Preface to the Special Issue on Time-Frequency Analysis

While many of us know about the Bunson burner and Kirchhoff's circuit laws, Bunson and Kirchhoff (jointly in 1854) are the originators of one of the major discoveries in science. They came up with the idea that the spectrum of light characterizes the substance emitting the light and many of its physical properties. That idea was revolutionary and the key to our understanding of matter and the universe. While we take it for granted now, think for a moment that we have discovered the nature of stars millions of light years away and the nature of atoms and molecules by merely studying the light they emit!

Although the explanation for why and how atoms and molecules produce spectra was not understood for about 80 years after their initial discovery, their idea was immediately successful and was the origin of what we now call spectral analysis. The foundation, methodology, and instrumentation of spectral analysis were developed in those fields where its use became most important, namely astronomy, physics, chemistry, and engineering. The mathematical expression of the spectrum is, of course, Fourier analysis, developed some 50 years before Bunsen and Kirchhoff's idea.

Spectral analysis was extended to many situations besides light, most importantly to acoustics. Historically, most of the applications were for stationary situations, where the spectral characteristics remain unchanged over time. However, what we now call time-varying spectral analysis or time-frequency analysis arose out of necessity. There are signals in nature where the spectral content is changing in time, as exemplified by the areas addressed in this issue. Perhaps the prime example is human speech, where the frequencies are changing as we speak, and indeed it is the changing frequencies and their temporal order which carries the message.

These types of signals are sometimes called "nonstationary," the phrase being used in a loose way to indicate that the basic spectral properties are changing in time. The goal of time-frequency analysis is to describe the time-varying spectral content of these signals and how their time-varying spectral properties are related to the physical source and medium of propagation. The mathematical and physical ideas necessary to achieve this goal have been developing for several decades, and continue very actively today.

The historical antecedents to the development of time-varying spectral analysis are many. However, a strong impetus came with the invention of the radio, since amplitude modulation and frequency modulation are inherently nonstationary. This necessitated an understanding of such situations and led to the development of many new ideas regarding the fundamental nature of signals, such as the concept of instantaneous frequency.

The basic difference between standard spectral analysis or Fourier analysis and time-varying spectral analysis can be simply stated. Fourier analyzing a signal, stationary or not, tells us what frequencies existed for its duration. However, it does not tell us when the frequencies occurred, that is, it does not tell us if or how the spectrum is evolving in time.

One approach to time-frequency analysis is to try to generalize standard Fourier analysis. The basic idea is to extend the concept of the energy density spectrum $|S(\omega)|^2$ (where $S(\omega)$ is the Fourier transform), which indicates the intensity per unit frequency, to that of a two-dimensional function $P(t, \omega)$ which indicates the intensity per unit frequency and per unit time. Such a function would then describe how the spectrum is evolving in time and is called a time-frequency distribution, density, or representation. There have been many methods proposed over the last 50 years that in some sense work very well and meet our intuitive notion of what a time-varying spectrum should be.

One of the original approaches was developed in the early 1940's for the specific purpose of understanding the nature of human speech. The approach was simple and powerful: break up a signal into short time intervals and Fourier analyze each interval (a procedure now called short-time Fourier analysis). Then, for each interval, take the magnitude squared to obtain an energy density spectrum which indicates the frequencies that existed in that time interval. The collection of these spectra constitute a time-frequency density that is commonly called the spectrogram.

In 1946, the milestone paper of Gabor crystallized the necessity of describing a signal jointly in time and frequency. He introduced a number of new ideas, such as the analytic signal, the uncertainty principle for signals, and also devised what is essentially the sampled spectrogram with a Gaussian window. Ville (1948) came up with another approach that

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we now call the Wigner–Ville distribution. (The Wigner distribution was developed considerably earlier (1932) as a bridge between classical and quantum mechanics.) In subsequent years, many distributions and approaches were developed although the spectrogram remained the main tool in applications. The reason was two-fold: It is conceptually easy to understand, and it is straightforward to calculate. However, it has a number of shortcomings and this gave continuous impetus to study other methods and distributions.

There has been great progress made in the past 20 years or so. The papers of Claasen and Mecklenbrauker (1980) were particularly important in that they showed how the Wigner distribution could be effectively used and also addressed many important issues relevant to signal analysis. Subsequently, many new ideas and methods have emerged that shed light on the fundamental nature of a time-varying spectrum and its production. Among them are signal-dependent methods, manifestly positive distributions, kernel design, extensions to variables other than time and frequency, operator methods, group theory, and the nature of multicomponent signals. Of particular importance is the work of Williams and his coworkers, who devised new approaches for obtaining time-frequency densities with desirable properties. While we do not yet have a complete and definitive theory of what a time-varying spectrum is, this special issue attests to the fact that the ideas and methods that have been developed have been applied with immense success to many fields.

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