**, 1966, pp. 515-517.

111. T. B. A. Senior and J. L. Volakis, *Approximate Boundary Conditions in Electromagnetics*, London, UK, The Institution of Electrical Engineers, 1995.

112. D. A. Hill and J. R. Wait, "Ground Wave Attenuation Function for a Spherical Earth with Arbitrary Surface Impedance," *Radio Science*, **15**, 1980, pp. 637-643.

113. W. J. Hughes and J. R. Wait, "Effective Wave Tilt and Surface Impedance Over a Laterally Inhomogeneous Two-Layer Earth, *Radio Science*, **10**, 1975, pp. 1001-1008.

114. J. R. Wait and K. P. Spies, "Range Dependence of the Surface Impedance and Wave Tilt for a Line-Source Excited Two-Layer Earth," *IEEE Transactions on Antennas and Propagation*, **AP-21**, 1973.

115. J. R. Wait, "Illumination of an Inhomogeneous Spherical Earth by an LF Plane Electromagnetic Wave," *Radio Science*, **2**, 1967, pp. 111-118.

116. J. R. Wait, "Influence of a Thin Inhomogeneous Surface Layer on Electromagnetic Ground Wave Propagation," *IEEE Proceedings*, **55**, 1967, pp. 568-569.

117. J. R. Wait, "Influence of a Sub-Surface Insulating Layer on Electromagnetic Ground Wave Propagation," *IEEE Transactions on Antennas and Propagation*, **AP-14**, 1966, pp. 755-759.

118. K. P. Spies and J. R. Wait, "On the Calculation of the Ground Wave Attenuation Factor at Low Frequencies," *IEEE Transactions on Antennas and Propagation*, **AP-14**, 1966, pp. 515-517.

119. J. R. Wait, "A Note on Surface Waves and Ground Waves," *IEEE Transactions on Antennas and Propagation*, **AP-13**, 1965, pp. 996-997.

120. P. R. Bannister, "New Formulas that Extend Norton's FarField Elementary Dipole Equations to the Quasi-Nearfield Range," Tech. Report 6883, US Naval Underwater Systems Center, New London, CT, 1984.

121. D. A. Hill, "HF Ground Wave Propagation Over Forested and Built-Up Terrain," NTIA (US Department of Commerce) Report 82-114, 1982.

122. J. R. Wait and D. A. Hill, "Excitation of the HF Surface Wave by Vertical and Horizontal Apertures," *Radio Science*, **14**, 1979, pp. 767-780.

123. IEEE Std. 211-1990 (R1997), *IEEE Standard Definitions of Terms for Radio Wave Propagation.*

Editor's Comments *Continued from page 6*

languages, using something close to the best of the currently available compilers or interpreters? You should read the contribution by J. E. Moreira, S. P. Midkiff, and M. Gupta in John Volakis' EM Programmer's Notebook. I think you'll find the information there interesting.

I have been meaning to get a current photograph above this column for more years than I'm going to admit. After reading Randy Haupt's Ethically Speaking column, I really am going to have to do it. It's a matter of time, you see. To get a photo in which you don't disappear into the background requires something more than going to the nearest passport-photo place. I'm also making excuses. Randy's column has some interesting perspectives on the importance of "appearances." It also contains some feedback relating to the issues of duplicate submissions, papers "trapped" in the review process, and some of the results these can have.

More sad news. This has been a very bad year for the AP-S/URSI family, in terms of members lost. Two "in memoria" for Jim Wait appear in this issue. Jim was a good friend. He was also an important part of AP-S and this *Magazine*, often suggesting articles, offering advice, and providing encouragement. He was a very important part of URSI and URSI-related publications, too, having been instrumental in the founding of *Radio Science*, and a major solicitor of articles and an Associate Editor for *the Radio-Scientist* and its successor publications.

Georges Deschamps died June 20; an "in memoriam" appears in this issue. In addition to having a brilliant grasp of electromagnetic theory, he was a great educator. He was also part of the University of Illinois team that made the Antenna Laboratory so great, having been its Director for many years. I first came across his name on a fundamental paper that gave insight into how log-periodic antennas worked. He also made many contributions to USNC/URSI.

As this was going to press, word came that Korada Umashankar of the University of Illinois at Chicago has died. We will try to have a contribution remembering him in the next issue.

An IEEE Web course. The IEEE is trying an experiment in Internet-based education. "An Introduction to Antennas" will be taught through the month of April, 1999, by Eric Michielssen. The stated purpose of the course is to provide an analytical and intuitive understanding of antenna physics, some exposure to computer-aided-design software for antennas, and an introduction to a variety of practical antenna structures. A full-page announcement for the course appears in this issue. Continuing Education Units can be earned. If you or your colleagues have any interest in this, I urge you to participate in the experiment. This should open a whole new realm of educational opportunities for the IEEE.

Some further comments regarding e-mail. In the last issue (pp. 44, 49), I commented on the variety of probably unintended formats and attachments some people seemed to be using when they sent e-mail. I suggested that you send yourself or a friend an e-mail message, and look at the result. I should probably have also pointed out that one reason some of this happens is that many people are now using Web browsers to send their e-mail, and they have not looked at some of the default settings. Of course, those defaults could be set by a variety of sources, including network administrators and Internet service providers who have supplied the browser software. Here are some of the things you should take a look at, if you use *Netscape* to send e-mail (some comments regarding *Internet Explorer* follow).

In *Netscape Navigator* (or *Communicator*) version 4.0 or higher, click on Edit...Preferences.... In the Category box, click on the "+" next to Mail and Newsgroups. Click on Formatting. You will then see your two choices for the editor used to create e-mail messages (text or HTML, and this also determines the output formatting of the message), and four choices for the format in which the message is to be sent. In particular, note that the HTML editor is often the default (at least, that was the default installed by the version of *Communicator* as downloaded from Netscape's Web site, at the time this was written). Also note that if the bottom choice for the format to be used in sending messages is chosen, you will automatically send *two* copies of each e-mail message: one in plain text, and the other in HTML. I find receiving two copies of e-mail like this very annoying. Since I don't use a browser to receive e-mail (and my e-mail software isn't HTML-enabled), receiving e-mail in HTML format is also quite annoying: you end up having to "manually decode" the message out of the HTML formatting.

After you are satisfied that the choices in the window discussed above reflect what you really want to do, click on Identity, under Mail and Newsgroups, in the Category box. See if the box next to "Attach my personal card to messages (as a Vcard)" is checked. If it is, then every e-mail message you send will have at least two parts: the body of the message, and a "binary attachment." This attachment is typically a few hundred bytes, and contains whatever information is present in the window displayed when you click on the "Edit Card" button. You may not want to send this information with every message. Even if you do, you may prefer to include it (e.g., by "pasting" it into the editing window) in the text portion of the message, rather than as a binary attachment. As I've discussed in previous "Screen of Stone" columns, such attachments can cause problems with some recipients' e-mail software, and because they are "binary," they should always be virus-checked before being "opened" by the recipient or his or her e-mail software. Note that in this window you can also elect to have a "signature" from a signature file included with your e-mail message.

You need to click on the "OK" button after making any changes to the above settings to have them take effect, of course.

Continued on page 39